1. (10%) Let $\vec{F}(x,y,z)$ be a vector field and $\hat{a}$ be an arbitrary unit vector. It can be shown that

$$\hat{a}\cdot\nabla\times\vec{F}=\lim_{\Delta s\to 0}\frac{1}{\Delta S}\oint_{C}\vec{F}\cdot d\vec{\ell}$$

where the loop C enclosing the area ΔS is perpendicular to $\hat{a}$. From this explain the physical meaning of $\nabla\times\vec{F}$ in terms of its magnitude and direction.

2. (15%) Let $H(\omega)$ be the Fourier transform of $h(t)$, defined as

$H(\omega)=\int_{-\infty}^{\infty}h(t)e^{-j\omega t}\,dt$. Define the convolution of $f(t)$ and $h(t)$ as

$$g(t)=f(t)\otimes h(t)\equiv\int_{-\infty}^{\infty}f(\tau)h(t-\tau)\,d\tau.$$

(a) If $f(t)=e^{+j\omega_1 t}+e^{-j\omega_2 t}$, express $g(t)$ in terms of ω_1, ω_2 and $H(\cdot)$.

(b) The correlation function between $f(t)$ and $h(t)$ is defined as

$R_{fh}(\tau)\equiv\int_{-\infty}^{\infty}f^{*}(t+\tau)h(t)\,dt$. Express $R_{hh}(\tau)$ in terms of convolution.

(c) Let $S(\omega)$ be the Fourier transform of $R_{hh}(\tau)$. Express $S(\omega)$ in terms of $H(\omega)$.

3. (15%) Let $\vec{R}=\hat{x}x+\hat{y}y+\hat{z}z$, where $\hat{x}$ is the unit vector in the x direction.

(a) Find an expression of $\nabla\frac{1}{|\vec{R}|}$ in terms of $\vec{R}$.

(b) Compute the line integral $\int_{(x,y,z)}^{(2x,2y,2z)}\frac{\vec{R}}{|\vec{R}|^3}\cdot d\vec{\ell}$

(c) Show that the surface area of a sphere of radius b is $4\pi b^2$.

4. Use the Laplace transform to solve the following initial- value problem :

(20%) $y'' - 4y' + 4y = \delta(t-1)$, $y(0)=0$, $y'(0)=1$

5. Find the general solution of the following ordinary differential equation :

(15%) $y' + 8x^3y^3 + 2xy = 0$

6. (15%) Consider two vector spaces V and W both over the field F. Let $\{\alpha_1,\cdots,\alpha_n\}$ be an ordered basis for V and let $\beta_1,\cdots,\beta_n$ be *any* vectors in W. Show that there exists a unique linear operator $\mathcal{A}:V\to W$ such that $\mathcal{A}\alpha_\ell=\beta_\ell$ for all $\ell=1,\ldots,n$.
(Notice that $\mathcal{A}$ is unknown yet. So, in the proof, you have to first derive $\mathcal{A}$, then show that it is linear and unique.)

7. (10%) Evaluate

$$\int_{C}\exp[(-1+i)\log z]dz$$

where $C=\{z:|z|=1\}$ and $0<\arg z<2\pi$.

1. Consider the phase-shift oscillator shown in Figure 1. (a)Derive the expression for the loop gain $T(s)$. (b)Derive the expression for the frequency of oscillation. (c)What is the condition for sustained oscillations? (5%*3)

2. (a)Determine the logic functions implemented by the circuits shown in Figure 2.a and 2.b (b)Given inputs A and B, design a CMOS circuit to implement the logic function $Y = AB + \overline{A}\overline{B}$. (3%*3)

3. (a)What the name of the circuit shown in Figure 3.a? (b)Determine I_O and V_O assuming that $Q1$, $Q2$, and $Q3$ are identical transistors and have the characteristics shown in Figure 3.b. (2%, 4%*2)

4. (a)Obtain the voltage transfer characteristic of the circuit given in Figure 4, assuming that $D1$ and $D2$ are identical and have parameters $V_\gamma \neq 0$ and $R_f = 0$. (b)Using (a), verify that the circuit is a two-level clipper. (5%*2)

5. (a)Draw the low-frequency small signal model of a NPN BJT and a NMOS FET respectively. (b)What elements must be added respectively to the models derived in (a) so that the equivalent circuits are valid at high frequency? (c) What is the physical origin of these elements added? (6%*3)

6. Consider the OP-Amp shown in Figure 6. (a) Identify and name the input stage, gain stage, output stage, and biasing circuit. (b) Identify the function of each transistor. (c)What the function of C_c? (8%*2,4%)

7. The Op-Amp in the circuit shown in Figure 7 has an open-loop gain A_V and output resistance R_O. (a) Determine A_{OL}, β, T, and A_F for the stage. (b)Determine R_{OF} of the circuit. (c)What the topology is employed? (3%*6)

Figure 1 :

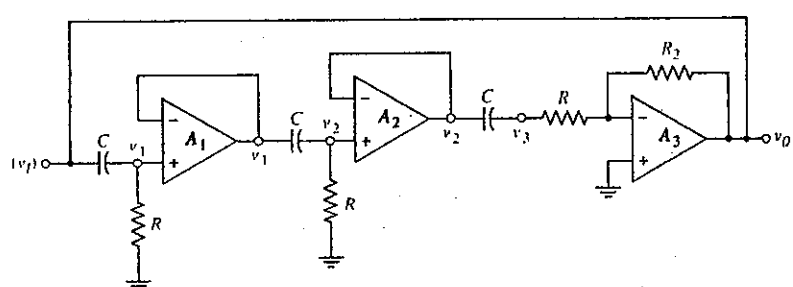
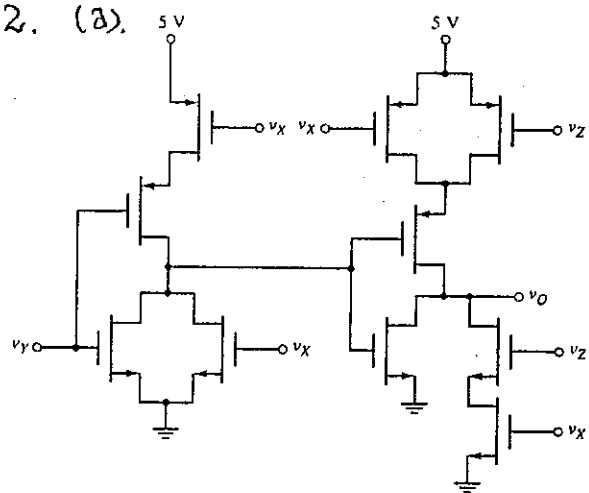


Figure 2. (a).



(b).

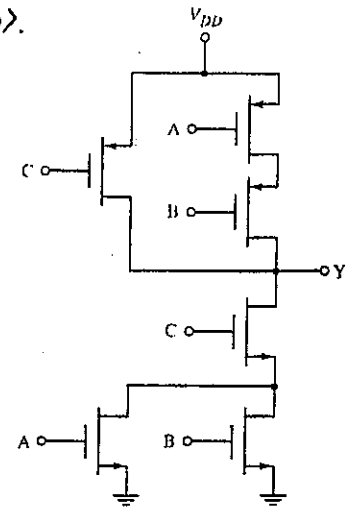
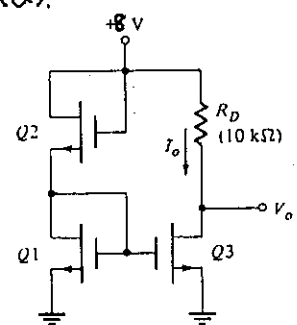


Figure 3. (a).



(b).

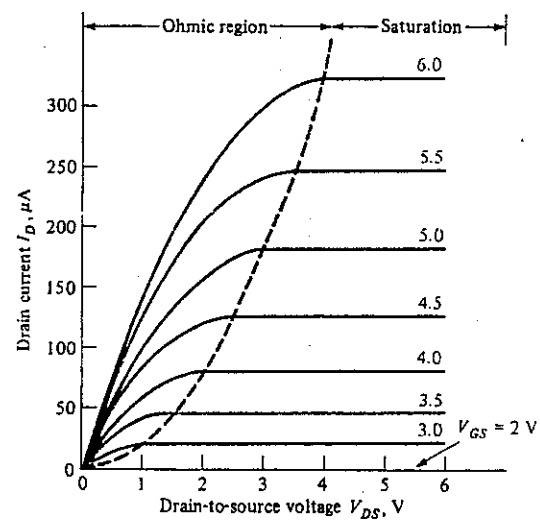


Figure 4.

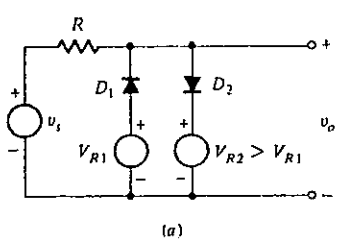


Figure 6.

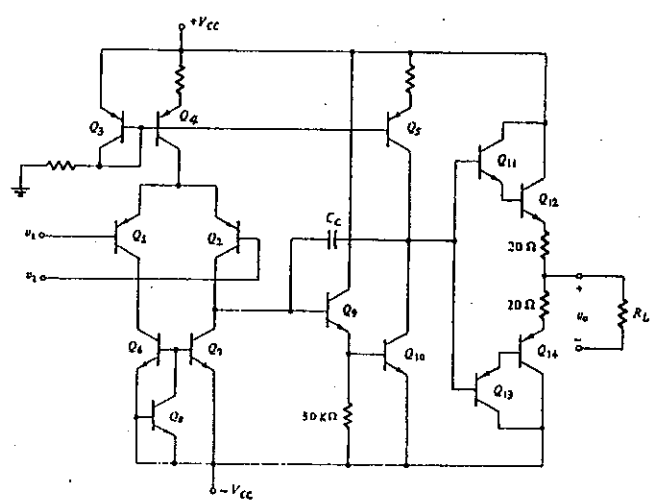
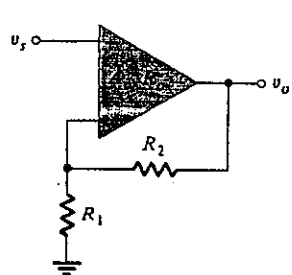


Figure 7.



1. Choose and explain the corrective answer.(15 %)
   (1) In the energy band diagrams for a Carbon (10^{14} cm^{-3}) doped silicon the Fermi energy is located at, a) closer to E_c above E_i, b) closer to E_V below E_i, c) in the middle between E_c and E_V d) none of the above

   (2) In addition to the original doped Ga impurities ,if we doped with phosphorus again, the Fermi level will, a) unchange b) move toward band edge, c) move toward middle gap d) none of the above

   (3) With increasing temperature, neglects the change in band gap, the fermi level will a) unchange, b) move toward band edge c) move to band edge and then toward middle gap d) none of the above

   (4) In a nonuniform doped semiconductor , assume the doping profile is linear increasing, i.e. $dn/dx > 0$, under thermal equilibrium, a) $dE_c/dx > 0$, b)$dE_c/dx < 0$, c) $dE_f/dx > 0$, d) $dE_f/dx < 0$, e) none of the above

   (5) Which is the site of intrinsic fermi level in the band gap, a) $E_f > E_i$; b) $E_f = E_i$; c) $E_f < E_i$; d) none of the above

2. How to design a diode with a ideal I-V characteristics ? (15%)

3. How to obtain a high gain BJT? (15 %)

4. (a) Drive the I-V curve of MOS FET (10%)
   (b) Describe the effect of following factors (for MOS FET)
   (1) Subthreshold current (4%)
   (2) Velocity limitation of the carriers in channel (4%)
   (3) Hot carrier effect (4%)
   (4) Short channel effect (3%)

5. Describe the comparison of properties of the p-n diode and the Schottky diode. (10%)

6. Consider a GaAs p-n diode is used as an LED with the following parameters at 300 K:
   D_n=30 $cm^2/V\text{-}s$; D_p=15 $cm^2/V\text{-}s$; Na = 5 x 10^{16} cm^{-3} ; Nd = 5x 10^{17} cm^{-3};
   $\tau_n = 10^{-8}$ s ; $\tau_p = 5 \times 10^{-7}$ s
   (a) Calculate the injection efficiency of LED assuming no recombination due to traps. (10%)
   (b)Assume that all the electrons injected into the top p region combine with holes to generate photons, Calculate the photon generation rate (5%)
   (c) Optical power coming from the LED (5%)

   If the device has the parameters as above and is forward biased at 1.0 V, the diode area is 0.1 mm^2 . Each photon has an energy of 1.43 eV.

國立中山大學八十七學年度碩博士班招生考試試題

科 目：工程數學乙（電機所乙組）

1. (10%) Let $\vec{F}(x,y,z)$ be a vector field and $\hat{a}$ be an arbitrary unit vector. It can be shown that

$$\hat{a}\cdot\nabla\times\vec{F}=\lim_{\Delta S\to 0}\frac{1}{\Delta S}\oint_C\vec{F}\cdot d\vec{\ell}$$

where the loop C enclosing the area ΔS is perpendicular to $\hat{a}$. From this explain the physical meaning of $\nabla\times\vec{F}$ in terms of its magnitude and direction.

2. (15%) Let $H(\omega)$ be the Fourier transform of $h(t)$, defined as

$H(\omega)=\int_{-\infty}^{\infty}h(t)e^{-j\omega t}\,dt$. Define the convolution of $f(t)$ and $h(t)$ as

$$g(t)=f(t)\otimes h(t)\equiv\int_{-\infty}^{\infty}f(\tau)h(t-\tau)\,d\tau.$$

(a) If $f(t)=e^{+j\omega_1 t}+e^{-j\omega_2 t}$, express $g(t)$ in terms of ω_1, ω_2 and $H(\cdot)$.

(b) The correlation function between $f(t)$ and $h(t)$ is defined as

$R_{fh}(\tau)\equiv\int_{-\infty}^{\infty}f^{*}(t+\tau)h(t)\,dt$. Express $R_{hh}(\tau)$ in terms of convolution.

(c) Let $S(\omega)$ be the Fourier transform of $R_{hh}(\tau)$. Express $S(\omega)$ in terms of $H(\omega)$.

3. Use the Laplace transform to solve the following initial- value problem :

(15%) $y'' - 4y' + 4y = \delta(t-1)$, $y(0)=0$, $y'(0)=1$

4. Find the general solution of the following ordinary differential equation :

(15%) $y' + 8x^3y^3 + 2xy = 0$

5. (15%) Consider two vector spaces V and W both over the field F. Let $\{\alpha_1,\cdots,\alpha_n\}$ be an ordered basis for V and let $\beta_1,\cdots,\beta_n$ be *any* vectors in W. Show that there exists a unique linear operator $\mathcal{A}:V\to W$ such that $\mathcal{A}\alpha_\ell=\beta_\ell$ for all $\ell=1,\ldots,n$.
(Notice that $\mathcal{A}$ is unknown yet. So, in the proof, you have to first derive $\mathcal{A}$, then show that it is linear and unique.)

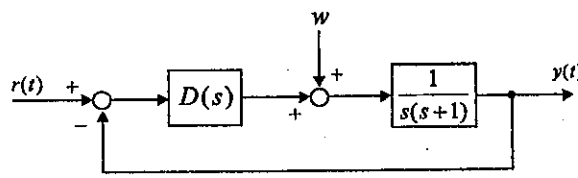
6. (15%) Let V be a vector space over the field F with $\dim V=n$ (i.e. dimension of V is n). Let $V=W\oplus Y$ with $\dim W=\ell$. Let $\mathcal{A}:V\to V$ be a linear operator. Show that if W is invariant under $\mathcal{A}$ (i.e. for each vector $v\in W$, the vector $\mathcal{A}v\in W$) then $\mathcal{A}$ has a block matrix representation $\begin{pmatrix}A_{11}&A_{12}\\0&A_{22}\end{pmatrix}$. Also, what is the dimension of A_{11}?

7. Let C be a simple closed contour and let $f(z)$ be
(i) nonzero on C
(ii) analytic on C
(iii) analytic inside C except at its poles.
Now denote following three quantities as
$\Delta_C\arg f(z)$: the argument variation of $f(z)$ as z moving across C in the positive direction (i.e. c.c.w.)
Z : the total number of zeros of $f(z)$, counting multiplicities, inside C
P : the total number of poles of $f(z)$, counting multiplicities, inside C.
Then
(a) (5%) Write the relationship between the three quantities $\Delta_C\arg f(z)$, Z, and P.
(b) (10%) Prove the relationship of part (a).

1. Consider the following type I system. It is desired to design a controller $D(s)$ to meet the requirements: (1) The steady-state value of y due to a constant unit disturbance w should be less than 0.4, and (2) the damping ratio $\zeta = 0.7$.

   (a)(6%) Use **root-locus technique** to show that proportional control alone is not adequate.

   (b)(6%) Find the range of the gains k and k_d for PD controller $D(s) = k + k_d s$ that meets the design specifications.



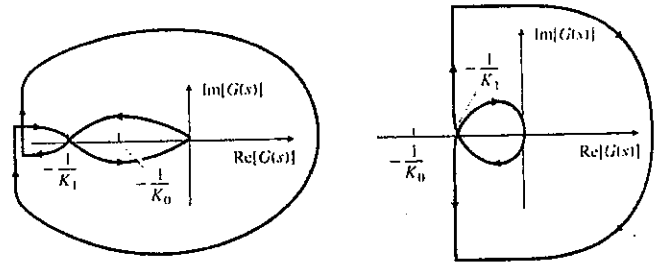
2. The Nyquist plots of two stable, open-loop systems are sketched in the following. The proposed operating gain is indicated as K_o, and arrows indicate increasing frequency. In each case give your best estimates of the following quantities for the closed-loop (unity feedback) system:

   (a)(4%) phase margin

   (b)(6%) damping ratio

   (c)(6%) range of gain for stability (i.e., the relationship between k_o and k_1)

   (d)(6%) system type (0, I, or II)



3. The sensitivity function $S(s)$ of a control system is given by

$$S(s) = \frac{1}{1+G(s)}$$

   where $G(s)$ is open-loop transfer function of a unity feedback system.

   (a)(6%) Prove that the sensitivity function $S(s)$ has magnitude greater than 1 inside a circle with radius of 1 centered at the -1 point.

   (b)(6%) If we want closed-loop control to out perform open-loop control at all frequency, then the Nyquist plot of $G(s)$ should lie inside or outside the circle of radius one centered at the -1 point? Explain.

4. Consider a control system with the loop transfer function

$$L(s) = \frac{K(s-1)}{s(s+1)}, \qquad -\infty \le K \le \infty$$

   (a)(6%) Sketch the Nyquist Plot of this system for $K > 0$.

(b)(7%) Using Nyquist criterion, determine the range of $K(>0)$ such that the closed-loop system is stable. If the system is unstable due to the range of K, find the number of closed-loop poles in the right-half of $s-$plane.

(c)(8%) Using Nyquist criterion and the Nyquist plot of $K>0$, determine the range of $K(<0)$ such that the closed-loop system is stable. If the system is unstable due to the range of K, find the number of closed-loop poles in the right-half of $s-$plane.

5. A unity feedback system, with a minimum phase open-loop transfer function $F(s)$, has Bode plot shown in the following when the loop gain is set at its nominal value.

(a)(4%) Determine the gain margin and phase margin.

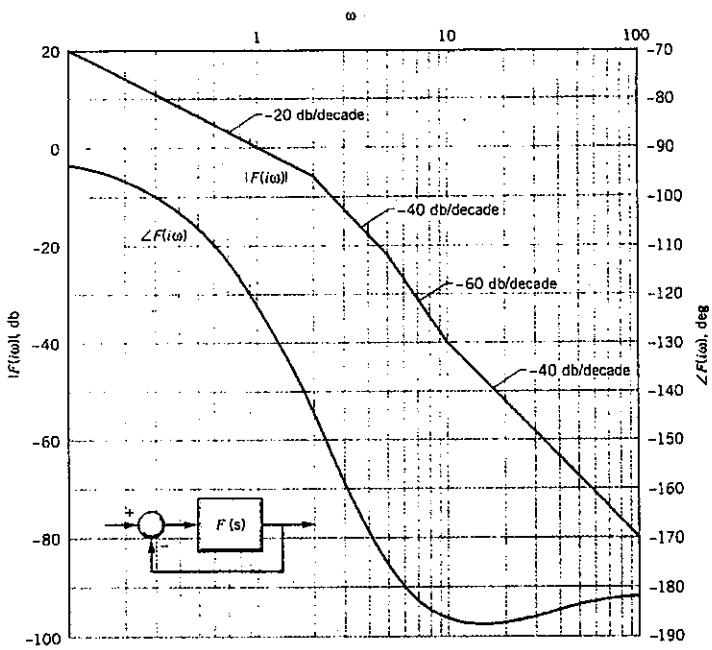
(b)(3%) Is the closed-loop system stable?

(c)(5%) Determine $F(s)$.

(d)(4%) Find out how much the gain (**do not** use dB unit) must be multiplied to the nominal gain so that the closed-loop gain margin is to be 40dB?

(e)(4%) Find out how much the gain (**do not** use dB unit) must be multiplied to the nominal gain so that the closed-loop phase margin is to be 70°?

(f)(3%) Find the steady-state error of the system if the reference input to the system is an unit-ramp input.



6.(10%) Consider a controllable n-dimensional system $(\mathbf{A}, \mathbf{b})$. Let $\mathbf{F} \in R^{n\times n}$ be an arbitrary matrix and let $\mathbf{k} \in R^{1\times n}$ be an arbitrary vector. Show that if the solution $\mathbf{T}$ of $\mathbf{AT}-\mathbf{TF}=\mathbf{bk}$ is nonsingular, then $(\mathbf{A}-\mathbf{bkT}^{-1})$ has the same eigenvalues of $\mathbf{F}$.

考生請注意:必須寫出或描述求解過程，否則不予計分。

Possibly used formulas:

(1) $1/(1-x) = 1 + x + x^2 + x^3 + \cdots = \sum_{i=0}^{\infty} x^i$

(2) $1/(1+x)^n = \binom{-n}{0} + \binom{-n}{1}x + \binom{-n}{2}x^2 + \cdots$

$= \sum_{i=0}^{\infty} \binom{-n}{i} x^i$

$= 1 + (-1)\binom{n+1-1}{1}x + (-1)^2\binom{n+2-1}{2}x^2 + \cdots$

$= \sum_{i=0}^{\infty} (-1)^i \binom{n+i-1}{i} x^i$

(3) $1/(1-x)^n = \binom{-n}{0} + \binom{-n}{1}(-x) + \binom{-n}{2}(-x)^2 + \cdots$

$= \sum_{i=0}^{\infty} \binom{-n}{i} (-x)^i$

$= 1 + (-1)\binom{n+1-1}{1}(-x) + (-1)^2\binom{n+2-1}{2}(-x)^2 + \cdots$

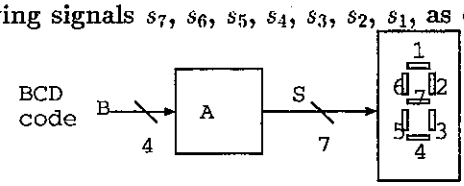
$= \sum_{i=0}^{\infty} \binom{n+i-1}{i} x^i$

1. A message is made up of 12 different symbols and is to be transmitted through a communication channel. In addition to the 12 symbols, the transmitter will also send a total of 45 spaces between the symbols, with at least three spaces between each pair of consecutive symbols. In how many ways can the transmitter send such a message? (15%)
2. How many positive integers between 1 and 30(inclusive) must we select in order to guarantee that we have two integers in our selection whose greatest common divisor is greater than 1? (15%)
3. (a) Let G=(V,E) be a directed graph with adjacency matrix M. How can one identify an isolated vertex of G from the matrix M? (10%)
   (b) For an equivalence relation R on a finite set, what is the structure of its associated graph?(In other words, what does its associated graph look like?) (10%)
4. Use generating function to solve: In how many ways can we select, with repetitions allowed, r objects from n distinct objects? (20%)
5. Please find the chromatic polynomial for $K_{2,3}$.What is its chromatic number? (15%)
6. Please explain the structures of group and ring in modern algebra. (15%)

注意事項: 1. 題目共六題. 2. 答題時務須將要點敘述清楚. 3. 答設計題時請節用試卷紙空間 (可運用試題紙空間)

1. (10%) Perform the conversion of the number $-7F6A_{16}$ to its octal number representation, its decimal number representation, and its 2's complement representation.

2. (10%) Design a 1-digit 7-segment display combinational circuit which accepts a 1-digit BCD (Binary Coded Decimal) code b_3, b_2, b_1, and b_0 as its input, and generate the 7-segment display driving signals s_7, s_6, s_5, s_4, s_3, s_2, s_1, as output.



3. (10%) A carry-look ahead adder computes the carry bits in parallel with combinational circuits. Given input signals of a 4-bit adder a_3, a_2, a_1, a_0, b_3, b_2, b_1, b_0, and c_0, derive the boolean equations of carry bits c_4, c_3, c_2 ,c_1, and draw the corresponding carry circuits.

4. (20%) Design a 4-digit BCD-to-binary code converter sequential circuit that accepts 4-digit BCD code $B3, B2, B1, B0$, each of which is represented as binary format b_3, b_2, b_1, b_0 and input *serially* from the most significant digit (B3) to the least significant digit (B0), and generates 14-bit binary code a_{13}, a_{12}, a_{11}, a_{10}, a_9, a_8, a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0 as output. You can use several 4-bit adder modules and shift registers in your design.

   Hint: This converter performs decimal to binary conversion. The conversion result A can be computed with the following equation

$$A = ((B3 * 10 + B2) * 10 + B1) * 10 + B0$$

   Thus, you can use the recurrence equation

$$A_{i+1} = A_i * 10 + B_{3-i}$$

   where $A_1 = B_3$ and $A = A_4$ to serially compute the result A_1, A_2, A_3, and A_4 (i.e. A). Note: You are required to design the circuit in this *serial* sequential circuit version.

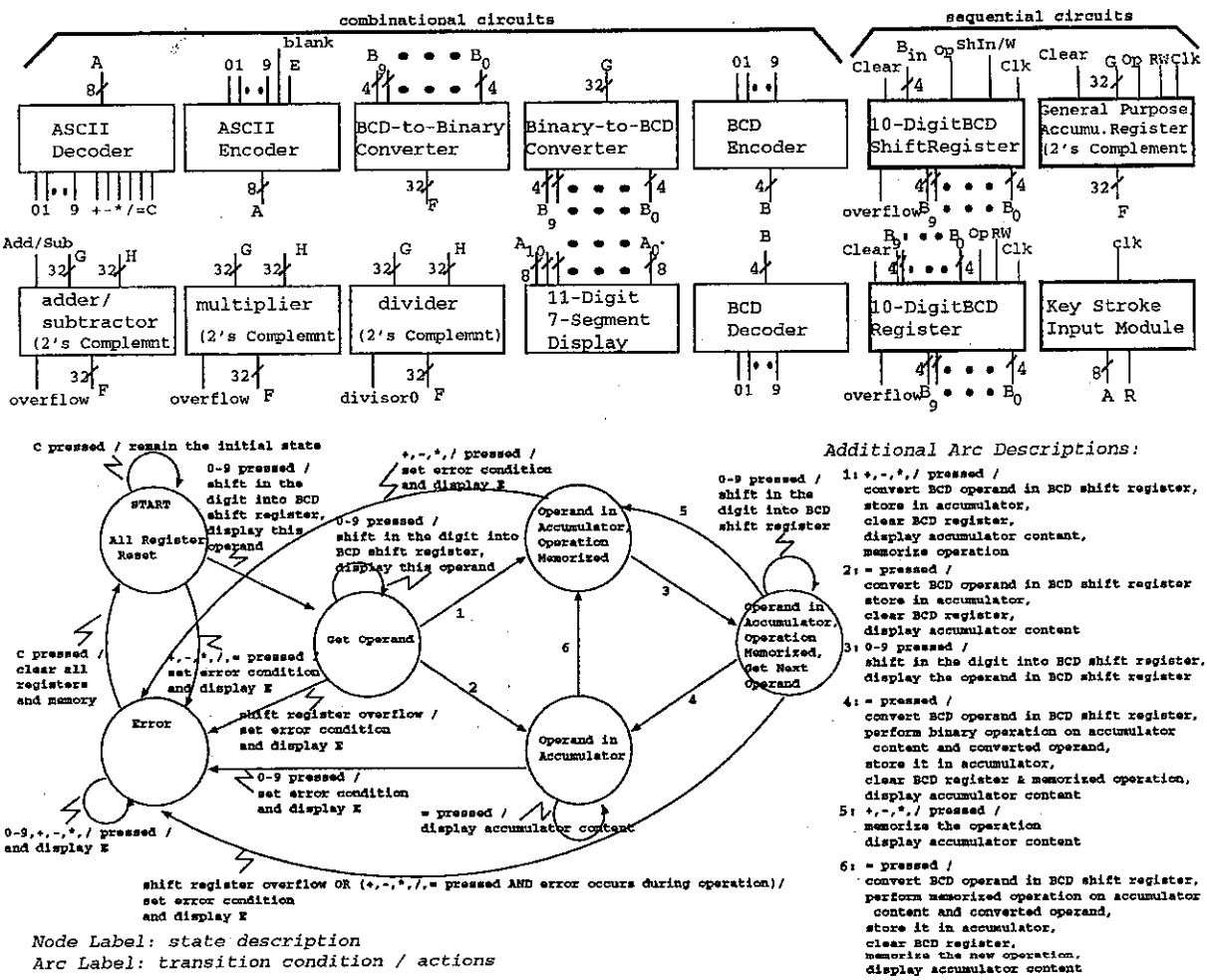
5. (20%) Use several D-type Flip-flop's to design an 8-bit shift register which can perform no-operation (Op=000), input to register (Op=001), output from register (Op=010), register reset (Op=011), logical right shift (Op=100), logical left shift (Op=101), right rotate (Op=110), and left rotate (Op=111) operations by 0 to 7 bit offset positions ($Offset$). The input signals include clk, 3-bit Op, 3-bit $Offset$, and 8-bit $data_{in}$. The output signals include 8-bit $data_{out}$.

6. (30%) Design a simplified *integer calculator*. User presses a stream of character strokes as input. (The character strokes include '9', '8', '7', '6', '5', '4', '3', '2', '1', '0', '+', '-', '*', '/', '=', and 'C' characters.) It performs conversion and integer operations on integer operands, and displays the input or the result on an 11-digit 7-segment display (A_{10}, A_9, ... , A_0). (A_{10} can display '-', ' ', or 'E' to represent negative, positive, or error, respectively. A_9, ... , A_0 can display '9', '8', ... , '0', or ' '.)

   You can use in the calculator the following hardware modules as shown in the figure:

   - a key stroke input module: The key stroke input module produces an 8-bit ASCII code output, and set a ready signal R to be 1 during the next clock period after a key is pressed.
   - an 11-digit 7-segment display: The 11-digit 7-segment display accepts 11 ASCII codes and display the corresponding signals '9', '8', '7', '6', '5', '4', '3', '2', '1', '0', '-', ' ', and 'E'.

- ASCII decoders, ASCII encoders, BCD decoders, and BCD decoders: The ASCII decoder/or encoder decodes/or encodes an input/or output ASCII code into/or from signals N_9, N_8, N_7, N_6, N_5, N_4, N_3, N_2, N_1, N_0, S_+, S_-, S_*, $S_/$, $S_=$, and S_C where N represents number and S represents symbol.
- a 10-digit BCD-code shift register: It has input signals Op(0: no operation, 1: shift in or register out), ShIn/W(0: shift in a digit, 1: register out), Clear (1: reset register content)
- a 10-digit-BCD-code-to-32-bit-unsigned-binary-number converter and a 32-bit-unsigne binary-number-to-10-digit-BCD-code converter
- several 32-bit general purpose registers (2's complement representation): It has input signals Op(0: no operation, 1: register in or out), RW(0: register in, 1: register out), Clear (1: reset register content)
- a 10-digit BCD operand register (BCD representation): It has input signals Op(0: no operation, 1: register in or out), RW(0: register in, 1: register out), Clear (1: reset register content)
- a 32-bit adder/subtractor, a 32-bit multiplier, a 32-bit divider: Assume these arithmetic circuits are all combinational circuits. The adder/subtractor needs an operation signal Add/Sub (0: addition, 1: subtraction).



(30%) Use the state transition diagram of the calculator as shown in the above figure, design and connect the hardware modules into a complete calculator circuit design.

Problem 1 : A bipartite graph, $G = (V, E)$, is an undirected graph whose vertices can be partitioned into two disjoint sets, V_1 and V_2, where $V_1 \cap V_2 = \emptyset$, $V_1 \cup V_2 = V$, with the properties :

a). no two vertices in V_1 are adjacent in G.

b). no two vertices in V_2 are adjacent in G.

Please do the following tasks.

(1). Assume a graph is represented by its adjacency lists. Develop a pseudo code or write a C function to determine whether a graph is bipartite or not. The only condition is that the computation complexity of your code must be $O(n+e)$ where n is the number of vertices, and e is the number of edges. (7%)

(2). Use a graph as an example to explain how your code works. (7%)

(3). Is a tree a bipartite graph? Please prove your answer. (6%)

(20 %)

Problem 2 : From time to time, sorting a group of records will be based on several keys. It is called **radix sorting**. Assume a group of records have three keys, K_2, K_1, K_0, where the degree of significancy is $K_2 > K_1 > K_0$. These keys are arranged as 3-digit integers. There are two sorting approaches : MSD, sorting begins with the most significant digit; LSD, sorting begins with the least significant digit. Given the following records with keys, 165, 39, 929, 823, 333, 367, 387, 5, 410, 601, please do the following tasks.

(1). Which sorting approach is the better one, MSD or LSD? Please explain.

(2). Use the approach you select in (1) to sort the given data in an ascending order. You have to show the result after each pass of the radix sorting.

(20 %)

Problem 3 : When we try to transmit a fixed length of data composed of a set of n letters by bits, a simple and straightforward scheme is to use $\log n$ bits to represent these letters. For instance, if the data to be transmitted are ABACCDA, which is composed of A, B, C, and D, we can use a table of A=00, B=01, C=10, D=11 to denote these four letters. Thus, the total number of bits to be transmitted is 14.

(1). Is this scheme the optimal method in terms of number of bits used in the transmission ? Why ?

(2). Given the following data to be transmitted, please develop a systematic method to encode each letter with bits such that the total bits of the data will be optimally small. (Hint : binary tree)

(20 %)

letter	how many times it shows in the data
A	15
B	6
C	7
D	13
E	20
F	2
G	3
H	8
I	14

Problem 4 : In Boolean algebra, 0 and 1 usually are the only constants to be used to evaluate the truth value of a given Boolean equation. (We ignore Z and don't care in order to simplify the problem.) It is likely that a Boolean equation can be represented by a binary tree. For instance, we set the the nodes of a binary tree to be the variables of the Boolean equation. Then, the right child of the node leads to the result setting the variable to be 1; while the left child of the node leads to the result setting the variable to be 0. The leaves will be the truth value of the equation. Hence, many of the binary tree processing methods can be applied to the Boolean equations.

(1). Construct the binary tree of $f_1 = \overline{x_1 \cdot x_3}$, and $f_2 = x_2 \cdot x_3$, respectively. (6%)

(2). Since Boolean equations can be computed with operators, the binary tree representation of Boolean equations should be able to be computed with operators. Please develop an algorithm to compute the $f = f_1 + f_2$. (7%)

(3). The resulted binary tree of (2) might not be a minimal form. Please also develop a reduction algorithm for the binary tree representation of Boolean equations. (7%)

(20%)

Problem 5 : Use the linear probing method to compete the following hash table. There are 10 buckets, 0,1,...,9, each of which has 2 slots. This hash table is assumed to be a circular table. The hashing priority is from the LSD to the MSD. The input sequence is 7333, 678, 789, 119, 5890, 123, 654, 55223344, 268, 737, 747, 987, 238, 499, 918, 337, 4780, 217, 150, 237.

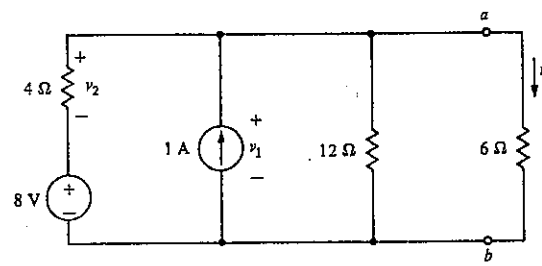
bucket	slot 1	slots searched	slot 2	slots searched
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				

(20%)

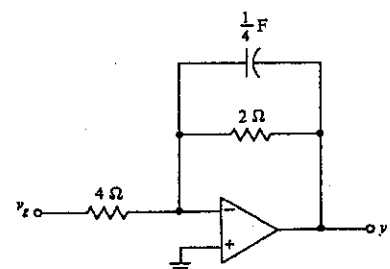
（橫書式）　國立中山大學八十七學年度碩博士班招生考試試題

科　目：電路學（電機所丁組）　　共 1 頁 第 1 頁

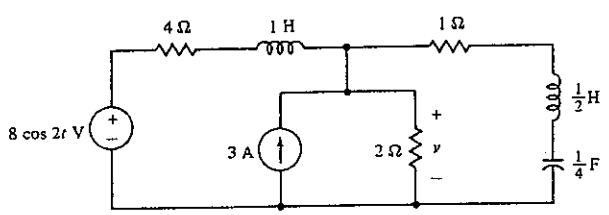
1. 利用戴維寧等效電路原理求 a、b 端點以左的等效電路，並據此求出通過 6Ω 之負載電流 i 及 a、b 點端電壓 v_1。 (20%)



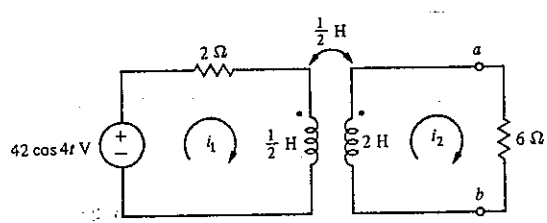
2. 假設下圖中，$v_s = 5\,e^{-t}u(t)$ 伏特，且沒有任何能量儲存於電容器中，求輸出電壓 v。 (20%)



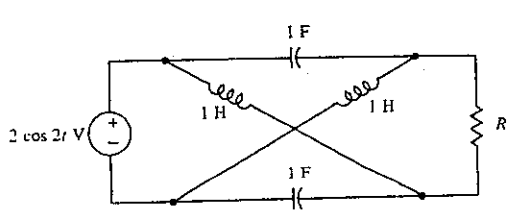
3. 如下圖所示，求穩態時跨於 2Ω 端電壓 v 爲何？ (20%)



4. 在下列變壓器圖中所示，求穩態電流 i_1 及 i_2。 (20%)



5. 如下圖所示，求 (a) 當 R=1Ω 時 R 所吸收的電功 (POWER) 爲何？ (10%) (20%)　(b) 電阻 R 可吸收最大電功爲何？在此情況下 R 爲多少Ω？ (10%)



I. (25%) Instruction Set Architectures

(1) (15%) Use **LOAD, MOVE, STOR, ADD, SUB, MUL,** and **DIV** to evaluate the arithmetic statement

$$X = (A - B) / (C + D * E)$$

(a) (5%) Use a general register computer with **three-address** instructions.
(b) (5%) Use a general register computer with **two-address** instructions.
(c) (5%) Use an accumulator type computer with **one-address** instruction.

(2) (10%) A two-word instruction is stored in memory at an address designated by the symbol **W**. The address field of the instruction (stored at **W + 1**) is designated by a symbol **Y**. The operand used during the execution of the instruction is stored at an address symbolized by **Z**. An index register contains the value **X**. State how **Z** is calculated from the other addresses, if the addressing mode of the instruction is
(a) (5%) **Indirect addressing mode**.
(b) (5%) **Indexed addressing mode**.

II. (25%) Computer Arithmetic Architectures

(1) (15%) Consider the two **8-bit signed** binary numbers **A = 10011001** and **B = 11100101**.
Add the two binary numbers together,
(a) (10%) Determine the Sign, Cary, Overflow, Zero, and Parity (assume even parity is 1) Flags.
(b) (5%) What is the calculation result stored in **16-bit** format?

(2) (10%) A **36-bit** floating point number consists of **26 bits** plus sign for the fraction and **8 bits** plus sign for the exponent.
(a) (5%) What is the **largest** positive quantity for normalized numbers?
(b) (5%) What is the **smallest** positive quantity for normalized numbers?

III. (25%) Parallelism and Pipelining Architectures

(1) (5%) **Grain size** is a measure of the amount of computation involved in a software process. The simplest measure is to count the number of instructions in a grain. Explain how does **grain size** affect
(i) **degree of parallelism**, (ii) **communication latency**, and (iii) **scheduling overhead**.

(2) (10%) Analyze the data dependency among the following statements in a given program.
Show flow dependency, output dependency and anti-dependency.

```
S1: A = B+C;
S2: C = B * A;
S3: S = 0;
S4: FOR (I=A; I <= 100: I++)
        S = S + X (I);
S5: IF (S > 1000)
        C = C * 2;
```

(3) (10%) Let **T** be the total time required for a nonpipelined sequential program of a given function. To execute the same program on a **K-stage** pipeline with an equal flow-through delay **T**, one needs a clock period of **P = T / (K + D)**, where **D** is the pipeline latch delay. Assume **C** represents the cost of all logic gates and **H** represents the cost of each latch.

(a) (5%) Derive the pipeline performance/cost ratio (**PCR**).

(b) (5%) What is the optimal number of desired pipeline stages.

IV. (25%) Multiprocessor Architectures

(1) (5%) Explain what is **cache coherence** in a shared memory multiprocessor architecture.
Identify one possible solution for prevent from cache inconsistency.

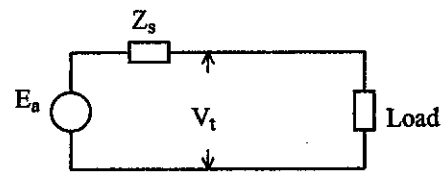
(2) (10%) Calculate the total number of permutations that can be implemented in a **single pass** for a **64 inputs** and **64 outputs IBM SP2**, which uses bi-directional **4x4** crossbar switch module as building blocks for its multistage interconnection networks. Assume the switch module supports only **crossover** and **straight** permutations.

(3) (10%) Answer the following questions for an **N-node** hypercube network:

(a) (5%) What is the node degree(d)?

(b) (5%) What is the Bisection width (B)?

1. Fig. 1 shows a simple power system consisted of one synchronous generator and a load, and the terminal voltage V_t is kept at 1.0 p.u. to supply the load at 1.0 p.u., 0.8 pf lagging. With the generator excitation current being fixed and the given generator parameters, how to increase the real power sent to the load (8%)? Calculate the maximum real power that can be supplied to the load (12%).



$Z_s = 0.01 + j0.05$ p.u.

Fig. 1. One-line diagram of a simple power system.

2. The synchronous impedance of a 3-Φ, 60 Hz generator is 10+j100 Ω. By maintaining its terminal phase voltage at the rated value of 120 V, calculate the voltage regulations and efficiencies of this machine for the following three load conditions (neglect mechanical losses) (30%).

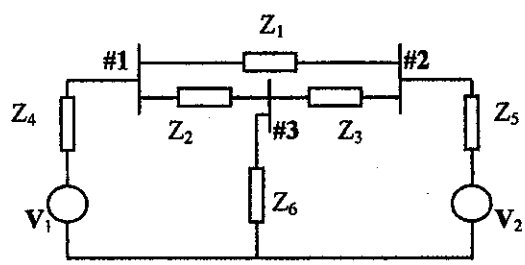
   (a) $Z_{load} = 400\ \Omega$, (b) $Z_{load} = j400\ \Omega$, (c) $Z_{load} = -j400\ \Omega$.

3. A 50 MVA, 12 kV generator having $X'' = X_2 = 15$ %, $X_0 = 6$ %, and a current-limiting reactor of 0.096 Ω in the neutral. Before the generator is connected to a power system, its voltage is adjusted to 11.5 kV when a line-to-line fault develops at terminal b and c. Please find the initial symmetrical **rms** current in the ground (10%) and in line **c** (15%).

4. Devise the $\mathbf{Y}_{bus}$ matrix of the following power system (10%), and determine the bus impedance matrix $\mathbf{Z}_{bus}$ directly without using $\mathbf{Y}_{bus}^{-1}$ (15%).

$\mathbf{V}_1 = 1.0$ p.u., $\mathbf{V}_2 = 1.0 + j0.2$ p.u.,

$Z_1 = j0.5$ p.u., $Z_2 = j0.25$ p.u., $Z_3 = j0.25$ p.u., $Z_4 = j0.2$ p.u., $Z_5 = j0.2$ p.u., and $Z_6 = j2.5$ p.u.



I. Questions (40%)

1. What do we mean by linear, homogeneous, and isotropic medium? (4%)

2. What are the boundary conditions for electrostatic and magnetostatic fields at an interface of two lossless dielectric media? What are the boundary conditions for electrostatic and magnetostatic fields at an interface between a perfect conductor and a lossless dielectric medium? (4%)

3. Write the differential and integral forms of Maxwell's equations. (4%)

4. State skin effects. Also express the skin depth of a conductor. (4%)

5. State Poynting's theorem. Also define Ponyting vector. (4%)

6. What is meant by the loss tangent of a medium? How do we define a good conductor and a low-loss dielectric? (4%)

7. What is meant by the dispersion of a signal? Give an example of a dispersive medium and show its dispersion relation. (4%)

8. Show the distributed circuit model for a transmission line. Express propagation constant and characteristic impedance of a transmission line in terms of R, L, G, C distributed parameters. (4%)

9. Express the relation between input impedance Z_{in} and reflection coefficient Γ at any reference position z' in a transmission line. (4%)

10. Express the group velocity, phase velocity and wave impedance for TEM, TM and TE modes as functions of the ratio of cut-off frequency to frequency. (4%)

II. Problems (60%)

1. Medium 1, comprising the region $r < a$ in spherical coordinates, is a perfect dielectric of permittivity ε_1, whereas medium 2, comprising the region $r > a$, is free space. The electric field intensities in the two media are given by

$$\mathbf{E}_1 = E_{01}(\mathbf{a}_r \cos\theta - \mathbf{a}_\theta \sin\theta)$$

$$\mathbf{E}_2 = E_{02}[\mathbf{a}_r(1+\frac{a^3}{r^3})\cos\theta - \mathbf{a}_\theta(1-\frac{a^3}{2r^3})\sin\theta]$$

respectively. Find ε_1. (10%)

2. Figure P2 shows a magnetic field electromechanical device in which the plunger is free to move in the x-direction between two nonmagnetic sleeves. The areas of cross section of all three legs are equal. Find the mechanical force $\mathbf{F}_e$ on the plunger. (15%)

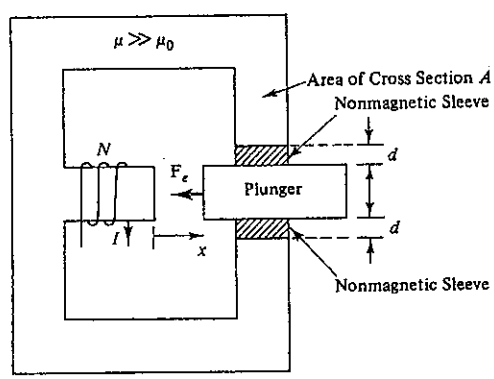


Figure P2

3. The arrangement shown in Figure P3 is that of a parallel-plate resonator made up of two perfect conductors coated with a dielectric, and in which uniform plane waves bounce back and forth normal to the plates. Find the characteristic equation for the lowest resonant frequency of this structure. (15%)

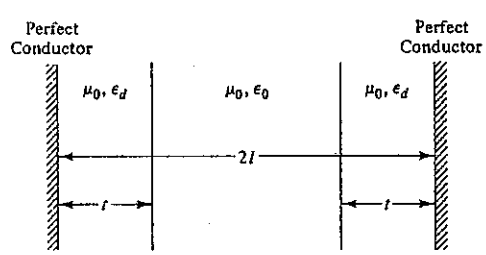


Figure P3

4. For an asymmetric dielectric slab waveguide, made up of a dielectric slab of thickness d and permittivity ε_1, sandwiched between two dielectric media of permittivities ε_2 ($< \varepsilon_1$) and ε_3 ($< \varepsilon_1$, ε_2), derive the characteristic equation for guiding of TE waves. (20%)

I.線性代數與機率之觀念與應用

各小題之計分如下，回答前請讀計分說明。(框線內爲公式計算，可以不讀)

本題計分將沿用一本科所創的"市場需求調整計分法簡化版"，即計分高低(價值)與獲解率成反比。

(獲解率可視爲市場需求率之倒數，愈易獲解，市場上需求愈低)。

所以考生除了要把握多數人會的題目外，還要著重於自己獨特的能力，解別人不容易會的問題。

在入學徵才錄取率不高的情況下，希望考生能發揮出自己卓越的特點。

你的得分(S_1",S_2",...,Sn")公式計算如下：令$R_1,R_2,...,Rn$爲小題各自原始分數給分範圍，

R_1",R_2",...,Rn"爲依市場需求調整後的分數給分範圍，$M_1,M_2,...,Mn$爲小題各自的原始平均得分，

$S_1,S_2,...,Sn$爲小題各自原始得分：

則 $S_1" = S_1 \times \frac{R_1"}{R_1}$

而 R_1",R_2",...,Rn"的計算，在$M_1,M_2,...,M_n$的各小題平均分數算出後，

依據 $R_1":R_2":....:R_n" = \frac{R_1}{M_1}:\frac{R_2}{M_2}:....:\frac{Rn}{Mn}$ 得到，且 $R_1"+R_2"+...+Rn" = 100$

注意R_1,R_2,R_3的原始範圍可以任意設定，爲著讓考生易所依循，我仍可以作一次不必要的設定：

$R_1 = R_2 = ... = Rn = 100/n$。　此框考生可以不證

簡言之，各小題的分數權重與獲解率成反比。

I. Concept problems

1. For the following T:R*R ->R*R, state which T is (are) linear. You must be all correct to get credits.
   (a) T(a1,a2) = (1,a2)
   (b) T(a1,a2) = (a1,a1*a2)
   (c) T(a1,a2) = (sin a1,cos a1)
   (d) T(a1,a2) = (0,0)
   (e) T(a1,a2) = (|a1|, |a2|)
   (f) T(a1,a2) =(a1 + 1, a2 + 3)

2. Prove or disprove that every linear transformation from n-dimensional vector space V to m-dimension vector space W can be represented by a matrix.

II. Computational problems:

3. Compute the eigenvalues for the following T defined by
   T(a1,a2,a3) = (4*a1+a3, 2*a1+3*a2+2*a3, a1+4*a3)

4. Two random variables, X and Y, have a joint probability density function given by
   p(x,y) = K　　0<x<1, 0<y<1
   =0　　elsewhere
   (a) What is the value K?
   (b) What is the conditional probability p(x/y) ?

III. Story problems:

5. In a certain communication system the message is coded into the binary digits 0 and 1. After coding, the probability of a 0 being transmitted is 0.45, while the probability of 1 is 0.55. In the communication channel the probability of transmitted 0 being distorted into 1 at the receiver is 0.1, while the probability of a 1 being distorted into a 0 is 0.2

(a) Find the probability that a received digit is correct.

(b) Find the probability that a received 0 is correct 0.

6. A multichannel microwave link is to provide telephone communication to a remote community of 10 subscribers, each of whom uses the link 25 percent of the time during peak hours. How many channels are needed to make the link available during peak hours to:

(a) Ninety percent of the subscribers all the time.

(b) All the subscribers 90 percent of the time.

(c) Which of (a) and (b) you would like to chose to meet the satisfaction rate if you are the engineer?

IV. Reading comprehension problems:

7. Please read the following theorem and its proof. Then, answer the question followed

Let V and W be vector spaces, and let T and U are linear transformation from V to W. Then for all a in the scalar field F

(a) aT + U is linear.

(b) Using the operations of addition and scalar multiplication **as defined above (A),** the collection of all linear transformations from V into W is a vector space over F. This vector space is denoted by L(V,W).

Proof. (a) Let x, y in V and c in F. Then,

$$(aT + U)(cx+y) = aT(cx+y) + U(cx+y)$$
$$= a[cT(x) + T(y)] + cU(x) + U(y)$$
$$= acT(x) + cU(x) + aT(y) + U(y)$$
$$= \mathbf{B}$$

So aT + U is linear.

(b) Noting that To, the zero transformation, play the role of the zero element in L(V,W), it is easy to show that L(V,W) is a vector space over F.

Question:

(a) What is A?

(b) What is B, i.e., the definitions of addition and multiplication?

(c) What is the role of (a) for the proof of (b)?

8. Image enhancement is an important but simple skill for image processing and is commonly seen in some popular commercial tools like Photoshop and Imagepal. The following concept is originally from the application of probability density function.

The histogram of gray levels in an image is defined by h(i) = n(i)/n where n(i) = sum of gray levels in the image having the value i and n = total number of gray levels in the image. We can enhance the image

by modifying the histogram. Histogram transformations can be obtained, in approximate form, by considering the continuous probability density function pr(r) in place of h(i). What is then required is to obtain a transformation that maps the gray levels in the original image, represented by the variable r, to a new variable s such that the distribution of gray levels in the transformed image follows some prescribed form to make the image appear better. Namely, s = T(r). If ps(s) denotes the probability density function of the transformed image, then from the theory of probability one can write:

$$Ps(s) = [pr(r)\, dr/ds] \text{ at } r = T^{-1}(s) \quad \ldots\ldots\ldots \quad \text{Eq. 1}$$

We are assuming, for the moment, that the original and mapped image are continuous functions in the two independent space variables x and y. Now consider the transformation

$$s = T(r) = \int_0^r Pr(w)dw$$

where the right-hand side is known as the distribution function Eq. We can write

$$ds/dr = \mathbf{A} \quad \ldots\ldots\ldots \quad \text{Eq. 2}$$

Substituting Eq 2 in Eq. 1 We get

$$ps(s) = \mathbf{B}$$

Hence, the transformation given by T provides an image with a special distribution in gray levels.

Question:

(a) What are the expressions of A and B?

(b) What is the special distribution for the gray levels of the mapped image by T?

(橫書式) 國立中山大學八十七學年度碩博士班招生考試試題

科 目：通訊理論（電機所己組） 共 3 頁 第 1 頁

共十題，每題10分

1. 什麼叫做 completely orthonormal basis functions？
你認為它們在通訊系統中可以扮演什麼樣的角色？
為什麼？ 如果 basis functions 不是 orthogonal，你
有什麼方法或學過什麼策略，可以將它們轉變為 orthonormal？

2. 傅氏級數展開（Fourier Series Expansion），傅氏轉換（Fourier Transform）
拉氏轉換（Laplace Transform），及 Z 轉換 在通信科學，信號
分析，系統設計中各有其運用之道理，試書述它們四者
運用之原則，範疇，並清楚說明其間相異相似之所在？

3. 頻譜密度函數（Spectral Density）與 相關函數（Correlation）
二者有何關係？在什麼樣的隨机環境下，此關係才能
夠確實成立？什麼叫做 Band-limited Additive White Gaussian Noise.
試以此雜音之每个字（英文字）的通訊含意，解說其大意。此
Noise 在設計通訊系統時，您認為它有何重要性？如果
雜音不是白色的（nonwhite），有什麼通訊程序的理論，
您可以將它轉變為白色的（white）？

4. 什麼叫做 Vestigial-Sideband （VSB） Modulation，它有何重要
用途？在硬体設計中，如何能由 DSB，（Double-Sideband）之
信号得到 VSB？您是否知道家中商用 AM，FM 之頻率範圍
及每个电台的頻寬，並告訴我商用 TV 每个點的頻寬為何？

5. FM (Frequency Modulation) 屬於相角調變的一種 (Angle Modulation)，在FM接收机中，为什麼会使用到 de-emphasis 的过程，請就您对 Commercial FM 收發机 (transmitter & receiver) 的了解，敘明理由，並提出实体之硬体收發机在这个論点之解決方案。

6. 什麼叫做 Matched Filter？有何通訊上之用途？設計此 Filter 的要領为何？所能達到的最大功效为多少？

7. 什麼叫做 Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA) 与 Code Division Multiple Access (CDMA)，試舉例說明之。並绘製一个可行的，number of stages $m=3$ 的 PN (pseudo-noise) Sequency Generator. 此通信設備產生之 Pulse modulation code 之週期是多少？

8. 机率与統計常運用在通信系統之設計中，試就您对 mean 以及變數變異量 (variance) 与互變異矩陣 (covariance matrix) 之基本物理意義，解釋之。假如有二个隨机变数 X 与 Y，均为一维常態分佈，即 $X \sim N(3, 3^2)$，$Y \sim N(4, 4^2)$，X 与 Y 之平均值分別为 3 与 4，而變異量分別为 9 与 16，試問若 X 与 Y 为互相獨立之 random variable，有無可能二人会相関 (correlated)，若 $Z = 3X + 4Y$，請問 Z 的平均值与變異量为若干？

9. 什麼是 source coding，什麼是 channel coding？二者有何具体差別？什麼是 Huffman coder？什麼叫做 channel capacity，它能做什麼？

10. Consider a continuous input signal having the following probability density function:

$$p(x) = \begin{cases} 1-|x| & ; \quad -1 \le x \le +1, \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

(a) Determine the quantizer step size and levels if a linear (uniform) eight-level ($n = 8$) quantizer is used.

(b) Determine the nonlinear quantizer step levels required to make the quantized signal levels equiprobable. Also plot the required compressor characteristic to precede a linear quantizer.