

國立中山大學 115 學年度 碩士班考試入學招生考試試題

科目名稱：電子學(甲組)【電機系碩士班甲組】

— 作答注意事項 —

考試時間：100 分鐘

- 考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。請先檢查答案卷(卡)之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：電子學(甲組)【電機系碩士班甲組】

題號：431006

※本科目依簡章規定「可以」使用計算機(廠牌、功能不拘)(問答申論題) 共2頁第1頁

1. (20%) It is required to design a Zener shunt regulator to provide an output voltage of 10 V as shown in **Figure 1**. The raw supply available varies between 15 V and 25 V and the load current varies over the range of 0 to 20 mA. The available 10 V Zener diode is specified to have 10 V drop at a test current of 25 mA. At this current its r_z is 7Ω . Design for a minimum Zener current (I_{zk}) of 5 mA. (a) Find the required value R. (b) Find the line regulation ($\Delta V_o/\Delta V_s$)

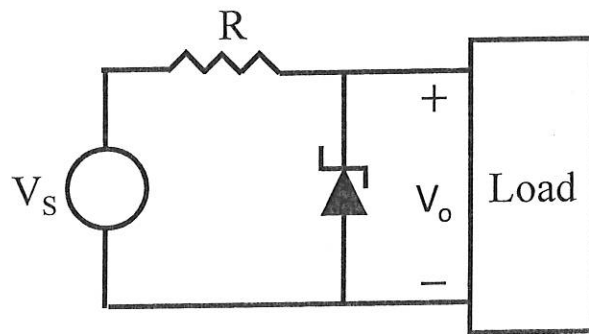


Figure 1

2. (20%) For the common-emitter amplifier shown in **Figure 2**, let $V_{CC}=15\text{ V}$, $R_1=27\text{ k}\Omega$, $R_2=15\text{ k}\Omega$, $R_E=2.4\text{ k}\Omega$, and $R_C=3.9\text{ k}\Omega$. The transistor has $\beta=100$. If the amplifier operates between a source for which $R_{sig}=2\text{ k}\Omega$ and a load of $2\text{ k}\Omega$, find the value of R_{in} and the overall voltage gain (v_o/v_{sig}).

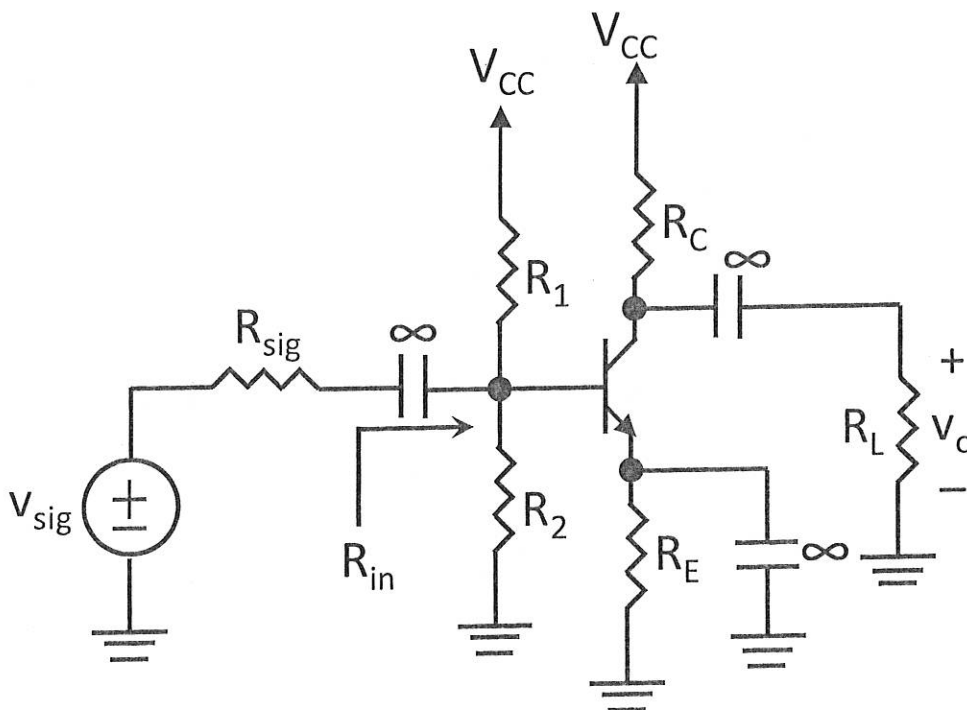


Figure 2

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：電子學(甲組)【電機系碩士班甲組】

題號：431006

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Q3. (20%) In common-gate amplifier circuit shown in **Figure 3**, Q_2 and Q_3 are matched. $k_n'(W/L)_n = k_p'(W/L)_p = 4\text{mA/V}^2$, and all transistors have $|V_t| = 0.8\text{ V}$ and $|V_A| = 20\text{ V}$. The signal v_{sig} is a small sinusoidal signal without dc component. Please answer the following questions.

- (a) Find the value of R_{in} .
- (b) Find the value of R_{out} .
- (c) Calculate the voltage gain (v_o/v_i).
- (d) How large can v_{sig} be (peak-to-peak) while maintaining saturation-mode operation for Q_1 and Q_2 .

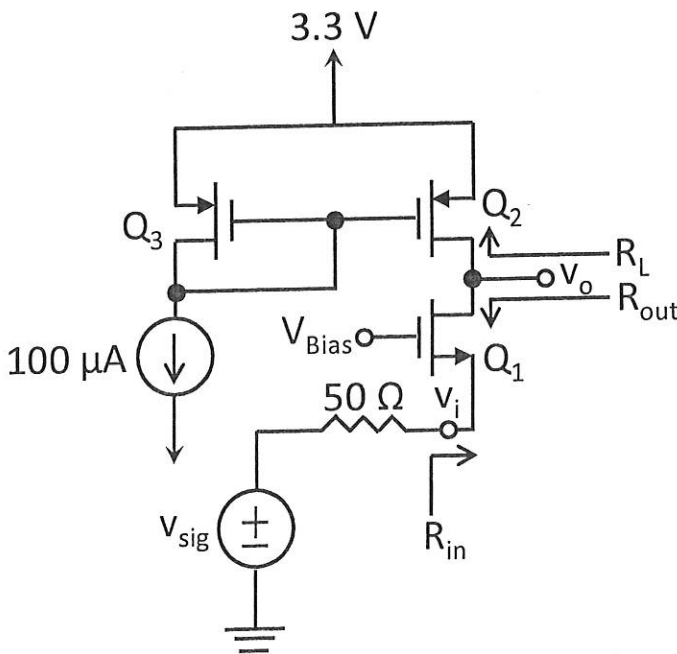


Figure 3

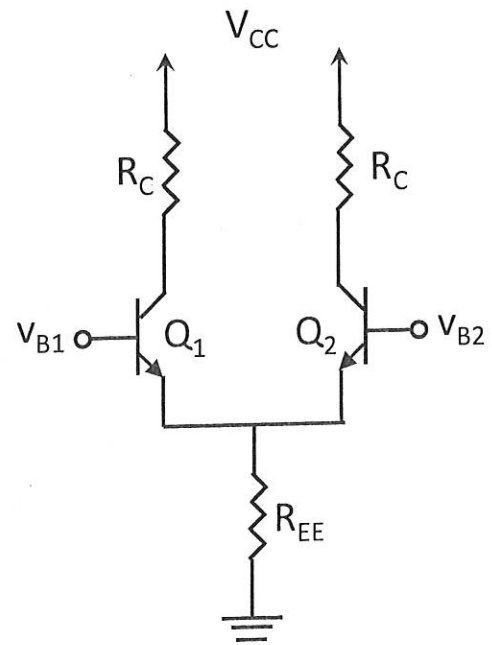


Figure 4

Q4. (20%) A bipolar differential amplifier shown in **Figure 4** with $I = 0.5\text{ mA}$ uses transistors for which $V_A = 50\text{ V}$ and $\beta = 100$. The collector resistance $R_C = 5\text{ k}\Omega$ and are matched to within 5%. Find the common-mode gain and the CMRR if the bias current I is generated using a simple current source.

Q5. (20%) A discrete MOSFET common-source amplifier has $R_G = 2\text{ M}\Omega$, $g_m = 5\text{ mA/V}$, $r_o = 100\text{ k}\Omega$, $R_D = 20\text{ k}\Omega$, $C_{gs} = 3\text{ pF}$, and $C_{gd} = 0.5\text{ pF}$. The amplifier is fed from a voltage source with an internal resistance of $500\text{ k}\Omega$ and is connected to a $20\text{ k}\Omega$ load. Find (a) the upper 3-dB frequency (f_H) and (b) the frequency of the transmission zero (f_z).

國立中山大學 115 學年度 碩士班考試入學招生考試試題

科目名稱：半導體概論【電機系碩士班甲組】

—作答注意事項—

考試時間：100 分鐘

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國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：半導體概論【電機系碩士班甲組】

題號：431007

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題） 共 1 頁第 1 頁

The dielectric constants of silicon and silicon dioxide (SiO_2) are 11.7 and 3.9, respectively.

The permittivity of vacuum is 8.85×10^{-14} F/cm.

Energy bandgap of Si: $E_g = 1.12$ eV, charge $q = 1.6 \times 10^{-19}$ C, $kT = 25.9$ meV at $T = 300$ K

Electron affinity of Si: $\chi = 4.01$ eV. Intrinsic concentration of Si at $T = 300$ K: $n_i = 1.5 \times 10^{10}$ cm^{-3} .

- (10%) Consider a semiconductor sample at $T = 300$ K in which donor concentration $N_d = 2.5 \times 10^{14}$ cm^{-3} and acceptor concentration $N_a = 1.5 \times 10^{14}$ cm^{-3} . Assume that $n_i = 5 \times 10^{13}$ cm^{-3} . Calculate the thermal-equilibrium electron and hole concentrations.
- (5%) Assume that the donor concentration in an n-type semiconductor at $T = 300$ K is given by $N_d(x) = 10^{16} - 10^{19}x$ cm^{-3} , where x is given in cm and ranges between $0 < x < 1$ μm . Determine the induced electric field in a semiconductor in thermal equilibrium at $x = 1$ μm .
- (5%) Consider n-type GaAs doped at $N_d = 10^{16}$ cm^{-3} . Assume that 10^{14} electron-hole pairs have been uniformly created per cm^3 at $t = 0$, but assume that generation rate $g' = 0$ for $t > 0$, and assume the minority carrier hole lifetime is $\tau_{p0} = 50$ ns. Determine the time at which the minority carrier hole concentration reaches 5% of its initial value.
- (10%) Consider silicon at $T = 300$ K doped at concentrations of $N_d = 10^{16}$ cm^{-3} and $N_a = 0$. Assume that $n' = p' = n_i = 1.5 \times 10^{10}$ cm^{-3} in the excess carrier recombination rate equation and assume parameter values of $\tau_{n0} = \tau_{p0} = 5 \times 10^{-7}$ s. Calculate the recombination rate of excess carriers if $\delta n = \delta p = 10^{14}$ cm^{-3} .
- (20%) A silicon pn junction at $T = 300$ K has the following parameters: $N_a = 5 \times 10^{16}$ cm^{-3} , $N_d = 1 \times 10^{16}$ cm^{-3} , $D_n = 25$ cm^2/s , $D_p = 10$ cm^2/s , $\tau_{n0} = 5 \times 10^{-7}$ s, and $\tau_{p0} = 1 \times 10^{-7}$ s. The cross-sectional area is $A = 10^{-3}$ cm^2 and the forward bias voltage is $V_a = 0.625$ V. Calculate the (a) (5%) depletion width in n-type silicon, (b) (5%) maximum electric field, (c) (5%) junction capacitance, (d) (5%) total current in the pn junction diode.
- (10%) Please explain the physical origin of the energy bandgap in a semiconductor and describe its relationship to crystal bonding from the perspective of atomic orbital overlap.
- (10%) Describe the physical mechanisms through which lattice defects in a semiconductor influence the electrical behavior of a pn diode, particularly under forward and reverse bias conditions, and explain how these effects are incorporated into the I-V characteristic equation.
- (10%) Please describe the fundamental differences between a Schottky diode and a pn diode, with emphasis on carrier transport mechanisms and the magnitude and origin of reverse leakage current.
- (10%) (a) (5%) Please describe the differences between the high-frequency and low-frequency C-V characteristics of a MOS capacitor, and explain the underlying physical mechanisms responsible for these differences. (b) (5%) Please compare the high-frequency C-V characteristics of a MOS capacitor under two-terminal biasing and those of a MOSFET under four-terminal biasing, and explain the physical reasons for the observed differences.
- (10%) Please describe the phenomena of (a) (5%) the deep depletion effect and (b) (5%) the polysilicon-gate depletion effect in a MOS capacitor, and explain the physical mechanisms responsible for each effect.

國立中山大學 115 學年度 碩士班考試入學招生考試試題

科目名稱：電子學【電機系碩士班乙組選考、戊組選考、通訊所碩士班乙組選考、電波聯合碩士班選考】

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國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：電子學【電機系碩士班乙組選考、戊組選考、通訊所碩士班乙組選考、電波聯合碩士班選考】題號：431005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題） 共 2 頁第 1 頁

1. (30 pt) For the circuit in Fig. 1, it is required to determine the value of the voltage V_{BB} that results in the transistor operating
- in the active mode with $V_{CE} = 5\text{ V}$ (10 pt)
 - at the edge of saturation (10 pt)
 - deep in saturation with $\beta_{\text{forced}} = 10$ (10 pt)
- For simplicity, assume that V_{BE} remains constant at 0.7 V . The transistor β is specified to be 50.

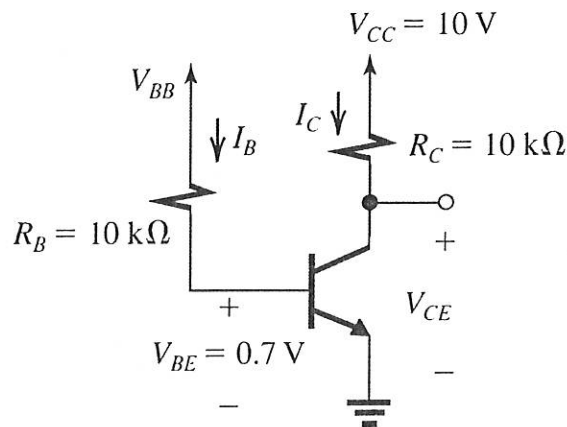


Figure 1.

2. (20 pt) For the circuit in Fig. 2, assume that $\beta_{\text{min}} = 30$, and find (a) V_E (5 pt), (b) V_B (5 pt), (c) I_{C1} (5 pt), and (d) I_{C2} (5 pt).

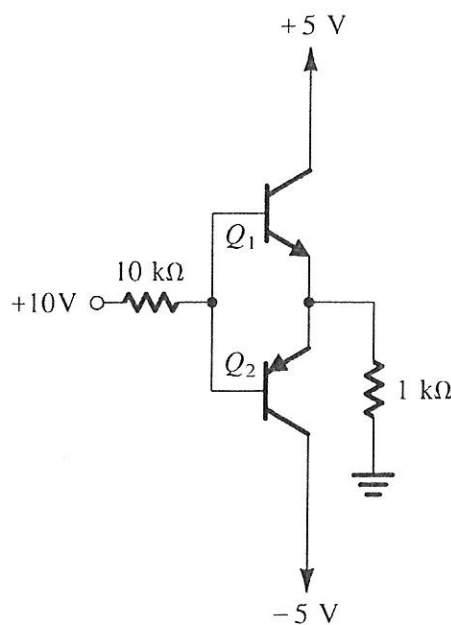


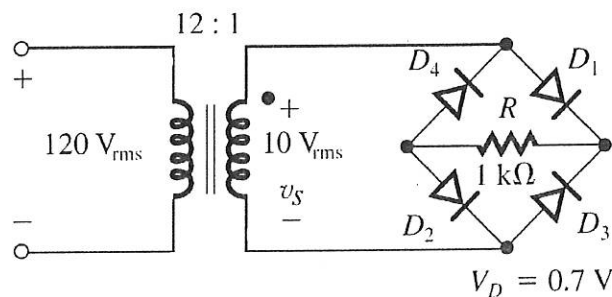
Figure 2.

國立中山大學 115 學年度碩士班考試入學招生考試試題

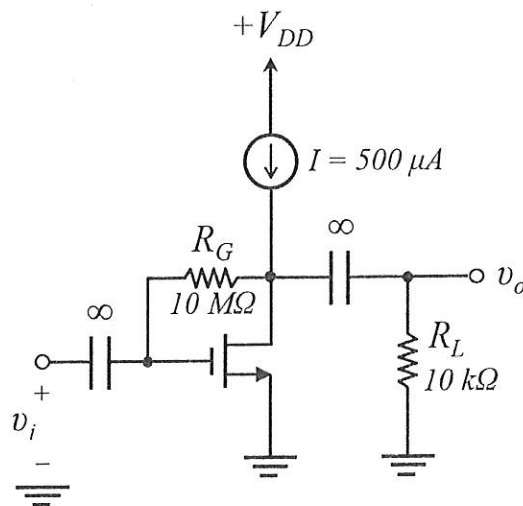
科目名稱：電子學【電機系碩士班乙組選考、戊組選考、通訊所碩士班乙組選考、電波聯合碩士班選考】題號：431005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題） 共 2 頁第 2 頁

3. (20 pt) For the circuit in Fig. 3, a full-wave bridge-rectifier circuit with a 1-k Ω load operates from a 120-V (rms) 60-Hz household supply through a 12-to-1 step-down transformer having a single secondary winding. It uses four diodes, each of which can be modeled to have a 0.7-V drop for any current. (a) What is the peak value of the rectified voltage across the load? (5 pt) (b) For what fraction of a cycle does each diode conduct? (5 pt) (c) What is the average voltage across the load? (5 pt) (d) What is the average current through the load? (5 pt)



4. (30 pt) In the circuit of Fig. 4, the NMOS transistor has $|V_t| = 0.9$ V, and $V_A = 50$ V, and operates with $V_D = V_{GS} = 2$ V. (a) What is the voltage gain v_o/v_i ? (10 pt) (b) What does V_D become when I increase to 1 mA? (10 pt) And (c) what is the new gain? (10 pt)



國立中山大學 115 學年度 碩士班考試入學招生考試試題

科目名稱：工程數學乙【電機系碩士班乙組選考】

— 作答注意事項 —

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國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：工程數學乙【電機系碩士班乙組選考】

題號：431001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）

共 5 頁第 1 頁

下面 1-20 題為複選題，每題 5 分，總分 100 分。每題有五個選項，其中至少有一個是正確答案。答錯 1 個選項者，得 3 分，答錯 2 個選項者，得 1 分，答錯多於 2 個選項或未作答者，該題以零分計算。

1. Let $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{d} \in \mathbb{R}^n$. Define the matrices

$$\mathbf{B} = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3] \in \mathbb{R}^{n \times 3}, \mathbf{C} = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4] \in \mathbb{R}^{n \times 4}.$$

Assume that the linear system $\mathbf{B}\mathbf{x} = \mathbf{d}$ has no solution for $\mathbf{x} \in \mathbb{R}^3$, and the linear system $\mathbf{C}\mathbf{y} = \mathbf{d}$ has at least one solution for $\mathbf{y} \in \mathbb{R}^4$. Let $[\mathbf{B}, \mathbf{d}] = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{d}] \in \mathbb{R}^{n \times 4}$. Which of the following statements are true? (Select all that apply.)

- (A) The set $\{\mathbf{a}_4, \mathbf{d}\}$ is linearly dependent.
- (B) The set $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{d}\}$ is linearly dependent.
- (C) The set $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$ is linearly independent.
- (D) $\text{rank}(\mathbf{C}) > \text{rank}(\mathbf{B})$.
- (E) $\text{rank}(\mathbf{C}) = \text{rank}([\mathbf{B}, \mathbf{d}])$.

2. Let $\mathbf{A} \in \mathbb{R}^{m \times n}$. For any matrix \mathbf{M} , let $R(\mathbf{M})$ denote the column space and $N(\mathbf{M})$ denote the null space. Which of the following statements are true? (Select all that apply.)

- (A) If $\mathbf{y} \in R(\mathbf{A}\mathbf{A}^T)$ and $\mathbf{y} \neq \mathbf{0}$, then $\mathbf{A}^T\mathbf{y} \neq \mathbf{0}$.
- (B) If $\mathbf{y} \in R(\mathbf{A})$, then $\mathbf{A}^T\mathbf{y} = \mathbf{0}$.
- (C) If $m = n$, then $R(\mathbf{A}) = R(\mathbf{A}^T)$.
- (D) If $m = n$, then $N(\mathbf{A}) = N(\mathbf{A}^T)$.
- (E) If $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{A}\mathbf{x} \neq \mathbf{0}$, then $\mathbf{x} \in R(\mathbf{A}^T)$.

3. Let $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathbf{B} \in \mathbb{R}^{m \times k}$, and $\mathbf{C} \in \mathbb{R}^{k \times n}$ such that $\mathbf{A} = \mathbf{B}\mathbf{C}$. For any matrix \mathbf{M} , let $R(\mathbf{M})$ denote the column space and $N(\mathbf{M})$ denote the null space. Which of the following statements are true? (Select all that apply.)

- (A) $R(\mathbf{B})$ is a subset of $R(\mathbf{A})$.
- (B) $N(\mathbf{A})$ is a subset of $N(\mathbf{C})$.
- (C) $k \geq \text{rank}(\mathbf{A})$.
- (D) If $k > \text{rank}(\mathbf{A})$, then $\text{rank}(\mathbf{B}) = \text{rank}(\mathbf{C})$.
- (E) If $k = \text{rank}(\mathbf{A})$, then $\text{rank}(\mathbf{A}) = \text{rank}(\mathbf{B})$.

4. Let

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & 3 & 0 & 3 \\ 0 & 2 & 0 & 2 & 0 \\ 3 & 0 & 3 & 0 & 3 \\ 0 & 2 & 0 & 2 & 0 \\ 3 & 0 & 3 & 0 & 3 \end{bmatrix}.$$

Which of the following statements are true? (Select all that apply.)

- (A) \mathbf{A} is diagonalizable (over \mathbb{R}).
- (B) $\text{rank}(\mathbf{A}) = 2$.
- (C) 2 is an eigenvalue of \mathbf{A} .
- (D) 3 is an eigenvalue of \mathbf{A} .
- (E) The number of positive eigenvalues of \mathbf{A} is 2.

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：工程數學乙【電機系碩士班乙組選考】

題號：431001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）

共 5 頁第 2 頁

5. Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix},$$

and define the subspace $W = \text{span}\{\mathbf{v}_1, \mathbf{v}_2\} \subset \mathbb{R}^3$. Let \mathbf{p} be the orthogonal projection of \mathbf{x} onto W . For any vector \mathbf{u} , let $\|\mathbf{u}\|$ denote the Euclidean norm: $\|\mathbf{u}\| = \sqrt{\mathbf{u}^T \mathbf{u}}$. Which of the following statements are true? (Select all that apply.)

- (A) $\|\mathbf{p}\| = 2$.
- (B) $\|\mathbf{x} - \mathbf{p}\| = 3$.
- (C) $\mathbf{w}^T(\mathbf{x} - \mathbf{p}) = 0$ for every $\mathbf{w} \in W$.
- (D) $\mathbf{x} - \mathbf{p} \in W$.
- (E) \mathbf{p} is orthogonal to \mathbf{v}_1 .

6. Let

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \mathbf{b}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}.$$

Define a linear transformation $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ satisfying

$$L(\mathbf{b}_1) = 2\mathbf{b}_1, L(\mathbf{b}_2) = \mathbf{b}_2, L(\mathbf{b}_3) = -\mathbf{b}_3.$$

Let $L^5(\mathbf{x}) = L(L(L(L(L(\mathbf{x}))))))$. Suppose that, in standard coordinates (with respect to the standard basis of \mathbb{R}^3), $L^5(\mathbf{x}) = [y_1, y_2, y_3]^T \in \mathbb{R}^3$. Let \mathbf{A} be the matrix representation of L with respect to the ordered basis $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$. Which of the following statements are true? (Select all that apply.)

- (A) $y_1 = 33$.
- (B) $y_2 = 31$.
- (C) $y_3 = -3$.
- (D) $\det(\mathbf{A}) = -2$.
- (E) $\text{rank}(\mathbf{A}) = 2$.

7. Let $\mathbf{A} \in \mathbb{R}^{4 \times 4}$. Suppose the following ranks are given:

$$\text{rank}(\mathbf{A} - \mathbf{I}) = 3, \text{rank}(\mathbf{A} - 2\mathbf{I}) = 2, \text{rank}(\mathbf{A}^2 - \mathbf{I}) = 2.$$

Which of the following statements are true? (Select all that apply.)

- (A) \mathbf{A} is symmetric.
- (B) \mathbf{A} is not diagonalizable (over \mathbb{R}).
- (C) $\det(\mathbf{A}) = 4$.
- (D) $\text{trace}(\mathbf{A}) = 4$.
- (E) $\text{rank}(\mathbf{A}) = 4$.

8. Let $\mathbf{K} \in \mathbb{R}^{3 \times 3}$ and define a linear mapping $L: \mathbb{R}^{1 \times 3} \rightarrow \mathbb{R}^{1 \times 3}$ by $L(\mathbf{x}) = \mathbf{x}\mathbf{K}$. Let \mathbf{c} be such that the ordered set $E = \{\mathbf{c}, \mathbf{c}\mathbf{K}, \mathbf{c}\mathbf{K}^2\}$ is a basis of $\mathbb{R}^{1 \times 3}$. Suppose \mathbf{c} satisfies $\mathbf{c}\mathbf{K}^3 - 5\mathbf{c}\mathbf{K}^2 + 7\mathbf{c}\mathbf{K} - 9\mathbf{c} = \mathbf{0}$. Let \mathbf{A} be the matrix representation of L with respect to E . \mathbf{A} is defined by $[L(\mathbf{x})]_E = \mathbf{A}[\mathbf{x}]_E$, where $[\mathbf{x}]_E \in \mathbb{R}^3$ is the column coordinate vector of \mathbf{x} in the basis E . Denote the last two rows of \mathbf{A} by $[a_{21}, a_{22}, a_{23}]$ and $[a_{31}, a_{32}, a_{33}]$, respectively. Which of the following statements are true? (Select all that apply.)

- (A) $a_{22} = 0$, (B) $a_{23} = 1$, (C) $a_{31} = 0$, (D) $a_{32} = -7$, (E) $a_{33} = 5$.

國立中山大學 115 學年度碩士班考試入學招生考試試題

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9. Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ be nonzero vectors with $n \geq 2$ and $\mathbf{u}^T \mathbf{v} \neq 0$. Define

$$\mathbf{M} = \mathbf{I} - \frac{2}{\mathbf{u}^T \mathbf{v}} \mathbf{u} \mathbf{v}^T.$$

Which of the following statements are true? (Select all that apply.)

- (A) \mathbf{v} is an eigenvector of \mathbf{M} .
 - (B) 1 is an eigenvalue of \mathbf{M} .
 - (C) \mathbf{M} is invertible and $\mathbf{M}^{-1} = \mathbf{M}$.
 - (D) Every eigenvalue of \mathbf{M} is real.
 - (E) \mathbf{M} is symmetric.
10. Let $V = \mathbb{R}^{2 \times 2}$ with inner product $\langle \mathbf{A}, \mathbf{B} \rangle = \text{trace}(\mathbf{A}^T \mathbf{B})$ and define $\|\mathbf{A}\| = \sqrt{\langle \mathbf{A}, \mathbf{A} \rangle}$. Consider the following set $\{\mathbf{U}_1, \mathbf{U}_2, \mathbf{U}_3, \mathbf{U}_4\} \subset V$, which is orthogonal with respect to the given inner product:

$$\mathbf{U}_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \mathbf{U}_2 = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}, \mathbf{U}_3 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \mathbf{U}_4 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

Suppose $\mathbf{M} \in V$ satisfies

$$\langle \mathbf{M}, \mathbf{U}_1 \rangle = 8, \langle \mathbf{M}, \mathbf{U}_2 \rangle = -4, \langle \mathbf{M}, \mathbf{U}_3 \rangle = 0, \langle \mathbf{M}, \mathbf{U}_4 \rangle = 4.$$

Let \mathbf{P} be the orthogonal projection of \mathbf{M} onto $\text{span}\{\mathbf{U}_1, \mathbf{U}_2\}$. Which of the following statements are true? (Select all that apply.)

- (A) $\mathbf{P} = 2\mathbf{U}_1 - \mathbf{U}_2$.
 - (B) $\|\mathbf{P}\|^2 = 20$.
 - (C) $\mathbf{M} = 2\mathbf{U}_1 + \mathbf{U}_2 - \mathbf{U}_4$.
 - (D) $\|\mathbf{M}\|^2 = 20$.
 - (E) $\langle \mathbf{M}, \mathbf{P} \rangle = 24$.
11. Given the differential equation (DE) $(x^2 + x)y' + (xy - 1) = 0$, where x is the independent variable. Which of the following is/are INCORRECT?
- (A) This DE is nonlinear.
 - (B) This DE is nonhomogeneous.
 - (C) The solution of this DE is $y = \frac{\ln(x+1)}{x+1} + \frac{C}{x+1}$.
 - (D) The solution interval is $(-1, \infty)$.
 - (E) Provided the initial condition $y(e) = 1$, there exists a unique solution.
12. Given the differential equation (DE) $(x^2 + 2y^2)dx - (xy)dy = 0$, where x is the independent variable. Which of the following is/are INCORRECT?
- (A) This DE is exact.
 - (B) Consider $M(x, y) = x^2 + 2y^2$ and $N(x, y) = -(xy)$. M and N are homogeneous functions.
 - (C) One of the solutions of this DE is $x^4 = (x^2 + y^2)/2$, which is an explicit solution.
 - (D) Consider the initial condition $y(-1) = 1$, the solution interval is $(-\infty, \infty)$.
 - (E) There exists a singular point.

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共 5 頁第 4 頁

13. Given $X' = \begin{pmatrix} 12 & -9 \\ 4 & 0 \end{pmatrix} X$, which of the following is correct? Assume t is the independent variable.
- (A) There are two distinct eigenvalues.
 - (B) The stationary point is a repeller.
 - (C) All solutions will converge onto the origin.
 - (D) One of the eigenvalues is $\lambda_2 = 1$.
 - (E) One of the solution vectors is $K_1 = \begin{pmatrix} 3t+2 \\ 2t+1 \end{pmatrix} e^{-6t}$.
14. Consider the differential equation (DE) $y' = y^2 - 3y + 2$. Which of the following is/are correct?
- (A) There is an attractor.
 - (B) $y(x) = 2$ is a singular solution.
 - (C) $y(x) = (ce^x - 2)/(ce^x - 1)$ is a one-parameter family of solutions, where c is an arbitrary constant.
 - (D) Provided $y(0) = -1$, the largest interval is $[0, \infty)$.
 - (E) Given $y(0) = 4$, there exists an interval centered at 0 on which the DE has a unique solution.
15. Here is the differential equation $y' = y^2 e^{-x}$, with x being the independent variable. Which of the following is/are correct?
- (A) This DE is exact.
 - (B) There is no singular solution.
 - (C) The solutions $y(x) > 0$ or $y(x) < 0$.
 - (D) The solution of this DE is $y = \frac{1}{e^{-x} + c}$.
 - (E) Given the initial condition $y(0) = 1$, the solution interval is $(-\infty, \infty)$.
16. Which of the following is/are INCORRECT?
- (A) Consider $xy'' + (\sin x)y = 0$. $x = 0$ is a regular singular point.
 - (B) Consider $x^3y'' - xy' + y = 0$. $x = 0$ is a regular singular point.
 - (C) Consider $x^3y'' - xy' + y = 0$. There exists one solution that is analytic at $x = 0$.
 - (D) Consider $y'' + \ln(x+1)y' + y = 0$. $x = 0$ is a regular singular point.
 - (E) Consider $y'' + \ln(x+1)y' + y = 0$. The solutions centered at $x = 2$ must converge between $[-1, 1]$.
17. Given the differential equation (DE) $2xy'' + (1+x)y' + y = 0$, where x is the independent variable. On the way to finding infinite series solutions around $x = 0$, which of the following is/are correct?
- (A) There is no trivial solution.
 - (B) One of the indicial roots is 0.
 - (C) One of the solutions is a power series.
 - (D) The solution interval is $(0, \infty)$.
 - (E) Given the form of a solution $y = \sum \frac{A^n}{B^n C!} x^D$, where n is a nonnegative integer, $A + B = 1$.
18. Consider the differential equation (DE) $2y'' - 3y^2 = 0$. Which of the following is/are correct?
- (A) This DE is linear.
 - (B) This DE is autonomous.
 - (C) There exists a trivial solution.
 - (D) Provide $y(0) = 1$ and $y'(0) = 1$, the solution $y = \frac{4}{(x-2)^2}$.
 - (E) Provide $y(0) = 1$ and $y'(0) = 1$, the solution $y = \frac{4}{(x+2)^2}$.

國立中山大學 115 學年度碩士班考試入學招生考試試題

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題號：431001

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共 5 頁第 5 頁

19. Consider the differential equation (DE) $y'' - y = (e^x + e^{-x})/2$, where x is the independent variable. Provide $y(0) = 2$ and $y'(0) = 12$, which of the following is/are correct?
- (A) The particular solution is $[Ax (Be^x + Ce^{-x})]/D$, where $A + B + C + D = 5$.
 - (B) The particular solution is $[Ax (B \sinh x + C \cosh x)]/D$, where $A + B + C + D = 4$.
 - (C) The complementary function is $Ae^x + Be^{-x}$, where $A + B = 12$.
 - (D) The complementary function is $A \sinh x + B \cosh x$, where $A + B = 14$.
 - (E) The general solution is $Ae^x + Be^{-x}$, where $A + B = 2$.
20. Consider the differential equation (DE) $x^2 y'' - 4xy' + 6y = g(x)$, where x is the independent variable. Which of the following is/are correct?
- (A) If $g(x) = 0$, $y(-2) = 8$ and $y'(-2) = 0$, the general solution is $Af_1(x) + Bf_2(x)$, where $A + B = 8$. $f_1(x)$ and $f_2(x)$ are functions in x .
 - (B) If $g(x) = 0$, the solution interval is $(-\infty, \infty)$.
 - (C) If $g(x) = \ln(x^2)$, the solution interval is $(-\infty, \infty)$.
 - (D) If $g(x) = \ln(x^2)$, the particular solution is $Af(x) + B$, where $A + B = 11$.
 - (E) If $g(x) = \ln(x^2)$, $y(1) = 5/18$ and $y'(1) = -2/3$, the general solution is $Af_1(x) + Bf_2(x) + Cf_3(x) + D$, where $A + B + C + D = 8/9$.

國立中山大學 115 學年度 碩士班考試入學招生考試試題

科目名稱：控制系統【電機系碩士班乙組】

— 作答注意事項 —

考試時間：100 分鐘

- 考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- 答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請斟酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

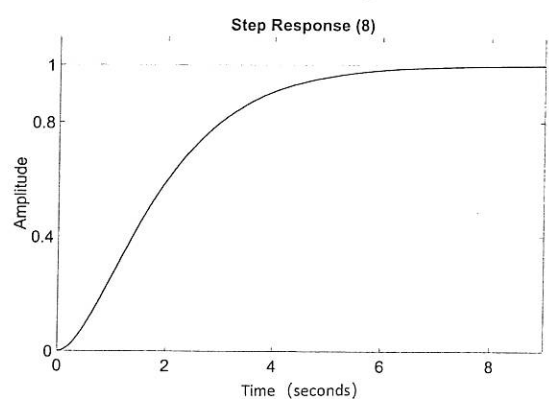
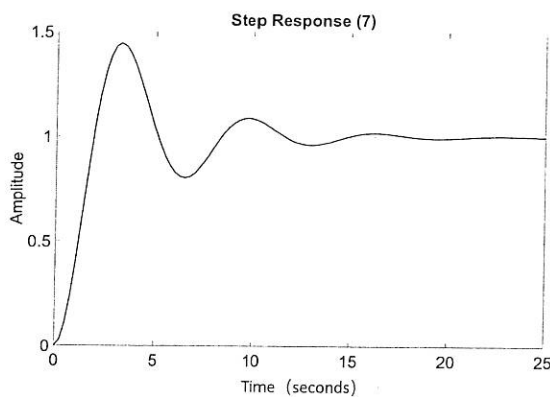
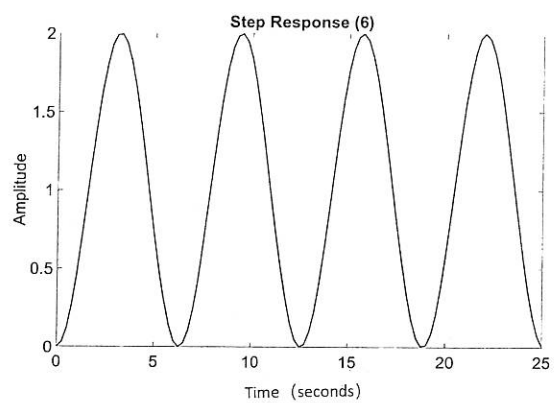
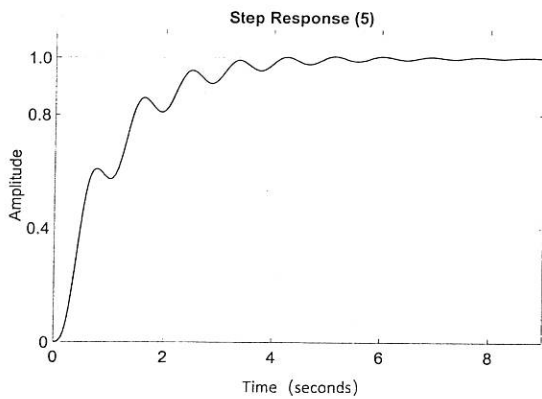
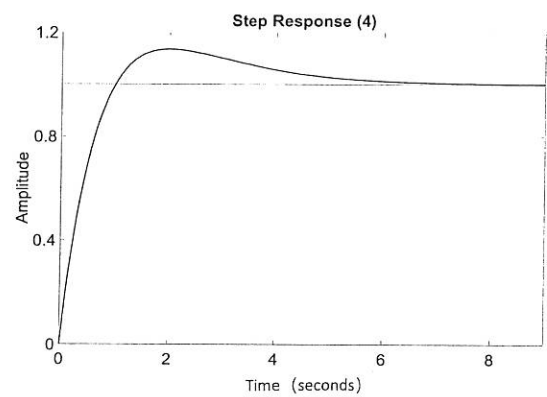
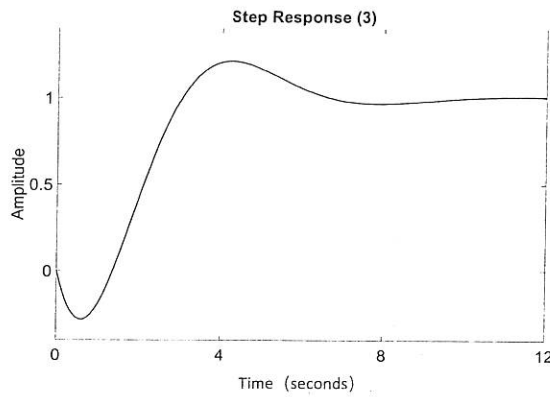
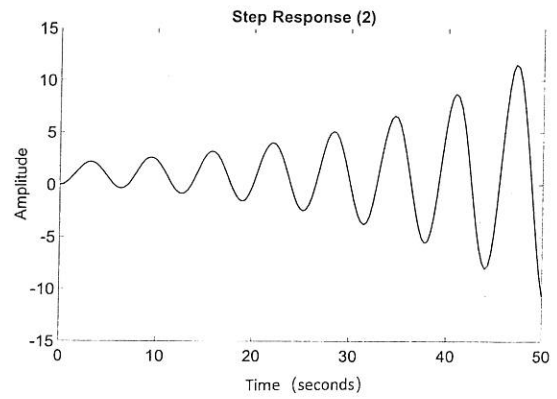
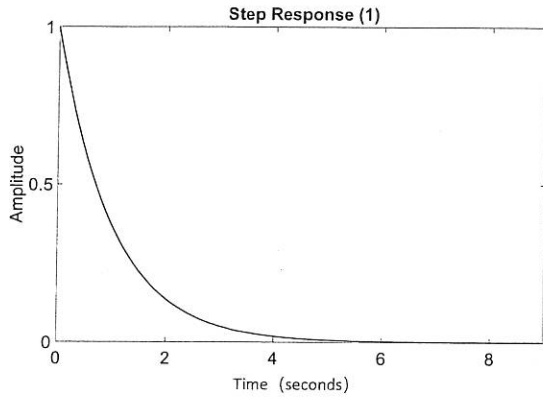
國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：控制系統【電機系碩士班乙組】

題號：431008

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題） 共 3 頁 第 1 頁

（問題一）（共 32 分）將下列步階響應圖與次頁之轉移函數配對，答對一題得 4 分。



試題請隨卷繳回，請留意背面是否有題

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- (A) $\frac{2}{s^2 + s - 2}$ (B) $\frac{s + 2}{s + 1}$ (C) $\frac{1}{s^2 + 2s + 1}$ (D) $\frac{s + 1}{s + 2}$ (E) $\frac{s}{s + 1}$ (F) $\frac{4}{s^2 + 5s + 2}$ (G) $\frac{1 - s}{s + 1}$
- (H) $\frac{s - 1}{s^2 + s + 2}$ (I) $\frac{1}{s^2 + 0.5s + 1}$ (J) $\frac{s + 1}{s}$ (K) $\frac{1}{s + 1}$ (L) $\frac{-s + 6}{(s + 1)(s^2 + 5s + 6)}$ (M) $\frac{s}{s^2 + 4}$
- (N) $\frac{1}{s^2 - 0.1s + 1}$ (O) $\frac{50}{(s + 1)(s^2 + s + 50)}$ (P) $\frac{1}{s^2 + 1}$ (Q) $\frac{s + 2}{s^2 + s + 1}$ (R) $\frac{2s + 1}{s^2 + 2s + 1}$
- (S) $\frac{10s + 1}{s^2 + 3s + 2}$ (T) $\frac{-s + 1}{s^2 + s + 1}$

(問題二) (共 48 分) 以下問題 (2a) 至 (2l) 考慮圖 Figure 2 之回授控制系統，其中 G 代表受控系統， C 代表控制器。請簡單回答問題並清楚標示答案對應之題號，無須提供計算或推理過程。答對一題得 4 分。

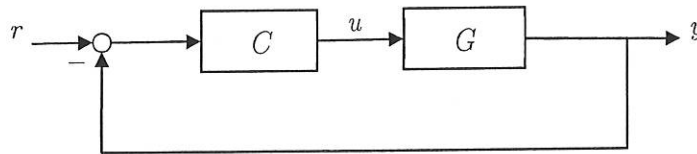
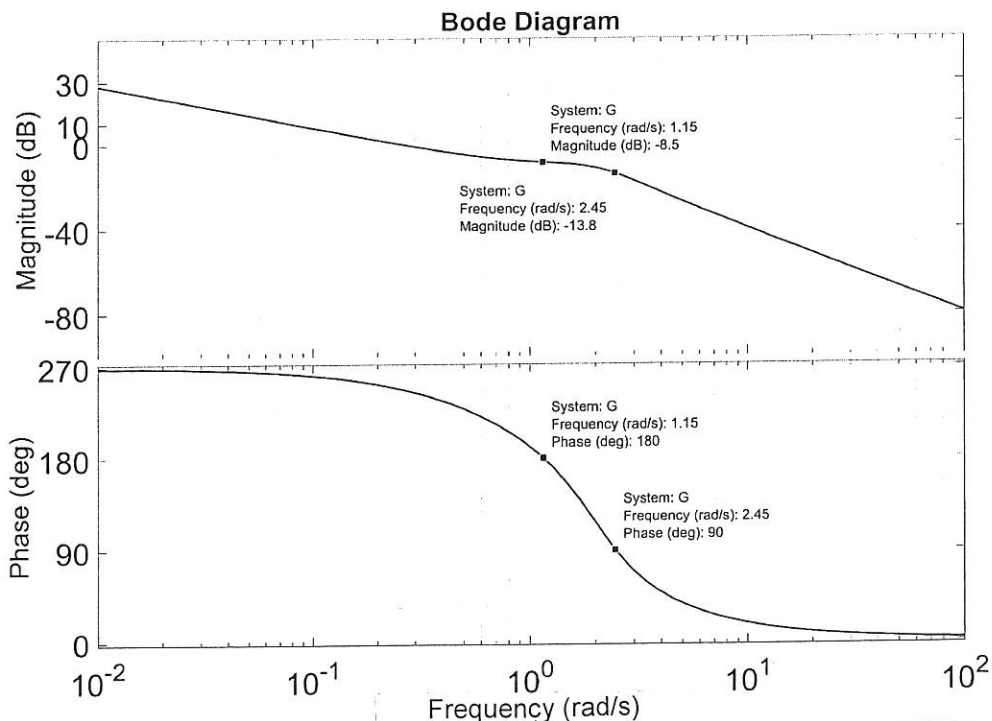


Figure 2: 回授控制系統

在以下問題(2a)至問題(2f)中，我們考慮一個三階的受控系統 G ，且該系統沒有極點在右半平面上；系統之 Bode plot 如下圖所示。



試題請隨卷繳回，請留意背面是否有題

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- (2a) 請問 G 有幾個極點在原點？
- (2b) 請問 G 的相對階數 (relative degree) 為何？
- (2c) 請問 G 有幾個零點在右半平面？（不計無窮大之零點）
- (2d) 令 C 為一個比例控制器（所謂的 P 控制器）。假設我們已知當 $C = 1$ 時閉迴路系統為穩定。請問此時 r 到 y 之轉移函數的頻寬大約為何？
- (2e) 承上題，請問讓閉迴路系統穩定的控制器的增益值範圍為何？
- (2f) 承上題，請問當閉迴路系統為穩定時，系統之單位步階響應的穩態誤差為何？

在以下問題(2g)至問題(2l)中，我們考慮一個受控系統 G ，其動態由下列之微分方程式所支配

$$\ddot{y}(t) + a_1\dot{y}(t) + a_0y(t) = b\dot{u}(t - \tau) + u(t - \tau)$$

其中 a_0, a_1, b, τ 為常數且 $\tau \geq 0$ 。

- (2g) 當 $b = 0$ 時，請問 a_0 與 a_1 需滿足何種條件方能使 G 為穩定且其步階響應無震盪現象？
- (2h) 當 $\tau \neq 0, b \geq 0$ ，且 G 為穩定時，請問 G 之增益邊界 (gain margin) 可否為無窮大？
- (2i) 假設 $\tau = 0$ 且控制器 C 為比例控制器（這裡我們僅考慮正增益值）。當 G 為極小相位系統時，請問 C 之增益是否可為任意值而閉迴路系統皆為穩定？
- (2j) 假設 $a_0 = a_1 = 0$ 且 $\tau \neq 0$ 。當 C 為比例控制器（這裡我們僅考慮正增益值）時，請問 b 需滿足何種條件閉迴路系統才有可能穩定？
- (2k) 假設 $a_0 = a_1 = b = 0$ 且 $\tau \neq 0$ 。請問下列哪些控制器有可能讓閉迴路系統穩定？
(1) I 控制器 (2) D 控制器 (3) PI 控制器 (4) PD 控制器 (5) PID 控制器
- (2l) 假設 $b = \tau = 0$ ，控制器 C 為一階且恰適 (proper)，並且已知閉迴路系統為穩定。請問在這些條件下，閉迴路系統是否有可能完美追蹤（即：達成零穩態誤差）下列訊號？
(1) 步階訊號 (2) 斜坡訊號 (3) 拋物線訊號

(問題三)（共 20 分）回答以下問題，且答案需包含嚴謹的說明、推導過程。只有答案但無說明或推導過程者，不予計分。

一個受控系統 G 的輸入 u 與輸出 y 之間有以下的關係：

$$y(t) = \int_0^t e^{-(t-\tau)} \cos(t-\tau) u(\tau) d\tau, \quad t \geq 0$$

- (3a)（10分）請問 G 對哪一個頻率的弦波訊號造成最大的振幅放大？
- (3b)（10分）針對 G 設計一（負）回授控制器 C ，使閉迴路系統之步階響應的穩態誤差為零。

【試題結束】

國立中山大學 115 學年度 碩士班考試入學招生考試試題

科目名稱：資料結構【電機系碩士班丙組】

— 作答注意事項 —

考試時間：100 分鐘

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- 答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請斟酌作答。
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- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
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- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：資料結構【電機系碩士班丙組】

題號：431004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題） 共 2 頁第 1 頁

1. 【此題 30 分；每小題 15 分】

舉凡可以儲存大量資料的資料結構皆可稱為「容器 (container)」，例：陣列 (array) 就是一種容器。假設陣列的容量 (capacity) 為 n ，裡頭有 $k < n$ 個元素；當 k 變小時，搜尋元素的平均時間就會減少。

Ⓐ 然而，當我們用 open addressing 的方式實作 hash table 時，卻可能發生裡頭的元素數量 k 減少時，搜尋元素的平均時間未必減少，請問這是為什麼？請舉例解說。

Ⓑ 請提出至少一種具體的方法來減緩上述問題。
2. 【此題 30 分；每小題 15 分】給定陣列 A ，容量為 n ，裡頭有 n 個正整數。我們打算用 insertion sort 對陣列 A 排序；並且我們要求排序的結果是陣列裡的元素「由小到大」排列。

Ⓐ 假設陣列 A 是「 k -亂序 (k -displaced)」，亦即每個元素距離其排序後的最終位置不超過 k 個索引值，其中 k 為常數。請問在這種情況下，insertion sort 的執行時間 $T(n)$ 為何？用 big- O 的方式表示。Ⓑ 考慮另一種情況：假設 n 為 3 的倍數，並且在陣列 A 裡頭，元素數值前三分之一大的元素全部落在 $A[0]$ 至 $A[(n/3)-1]$ 裡頭。請問在這種情況下，insertion sort 的執行時間 $T(n)$ 為何？用 big- O 的方式表示。

註 1：此題必須寫出推論過程；直接寫答案，此題 0 分。註 2：用 $O(f(n))$ 表示執行時間時， $f(n)$ 必須同時兼具 simplest 與 tightest 的型式。Tightest 的型式是指：若 $T(n)$ 為 $O(f(n))$ ，那麼 $T(n)$ 就不能寫成 $O(g(n))$ ，其中 $\lim_{n \rightarrow \infty} f(n)/g(n) = 0$ 。
3. 【此題 15 分】給定陣列 $A = \{3, 2, 1, 6, 5, 7, 9, 11, 14, 18\}$ ，請問執行函式 build_max_heap(A) 之後，陣列 A 的內容為何？註：給定陣列 A ，build_max_heap 函式的功能是将陣列 A 調整成 max-heap。此題必須有推導過程；直接寫答案，此題 0 分。
4. 【此題 15 分】一個 graph 是 simple graph，表示在該 graph 裡頭，任二點之間最多只有一條 edge 相連。小明學完資料結構課程之後，想到使用 divide-and-conquer 策略在 simple weighted connected graph G 上面尋找 minimum cost spanning tree T 的 recursive 方法，如下：令 V 和 E 分別表示 G 的 vertex set 和 edge set。集合的絕對值表示集合裡的元素數量。如果 $1 \leq |V| \leq 2$ ，那麼 T 等於 G ；否則將 V 分成二個沒有交集的子集合 V_1 和 V_2 ，並使得 $|V_1|$ 和 $|V_2|$ 最多相差 1。令 $E_1 = \{(u, v) \in E \mid u, v \in V_1\}$ 且 $E_2 = \{(u, v) \in E \mid u, v \in V_2\}$ 。假設 E_1 和 E_2 裡的 minimum cost spanning tree 分別為 T_1 和 T_2 。令 $E_3 = \{(u, v) \in E \mid u \in V_1 \text{ 且 } v \in V_2\}$ ；令 (α, β) 表示 E_3 裡頭 weight 最小的邊。那麼 G 裡頭的 minimum cost spanning tree T 為 $T_1 \cup T_2 \cup (\alpha, \beta)$ 。請問：小明這個尋找 minimum cost spanning tree T 的 recursive 方法是否正確？如果你認為小明的方法正確，寫「正確」即可，不必解釋理由；如果你認為小明的方法錯誤，請舉出反例，解說小明的方法為何有錯。如果你認為小明的方法有錯，但舉不出反例，此題只給 3 分。

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：資料結構【電機系碩士班丙組】

題號：431004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題） 共 2 頁第 2 頁

5. 【此題 10 分】假設有一棵 binary search tree，裡頭每個 node 代表一筆資料的 key 值，且 key 值皆為介於 100~1000 之間的正整數。假設我們想在這個 binary search tree 裡頭尋找 key 值為 350 的資料，那麼下列檢查次序，哪些是不可能發生的？找出答案，並解釋理由。
- Ⓐ 137→250→430→380→324→365→350
 - Ⓑ 820→125→765→269→542→311→350
 - Ⓒ 127→633→278→423→251→365→350
 - Ⓓ 935→198→430→248→521→264→350

國立中山大學 115 學年度 碩士班考試入學招生考試試題

科目名稱：離散數學【電機系碩士班丙組】

— 作答注意事項 —

考試時間：100 分鐘

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國立中山大學 115 學年度碩士班考試入學招生考試試題

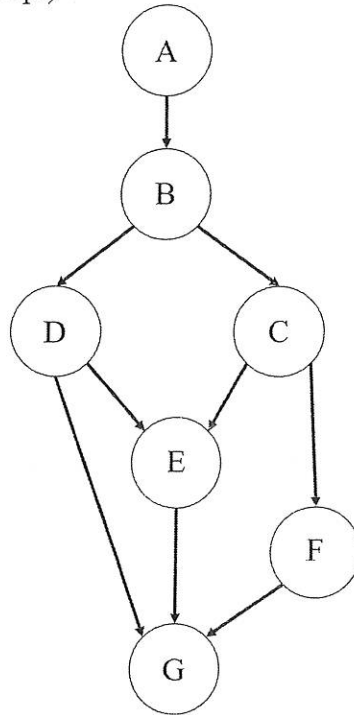
科目名稱：離散數學【電機系碩士班丙組】

題號：431010

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 2 頁第 1 頁

1. What is the probability that, among all possible topological sort starting from node A, node F is visited before node E while nodes F and E are not visited consecutively? Please justify your answer to receive full credit. Failure to provide justification will result in 0 points. Additionally, write your final answer at the leftmost part of the answer paper using “Ans:” on a new separate line. Failure to follow this instruction will result in a penalty of -5 points. (10pt)



2. Select any 2 digits from 1,3,5,7, and 9. Then, select any 2 digits from 0,2,4, and 6. How many distinct four-digit numbers can be formed using these digits without repetition? Please justify your answer to receive full credit. Failure to provide justification will result in 0 points. Additionally, write your final answer at the leftmost part of the answer paper using “Ans:” on a new separate line. Failure to follow this instruction will result in a penalty of -5 points. (20pt)

3. Given $x = 19$, what is $2^{2^{17}} \bmod x$? Please justify your answer to receive full credit. Failure to provide justification will result in 0 points. Additionally, write your final answer at the leftmost part of the answer paper using “Ans:” on a new separate line. Failure to follow this instruction will result in a penalty of -5 points. (20pt)

4. Let R be a ring. If there exists a positive integer n such that $na = 0$ for all $a \in R$, then the smallest positive integer n is called the characteristic of R . If no such positive integer exists, R has characteristic zero. The characteristic of R is denoted by $C(R)$. Let R be a commutative ring with identity, and let $C(R) = 3$. Compute and simplify $(a+b)^9$, $b \in R$. Please justify your answer to receive full credit. Failure to provide justification will result in 0 points. Additionally, write your final answer at the leftmost part of the answer paper using “Ans:” on a new separate line. Failure to follow this instruction will result in a penalty of -5 points. (20pt)

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：離散數學【電機系碩士班丙組】

題號：431010

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 2 頁第 2 頁

5. Compute $C_m^7 + C_{m+1}^7 + C_{m+2}^8 + C_{m+3}^9 + C_{m+4}^{10}$ if $(C_m^5)^{-1} - (C_m^6)^{-1} = 0.7 \times (C_m^7)^{-1}$. Please justify your answer to receive full credit. Failure to provide justification will result in 0 points. Additionally, write your final answer at the leftmost part of the answer paper using “Ans:” on a new separate line. Failure to follow this instruction will result in a penalty of -5 points. (10pt)

6. Find all the solutions in positive integer of the equation $x^n + y^n = z^n$, where $n > 1$. Please justify your answer to receive full credit. Failure to provide justification will result in 0 points. Additionally, write your final answer at the leftmost part of the answer paper using “Ans:” on a new separate line. Failure to follow this instruction will result in a penalty of -5 points. (10pt)

7. Find all the solutions in prime of the equation $p^3 = p^2 + q^2 + r^2$. Please justify your answer to receive full credit. Failure to provide justification will result in 0 points. Additionally, write your final answer at the leftmost part of the answer paper using “Ans:” on a new separate line. Failure to follow this instruction will result in a penalty of -5 points. (10pt)

國立中山大學 115 學年度 碩士班考試入學招生考試試題

科目名稱：電路學【電機系碩士班丁組】

— 作答注意事項 —

考試時間：100 分鐘

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國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：電路學【電機系碩士班丁組】

題號：431012

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題） 共 2 頁第 1 頁

1. (12 pt) Voltage divider depicted in Figure. 1 is commonly applied as an adjustable voltage source. The ideal output voltage is equated as (1). However, in fact there is power dissipation at output. Assuming that the current consumption I_o is fixed at 1 mA and $1\text{ V} < V_o < 11\text{ V}$, but the user still expects that output V_o is proportional to R_{bc}/R_{ac} . Suggest maximum R_{ac} such that the output voltage error ($|V_{o_ideal} - V_{o_real}|/V_{o_ideal}$) is lower than 2%.

$$V_{o_ideal} = 12 \times \frac{R_{bc}}{R_{ac}} \text{ V} \quad (1)$$

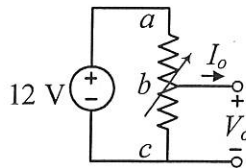


Figure. 1

2. (8 pt) In the circuit of Figure. 2, find the voltage v_a and current I_o .

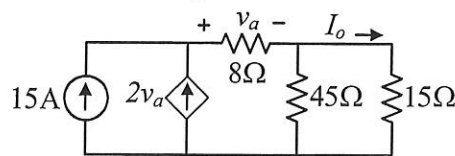


Figure. 2

3. (12 pt) Derive the output voltage v_o in the circuit of Figure. 3. Besides, assuming ideal output resistance of op amp is zero, find the input resistance R_{in} .

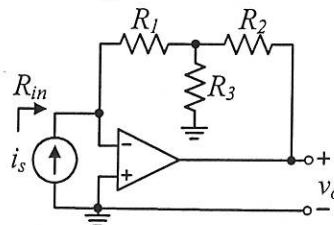


Figure. 3

4. (8 pt) For the circuit in Figure.4, determine the resistance r such that the maximum power transferred to load R_L is 12 mW. Note that please **use source transformation** to solve the problem!

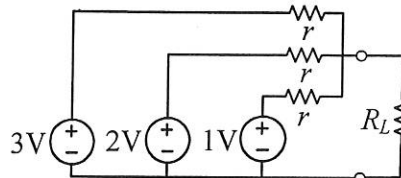


Figure. 4

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：電路學【電機系碩士班丁組】

題號：431012

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題） 共 2 頁第 2 頁

5. (10 pt) Find i_1 and i_2 in the circuit of Figure.5 for $t \geq 0$.

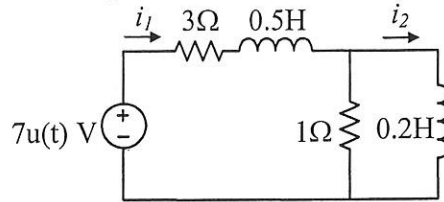


Figure. 5

6. (10 pt) A series RLC passive filter with $R = 1 \Omega$, $L = 1 \text{ H}$, and $C = 1 \text{ F}$. Using Scaling to calculate new values of R and L to accomplish the same quality factor but the center frequency at 1 kHz. Use a 10uF capacitance value.
7. (10 pt) A series RL low-pass filter with a cutoff frequency of 10 kHz. Using $R = 10 \text{ k}\Omega$, compute the inductance value and the voltage gain of the filter at 50 kHz.
8. (10 pt) A load works with the following voltage and current:
 $v(t) = 100\cos(50t) \text{ V}$
 $i(t) = 10\sin(50t + 30^\circ) + 10\cos(250t - 45^\circ) \text{ A}$
 Please find:
 (a) average power of the load. (5%)
 (b) power factor of the load. (5%)
9. (10 pt) A balanced three-phase power system supplies two loads in parallel connection: a wye-connected load consuming 12 kW at 0.886 power factor lagging and a delta-connected load consuming 10 kVA at 0.6 power factor leading. If line voltage is 220 V_{rms}, find line current and power factor of the combined loads.
10. (10 pt) Recently, a power system with 800V HVDC architecture was announced for the next generation high-power AI servers. 800V HVDC is generally supplied from a medium-voltage AC system around 11 kV or 22 kV by using the so-called solid-state transformer. Please explain and discuss the solid-state transformer compared with the traditional line-frequency transformer, including circuit topologies and efficiency.

國立中山大學 115 學年度 碩士班考試入學招生考試試題

科目名稱：通訊理論【電機系碩士班戊組選考、通訊所碩士班甲組、乙組選考、電波聯合碩士班選考】

— 作答注意事項 —

考試時間：100 分鐘

- 考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- 答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：通訊理論【電機系碩士班戊組選考、通訊所碩士班甲組、乙組選考、電波聯合碩士班選考】題號：437002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）

共 4 頁第 1 頁

一、選擇題(單選，計分方式:不倒扣，答對得該題全部分數，答錯及未作答得零分)

1. (5%) Suppose a communication channel has bandwidth W . Let the symbol (sampling) interval be $t_s = \frac{1}{W}$. Which of the following pulse waveforms does not satisfy the ISI-free (Nyquist) criterion?
 - (A) $p_1(t) = 2W \text{sinc}^2(2Wt) + W \text{sinc}(Wt)$
 - (B) $p_2(t) = 4W \text{sinc}(4Wt) - W \text{sinc}^2(Wt)$
 - (C) $p_3(t) = 2W \text{sinc}(Wt) \cos(2\pi Wt)$
 - (D) $p_4(t) = 2W \text{sinc}^2(2Wt) - W \text{sinc}^2(Wt)$
 - (E) None of the above

2. (5%) An M -ary communication system transmits at a rate of 2000 symbols per second. What is the equivalent bit rate in bits per second for $M = 16$?
 - (A) 4000
 - (B) 6000
 - (C) 8000
 - (D) 10000
 - (E) None of the above

3. (5%) Consider a 3-ary communication system in which each transmitted message is chosen from one of three symbols, m_{-1} , m_0 , and m_1 . These symbols are transmitted using the waveforms $-p(t)$, 0 , and $p(t)$, respectively, where the pulse $p(t)$ has duration T_M . At the receiver, a matched filter matched to $p(t)$ is employed. Let r denote the output of the matched filter sampled at time T_M . Assume that the messages are equiprobable, i.e., $P(m_{-1}) = P(m_0) = P(m_1)$. The energy of the pulse $p(t)$ is E_p and the channel noise is additive white Gaussian noise (AWGN) with two-sided power spectral density $N_0/2$. Determine the symbol error probability P_e using the optimum decision thresholds. (The Q-function is defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-z^2/2} dz$.)
 - (A) $P_e = \frac{4}{3} Q\left(\sqrt{\frac{E_p}{2N_0}}\right)$
 - (B) $P_e = \frac{1}{3} Q\left(\sqrt{\frac{E_p}{2N_0}}\right)$
 - (C) $P_e = Q\left(\sqrt{\frac{3E_p}{2N_0}}\right)$
 - (D) $P_e = Q\left(\sqrt{\frac{E_p}{N_0}}\right)$
 - (E) None of the above

4. (5%) Let $g(t)$ be a signal with Fourier transform $G(f)$. Define $G_1(t)$ as the inverse Fourier transform of the frequency-domain function $g(af - f_0)$. Which of the following expressions correctly gives $G_1(t)$?
 - (A) If $a > 0$, $G_1(t) = \frac{1}{a} G(-t/a) e^{j2\pi f_0 t/a}$
 - (B) If $a < 0$, $G_1(t) = \frac{1}{a} G(t/a) e^{j2\pi f_0 t/a}$
 - (C) If $a = 0$, $G_1(t) = g(f_0) \delta(t)$

試題請隨卷繳回，請留意背面是否有題

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：通訊理論【電機系碩士班戊組選考、通訊所碩士班甲組、乙組選考、電波聯合碩士班選考】題號：437002

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- (D) If $a = 0$, $G_1(t) = g(1/f_0)\delta(t)$
 (E) None of the above

5. (5%) Consider a narrowband FM signal that can be approximated by

$$s(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_m t) \sin(2\pi f_c t),$$

where A_c is the carrier amplitude, f_c is the carrier frequency, f_m is the modulating frequency, and $\beta \ll 1$ is the modulation index. Which of the following statements is correct?

- (A) The minimum value of the signal envelope is $A_c \sqrt{1 - \beta^2}$
 (B) The maximum value of the signal envelope is $A_c \sqrt{1 + \beta^2}$
 (C) The total average power of the FM is $\frac{(1+\beta^2)A_c^2}{2}$
 (D) The total average power of the FM is $\frac{(1+\beta^2)A_c^2}{4}$
 (E) None of the above

6. (5%) Consider a signal

$$x(t) = \begin{cases} 9 - t^2, & |t| \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

with its Fourier transform $X(f)$. Which of the following statements is wrong?

- (A) The imaginary part of $X(f)$ is zero.
 (B) The value of $\int_{-\infty}^{\infty} X(f) df$ is 9.
 (C) The value of $X(0)$ is 36.
 (D) The value of $\int_{-\infty}^{\infty} |X(f)|^2 df$ is 324.
 (E) The value of $\int_{-\infty}^{\infty} f \cdot X(f) df$ is 0.

7. (5%) The signals $x_i(t)$ undergo sampling with sampling period $T_s = 0.02$ second. Which of the following signals can be recovered perfectly from the sampled signal.

$$\begin{aligned} x_1(t) &= \frac{\sin(40\pi t)}{\pi t}, \\ x_2(t) &= \frac{\sin(40\pi t)}{\pi t} \cdot \frac{\sin(20\pi t)}{\pi t}, \\ x_3(t) &= t x_2(t), \\ x_4(t) &= x_1(t) \cdot \cos(20\pi t), \\ x_5(t) &= \frac{d}{dt} x_1(t), \end{aligned}$$

- (A) $x_1(t), x_3(t)$.
 (B) $x_1(t), x_5(t)$
 (C) $x_2(t), x_4(t)$.
 (D) $x_1(t), x_3(t), x_5(t)$.
 (E) $x_3(t), x_4(t), x_5(t)$.

8. (5%) Let $X(t) = A \cdot \cos(2\pi f_c t + \Theta)$ be a real-valued random process, where A is a constant, Θ is a random variable uniformly distributed between 0 and 2π . Which of the following is wrong.

- (A) $X(t)$ is wide-sense stationary (WSS).
 (B) Mean value of $X(t)$ is zero
 (C) Autocorrelation function of $X(t)$ is $R_X(\tau) = \frac{A^2}{2} \cos(2\pi f_c \tau)$
 (D) Power spectral density function of $X(t)$ is $S_X(f) = \frac{A^2}{4} [\delta(f - f_c) + \delta(f + f_c)]$
 (E) Average power of $X(t)$ is A^2

國立中山大學 115 學年度碩士班考試入學招生考試試題

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共 4 頁 第 3 頁

9. (5%) Consider an amplitude-modulated signal expressed by

$$s(t) = A_c(1 + k_a m(t)) \cos(2\pi f_c t).$$

Which of the following is wrong.

- (A) The amplitude sensitivity k_a can be any positive real number
- (B) The message $m(t)$ can be recovered from an envelop detector
- (C) We may apply DSB-SC (double sideband- suppress subcarrier) to reduce transmission power
- (D) We may apply SSB (single sideband) scheme to reduce required transmission bandwidth
- (E) The transmission bandwidth of VSB(vestigial-sideband) is between those of the SSB and DSB-SC.

10. (5%) Consider an Frequency-modulated signal expressed by

$$s(t) = A_c \cos \left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right],$$

where the message $m(t)$ has the maximum amplitude of A_m and the highest frequency F_m . Which of the following is wrong.

- (A) The instantaneous frequency of $s(t)$ is $f_c + k_f m(t)$
- (B) The transmission power of is $A_c^2/2$
- (C) If we connect an FM modulator with an integrator, we can generate a phase-modulated signal
- (D) The maximum frequency deviation is $k_f A_m$
- (E) The transmission bandwidth of $s(t)$ is approximately $2k_f A_m + 2F_m$

二、問答計算題：

1. (10%) Consider the following baseband signal $g(t)$:

$$g(t) = \begin{cases} t/2, & 0 \leq t < T \\ 0, & \text{otherwise} \end{cases}$$

- (A)(5%) Determine the impulse response of a filter $h(t)$ matched to the signal.
- (B)(5%) Plot the matched filter output $g_0(t) = g(t) * h(t)$ in the absence of noise.

2. (15%) Consider the frequency demodulation scheme shown as Fig.1, where the incoming FM signal is given by

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)].$$

The delay line produces a delayed signal $s(t - T)$, and leads to a phase-shift of $\pi/2$ radians at the carrier frequency f_c , i.e., $2\pi f_c T = \pi/2$. Assume that $\beta < 1$ and T is sufficiently small, s.t.

$$\begin{aligned} \cos(2\pi f_m T) &\approx 1 \\ \sin(2\pi f_m T) &\approx 2\pi f_m T. \end{aligned}$$

Find the output signal of the demodulator.

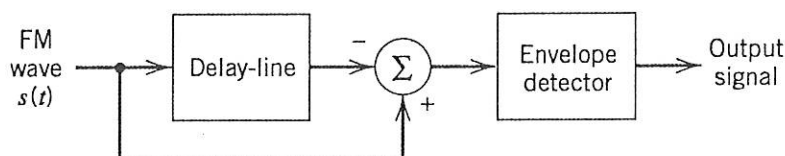


Fig.1 Frequency demodulator

國立中山大學 115 學年度碩士班考試入學招生考試試題

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3. (10%) Consider antipodal signaling with amplitude imbalance. That is, a logic-1 symbol is transmitted as a rectangular pulse of amplitude A_1 , and duration T , and a logic-0 symbol is transmitted as a rectangular pulse of amplitude $-A_2$, where $A_1 \geq A_2 > 0$. The receiver employs a fixed decision threshold at zero. Define the ratio $\rho = A_2/A_1$, and note that the average signal energy, for equally likely 1s and 0s, is

$$E = \frac{A_1^2 + A_2^2}{2} T.$$

The channel noise is additive white Gaussian noise (AWGN) with two-sided power spectral density $N_0/2$. Show that the resulting probability of bit error can be expressed as

$$P_e = \frac{1}{2} Q \left(\sqrt{\frac{2}{1 + \rho^2} \frac{2E}{N_0}} \right) + \frac{1}{2} Q \left(\sqrt{\frac{2\rho^2}{1 + \rho^2} \frac{2E}{N_0}} \right).$$

4. (15%) Consider a real-valued message signal $m(t)$ whose spectrum is nonzero over the frequency range 200 Hz to 3 kHz, as illustrated in Fig. 2. This message is SSB modulated to produce the transmitted signal

$$s(t) = A_c m(t) \cos(2\pi f_c t) + A_c \hat{m}(t) \sin(2\pi f_c t),$$

Where $\hat{m}(t)$ denotes the Hilbert transform of $m(t)$. At the receiver, $s(t)$ is demodulated using a carrier of the form $\cos(2\pi(f_c + \Delta f)t)$. Assume that the lowpass filter is ideal with unity gain and extends over $[-4k, 4k]$ Hz, and $f_c = 20$ kHz.

- (A) (5%) Derive the expression for the demodulated signal at the output of the lowpass filter.
- (B) (5%) Plot the spectrum of the demodulated signal when $\Delta f = 20$ Hz.
- (C) (5%) Plot the spectrum of the demodulated signal when $\Delta f = -10$ Hz.

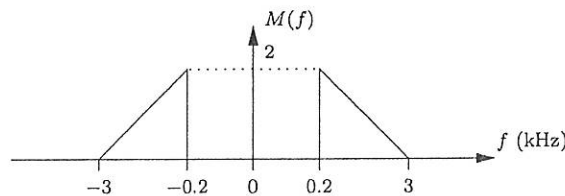


Fig.2 Message Spectrum

國立中山大學 115 學年度 碩士班考試入學招生考試試題

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—作答注意事項—

考試時間：100 分鐘

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- 答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：工程數學甲【電機系碩士班戊組選考、庚組、通訊所碩士班乙組選考、電波聯合碩士班選考】題號：431002

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下面 1-10 題為複選題，每題 5 分，總分 50 分。每題有五個選項，其中至少有一個是正確答案。答錯 1 個選項者，得 3 分，答錯 2 個選項者，得 1 分，答錯多於 2 個選項或未作答者，該題以零分計算。

1. Let $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{d} \in \mathbb{R}^n$. Define the matrices

$$\mathbf{B} = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3] \in \mathbb{R}^{n \times 3}, \quad \mathbf{C} = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4] \in \mathbb{R}^{n \times 4}.$$

Assume that the linear system $\mathbf{B}\mathbf{x} = \mathbf{d}$ has no solution for $\mathbf{x} \in \mathbb{R}^3$, and the linear system $\mathbf{C}\mathbf{y} = \mathbf{d}$ has at least one solution for $\mathbf{y} \in \mathbb{R}^4$. Let $[\mathbf{B}, \mathbf{d}] = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{d}] \in \mathbb{R}^{n \times 4}$. Which of the following statements are true? (Select all that apply.)

- (A) The set $\{\mathbf{a}_4, \mathbf{d}\}$ is linearly dependent.
- (B) The set $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{d}\}$ is linearly dependent.
- (C) The set $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$ is linearly independent.
- (D) $\text{rank}(\mathbf{C}) > \text{rank}(\mathbf{B})$.
- (E) $\text{rank}(\mathbf{C}) = \text{rank}([\mathbf{B}, \mathbf{d}])$.

2. Let $\mathbf{A} \in \mathbb{R}^{m \times n}$. For any matrix \mathbf{M} , let $R(\mathbf{M})$ denote the column space and $N(\mathbf{M})$ denote the null space. Which of the following statements are true? (Select all that apply.)

- (A) If $\mathbf{y} \in R(\mathbf{A}\mathbf{A}^T)$ and $\mathbf{y} \neq \mathbf{0}$, then $\mathbf{A}^T\mathbf{y} \neq \mathbf{0}$.
- (B) If $\mathbf{y} \in R(\mathbf{A})$, then $\mathbf{A}^T\mathbf{y} = \mathbf{0}$.
- (C) If $m = n$, then $R(\mathbf{A}) = R(\mathbf{A}^T)$.
- (D) If $m = n$, then $N(\mathbf{A}) = N(\mathbf{A}^T)$.
- (E) If $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{A}\mathbf{x} \neq \mathbf{0}$, then $\mathbf{x} \in R(\mathbf{A}^T)$.

3. Let $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathbf{B} \in \mathbb{R}^{m \times k}$, and $\mathbf{C} \in \mathbb{R}^{k \times n}$ such that $\mathbf{A} = \mathbf{B}\mathbf{C}$. For any matrix \mathbf{M} , let $R(\mathbf{M})$ denote the column space and $N(\mathbf{M})$ denote the null space. Which of the following statements are true? (Select all that apply.)

- (A) $R(\mathbf{B})$ is a subset of $R(\mathbf{A})$.
- (B) $N(\mathbf{A})$ is a subset of $N(\mathbf{C})$.
- (C) $k \geq \text{rank}(\mathbf{A})$.
- (D) If $k > \text{rank}(\mathbf{A})$, then $\text{rank}(\mathbf{B}) = \text{rank}(\mathbf{C})$.
- (E) If $k = \text{rank}(\mathbf{A})$, then $\text{rank}(\mathbf{A}) = \text{rank}(\mathbf{B})$.

4. Let

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & 3 & 0 & 3 \\ 0 & 2 & 0 & 2 & 0 \\ 3 & 0 & 3 & 0 & 3 \\ 0 & 2 & 0 & 2 & 0 \\ 3 & 0 & 3 & 0 & 3 \end{bmatrix}.$$

Which of the following statements are true? (Select all that apply.)

- (A) \mathbf{A} is diagonalizable (over \mathbb{R}).
- (B) $\text{rank}(\mathbf{A}) = 2$.
- (C) 2 is an eigenvalue of \mathbf{A} .
- (D) 3 is an eigenvalue of \mathbf{A} .
- (E) The number of positive eigenvalues of \mathbf{A} is 2.

國立中山大學 115 學年度碩士班考試入學招生考試試題

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5. Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix},$$

and define the subspace $W = \text{span}\{\mathbf{v}_1, \mathbf{v}_2\} \subset \mathbb{R}^3$. Let \mathbf{p} be the orthogonal projection of \mathbf{x} onto W . For any vector \mathbf{u} , let $\|\mathbf{u}\|$ denote the Euclidean norm: $\|\mathbf{u}\| = \sqrt{\mathbf{u}^T \mathbf{u}}$. Which of the following statements are true? (Select all that apply.)

- (A) $\|\mathbf{p}\| = 2$.
- (B) $\|\mathbf{x} - \mathbf{p}\| = 3$.
- (C) $\mathbf{w}^T(\mathbf{x} - \mathbf{p}) = 0$ for every $\mathbf{w} \in W$.
- (D) $\mathbf{x} - \mathbf{p} \in W$.
- (E) \mathbf{p} is orthogonal to \mathbf{v}_1 .

6. Let

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \mathbf{b}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}.$$

Define a linear transformation $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ satisfying

$$L(\mathbf{b}_1) = 2\mathbf{b}_1, L(\mathbf{b}_2) = \mathbf{b}_2, L(\mathbf{b}_3) = -\mathbf{b}_3.$$

Let $L^5(\mathbf{x}) = L(L(L(L(L(\mathbf{x})))))$. Suppose that, in standard coordinates (with respect to the standard basis of \mathbb{R}^3), $L^5(\mathbf{x}) = [y_1, y_2, y_3]^T \in \mathbb{R}^3$. Let \mathbf{A} be the matrix representation of L with respect to the ordered basis $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$. Which of the following statements are true? (Select all that apply.)

- (A) $y_1 = 33$.
- (B) $y_2 = 31$.
- (C) $y_3 = -3$.
- (D) $\det(\mathbf{A}) = -2$.
- (E) $\text{rank}(\mathbf{A}) = 2$.

7. Consider the surface $z = x^2 + y^2$ at the point $P(2, -2, 8)$. Which of the following is/are correct? Assume t below is a scalar. The $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors along the x -, y - and z -axis, respectively.

- (A) The gradient vector is $2x\mathbf{i} + 2y\mathbf{j} - \mathbf{k}$.
- (B) The tangent plane equals $4x - 4y - z = 8$.
- (C) The normal vector to the surface at P is $4\mathbf{i} - 4\mathbf{j} - 8\mathbf{k}$.
- (D) The greatest rate of increase in the direction of the normal vector is $\sqrt{32}$.
- (E) The parametric equations of the normal line are $x = 2 + 4t, y = -2 - 4t, z = 8 - t$.

8. Consider a space curve defined by the intersection of the surface $z = \cot^{-1}(x/y)$ and the cylinder $x^2 + y^2 = 1$. Assume \cot^{-1} denotes the principal value with the range $(0, \pi)$ and restrict to the branch where $y > 0$. The $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors along the x -, y - and z -axis, respectively. Which of the following is/are correct?

- (A) The arc length between $(1, 0, 0)$ and $(1, 0, \pi/2)$ is $\sqrt{2}\pi$.
- (B) The curvature is 2.
- (C) The radius of curvature is 2.
- (D) The unit tangent vector at $(0, 1, \pi/2)$ is $(-\mathbf{i} + \mathbf{k})/\sqrt{2}$.
- (E) The unit normal vector at $(0, 1, \pi/2)$ is \mathbf{j} .

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：工程數學甲【電機系碩士班戊組選考、庚組、通訊所碩士班乙組選考、電波聯合碩士班選考】題號：431002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）

共 5 頁第 3 頁

9. Find a solution $u(x, y)$ of the differential equation (DE) $xu_{xy} + 2yu = 0$. Assume $x > 0$. The c and k below are constants, and A, B, E and F are integers. Which of the following is/are INCORRECT?
- (A) $u = cx^{Ak}e^{(By^E)/(Fk)}$, where $A + B = 0$
(B) $u = cx^{Ak}e^{(By^E)/(Fk)}$, where $E + F = 3$
(C) This DE is parabolic.
(D) This DE is elliptic.
(E) This DE is homogeneous.
10. Consider the differential equation (DE) $y' = y^2 - 3y + 2$. Which of the following is/are correct?
- (A) There is an attractor.
(B) $y(x) = 2$ is a singular solution.
(C) $y(x) = (ce^x - 2)/(ce^x + 1)$ is a one-parameter family of solutions, where c is an arbitrary constant.
(D) Provided $y(0) = -1$, the largest interval is $[0, \infty)$.
(E) Given $y(0) = 4$, there exists an interval centered at 0 on which the DE has a unique solution.

下面 11-15 題為單選題，總分 20 分。每題答對 4 分，答錯或未作答者以 0 分計。

For the following problems, let $j = \sqrt{-1}$ denote the imaginary unit.

11. If $z = 3 - 4j$, what is $|z|$?
- (A) 1
(B) 5
(C) 7
(D) $\sqrt{7}$
12. The function $f(z) = \bar{z}$ is:
- (A) Analytic everywhere
(B) Analytic only at $z = 0$
(C) Not analytic anywhere
(D) Entire
13. Let $f(z) = z^2$, find $f'(1 + j)$.
- (A) $1 + j$
(B) $2 + 2j$
(C) $2 - 2j$
(D) $4j$
14. Let $z = x + jy$. Which of the following functions satisfies the Cauchy–Riemann equations?
- (A) $f(z) = x^2 + y^2$
(B) $f(z) = x^2 - y^2 + j(2xy)$
(C) $f(z) = x^2 + jy^2$
(D) $f(z) = |z|^2$

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：工程數學甲【電機系碩士班戊組選考、庚組、通訊所碩士班乙組選考、電波聯合碩士班選考】題號：431002

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共 5 頁第 4 頁

15. Evaluate

$$\oint_{|z|=2} \frac{1}{z} dz.$$

- (A) 0
(B) 1
(C) 2π
(D) $2\pi j$

下面 16-21 題為單選題，總分 30 分。每題答對 5 分，答錯或未作答者以 0 分計。

16. If $f(t) = \cos(3t) + 3t \sin(3t)$, then its Laplace Transform is given by?

- (A) $\frac{s+3}{s-3}$ (B) $\frac{27}{(s^2+9)^2}$ (C) $\frac{s(s^2+27)}{(s^2+9)^2}$ (D) Does not exist

17. Consider three continuous-time periodic signals whose Fourier series representations are as follows:

$$x_1(t) = \sum_{k=0}^{100} \left(\frac{1}{2}\right)^k e^{jk\frac{2\pi}{50}t}, x_2(t) = \sum_{k=-100}^{100} \cos(k\pi) e^{jk\frac{2\pi}{50}t}, x_3(t) = \sum_{k=-100}^{100} j \sin\left(\frac{k\pi}{2}\right) e^{jk\frac{2\pi}{50}t}.$$

Which of the three signal is/are even?

- (A) $x_1(t)$
(B) $x_2(t)$
(C) $x_3(t)$
(D) $x_2(t)$ and $x_3(t)$

18. Let $x(t)$ be a periodic signal with Fourier series coefficients defined by

$$a_k = \begin{cases} 2, & k = 0 \\ j\left(\frac{1}{2}\right)^{|k|}, & k \neq 0 \end{cases}$$

Which of the following statements is **true**?

- (A) $x(t)$ is real.
(B) $x(t)$ is even.
(C) $\frac{dx(t)}{dt}$ is even.
(D) None of the above.

19. The Fourier Transform of $e^{-|t|}$ is $\frac{2}{1+\omega^2}$. Then, what is the Fourier Transform of $e^{-2|t|}$?

- (A) $\frac{4}{4+\omega^2}$ (B) $\frac{2}{4+\omega^2}$ (C) $\frac{2}{2+\omega^2}$ (D) $\frac{4}{2+\omega^2}$

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：工程數學甲【電機系碩士班戊組選考、庚組、通訊所碩士班乙組選考、電波聯合碩士班選考】題號：431002

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共 5 頁第 5 頁

20. For each of the following Fourier transforms, use Fourier transform properties to determine which of the corresponding time-domain signal is purely imaginary and odd. Do this without evaluating the inverse of any of the given transforms.

(A) $X(j\omega) = u(\omega) - u(\omega - 2)$

(B) $X(j\omega) = \cos(2\omega) \sin\left(\frac{\omega}{2}\right)$

(C) $X(j\omega) = A(\omega)e^{jB(\omega)}$, where $A(\omega) = (\sin 2\omega)/\omega$ and $B(\omega) = 2\omega + \frac{\pi}{2}$

(D) $X(j\omega) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|k|} \delta\left(\omega - \frac{k\pi}{4}\right)$

21. Determine the inverse Laplace transform $x(t) = \mathcal{L}^{-1}\{X(s)\}$ for $t \geq 0$:

$$X(s) = \frac{s - 1}{s^2 + 3s + 2}.$$

(A) $x(t) = (3e^{-2t} - 2e^{-t})u(t)$

(B) $x(t) = (2e^{-t} - 3e^{-2t})u(t)$

(C) $x(t) = (3e^{-t} - 2e^{-2t})u(t)$

(D) $x(t) = (3e^{-2t} + 2e^{-t})u(t)$

國立中山大學 115 學年度 碩士班考試入學招生考試試題

科目名稱：電磁學【電機系碩士班戊組、通訊所碩士班乙組、電波聯合碩士班】

—作答注意事項—

考試時間：100 分鐘

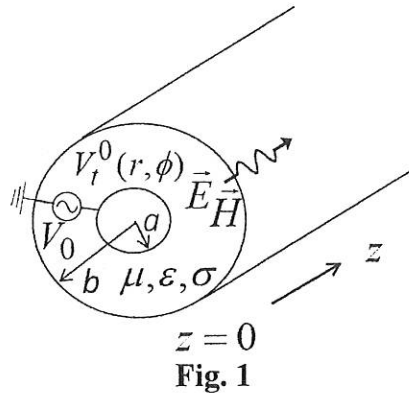
- 考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- 答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
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- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：電磁學【電機系碩士班戊組、通訊所碩士班乙組、電波聯合碩士班】題號：482004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題） 共 1 頁第 1 頁

1. (25%) In a charge-free region, the electric field intensity is $\mathbf{E} = (x + 3y - k_1z)\mathbf{a}_x + (k_2x + 5z)\mathbf{a}_y + (2x - k_3y + k_4z)\mathbf{a}_z$. Please determine the values of the constants $k_1, k_2, k_3,$ and k_4 .
2. (25%) A planar rectangular loop with length a and width b carries a direct current I . Find the magnetic flux density at the center of the rectangle.
3. (10%) Please describe in detail the boundary conditions of electromagnetic fields at the interface between two dielectric media.
4. (15%) In seawater, a uniform plane wave propagates in the $+z$ direction. At $z = 0$, the electric field intensity is given by $\mathbf{E} = \mathbf{a}_x 100 \cos(10^7 \pi t)$ (V/m). The relative permittivity ϵ_r of seawater is 72, the relative permeability μ_r is 1, and the conductivity σ is 4 S/m. Please derive in detail the polarization of this uniform plane wave, its phase velocity, and the instantaneous expression of the magnetic field intensity at $z = 0.8$ m.
5. (10%) As shown in Fig. 1,



Consider a coaxial cable that operates in the TEM mode. The inner conductor has radius a , and the outer conductor has radius b ; the thickness of both conductors can be neglected. The region between the inner and outer conductors is filled with a dielectric material with permittivity ϵ and permeability μ . Please calculate the surface current density on the two conductors and the characteristic impedance of this coaxial cable, respectively.

6. (15%) Consider a circuit that includes a signal generator with an internal resistance of 50Ω , producing a sinusoidal voltage of 10 V at 300 MHz. The signal generator is connected to a transmission line of length 2 m and characteristic impedance 50Ω . The end of the transmission line is terminated with a load impedance of $(30 - 40j) \Omega$. Please derive in detail the reflection coefficient at the load end of the transmission line and the expression for the voltage along the transmission line. In addition, to maximize the average power delivered to the load, what should the load impedance be set to?

國立中山大學 115 學年度 碩士班考試入學招生考試試題

科目名稱：電子學(含數位電路)【IC 設計領域聯招碩士班、電機系碩士班己組、IC 設計所碩士班】

— 作答注意事項 —

考試時間：100 分鐘

- 考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。請先檢查答案卷(卡)之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：電子學(含數位電路)【IC 設計領域聯招碩士班、電機系碩士班已組、IC 設計所碩士班】題號：430001

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Part I. Multiple-choice questions (Single answer).

Only one option is correct for each question in this part.

- (2%) Which component is the main contributor to power consumption in a CMOS circuit during normal operation?
 - Static leakage power
 - Dynamic switching power
 - Short-circuit power only
 - Gate-oxide tunneling power
- (2%) If the supply voltage V_{DD} is reduced by 50%, how does dynamic power change (all other parameters constant)?
 - Reduced by 50%
 - Reduced by 75%
 - Reduced by 40%
 - Reduced by 25%
- (2%) Why is static power consumption ideally very low in CMOS circuits?
 - NMOS and PMOS are never ON
 - There is no DC path between V_{DD} and GND in steady state
 - CMOS circuits operate at low frequency
 - CMOS transistors have zero resistance
- (2%) Which technique is most effective for reducing both dynamic and static power in CMOS systems?
 - Increasing transistor width
 - Clock gating only
 - Power gating idle blocks
 - Increasing supply voltage
- (2%) Lowering V_{DD} in a CMOS circuit generally leads to which trade-off?
 - Lower power and higher speed
 - Lower power and increased delay
 - Higher noise margin and lower power
 - No change in circuit behavior
- (2%) In a 2-input CMOS NAND gate, how are the NMOS transistors connected?
 - In parallel
 - In series
 - One in series, one in parallel
 - Cross-coupled
- (2%) Why are PMOS transistors typically sized wider than NMOS transistors in CMOS logic gates?
 - PMOS has higher mobility
 - NMOS has higher threshold voltage
 - PMOS has lower carrier mobility
 - PMOS must carry leakage current
- (2%) Which logic function is most expensive to implement using static CMOS in terms of transistor count?
 - NAND
 - NOR
 - XOR
 - NOT

國立中山大學 115 學年度碩士班考試入學招生考試試題

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9. (2%) What does propagation delay of a CMOS logic gate represent?
 - A. Time for input voltage to change
 - B. Time for output to reach 50% after an input transition
 - C. Clock-to-output delay
 - D. Setup time of a flip-flop
10. (2%) If the load capacitance of a CMOS gate doubles, the propagation delay will approximately:
 - A. Remain unchanged
 - B. Double
 - C. Be reduced by half
 - D. Increase logarithmically
11. (2%) Which design change most effectively reduces propagation delay?
 - A. Increasing threshold voltage
 - B. Increasing transistor width
 - C. Lowering supply voltage
 - D. Increasing logic depth
12. (2%) In a multi-stage CMOS logic path, total propagation delay is best approximated as:
 - A. Maximum delay of any single gate
 - B. Average delay of all gates
 - C. Sum of delays of gates on the critical path
 - D. Delay of the first gate only
13. (2%) Why do CMOS logic gates generally have larger noise margins than TTL gates?
 - A. CMOS operates at higher frequency
 - B. CMOS has lower threshold voltage
 - C. CMOS has rail-to-rail output swing
 - D. CMOS uses resistive pull-up networks
14. (2%) Which parameter directly affects the noise margin of a CMOS logic gate?
 - A. Load capacitance
 - B. Output resistance
 - C. Input threshold voltages
 - D. Switching frequency
15. (2%) Which design change most directly improves noise margin in a CMOS inverter?
 - A. Increasing load capacitance
 - B. Increasing transistor channel length
 - C. Balancing PMOS and NMOS strengths
 - D. Increasing switching frequency
16. (4%) Given the circuit in Fig. 1, transistors M_1 and M_2 have identical process parameters: $\mu_n C_{ox} \left(\frac{W}{L}\right) = 200 \mu\text{A}/\text{V}^2$, $V_{th} = 0.7 \text{ V}$, $\lambda = 0.01$. $R_S = 1 \text{ k}\Omega$, while V_1 and V_2 are DC bias voltages. Assume $I_0 = 400 \mu\text{A}$, and both M_1 and M_2 operate in the saturation region. What is the value of V_1 ?
 - A. 1.1 V
 - B. 4.0 V
 - C. 3.1 V
 - D. 2.0 V
17. (4%) Based on Question 16, what is the minimum value of V_2 ?
 - A. 5.1 V
 - B. 3.1 V
 - C. 2.7 V
 - D. 4.4 V

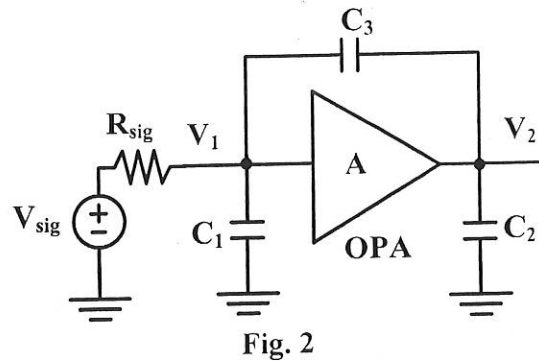
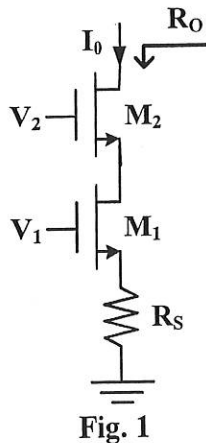
國立中山大學 115 學年度碩士班考試入學招生考試試題

科目名稱：電子學(含數位電路)【IC 設計領域聯招碩士班、電機系碩士班已組、IC 設計所碩士班】題號：430001

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18. (4%) Based on Question 16, which of the following is closest to the value of the small-signal output resistance R_o ?

- A. 100 M Ω
- B. 10 M Ω
- C. 1 M Ω
- D. 100 k Ω



19. (4%) In the circuit shown in Fig. 2, $R_{sig} = 10 \text{ k}\Omega$, $C_1 = 0.1 \text{ pF}$, $C_2 = 3 \text{ pF}$, $C_3 = 1 \text{ pF}$, and the amplifier gain is $A = -100$. Assume the operational amplifier (OPA)'s input resistance is infinite. which of the following is closest to the value of the small-signal equivalent capacitance seen at node V_1 ?

- A. 0.1 pF
- B. 1 pF
- C. 4 pF
- D. 100 pF

20. (4%) Based on Question 19, which of the following is closest to the value of the small-signal equivalent capacitance at node V_2 ?

- A. 0.1 pF
- B. 1 pF
- C. 4 pF
- D. 100 pF

21. (4%) For an NMOS transistor, which of the following statements is correct?

- A. The gate capacitance is mainly determined by the channel width W .
- B. Reducing the drain current I_D can increase the voltage gain.
- C. Increasing the channel width W can increase the voltage gain.
- D. The small-signal impedance seen looking into the source terminal is r_o .

22. (4%) In the circuit shown in Fig. 3, the input voltage is $V_1 = 1.0 \text{ V}$ and the output voltage is $V_O = 0.99 \text{ V}$. What is the magnitude of the voltage gain of the operational amplifier (OPA)? Choose the closest value.

- A. The voltage gain is infinite.
- B. The voltage gain is 100.
- C. The voltage gain is 10.
- D. The voltage gain cannot be determined.

國立中山大學 115 學年度碩士班考試入學招生考試試題

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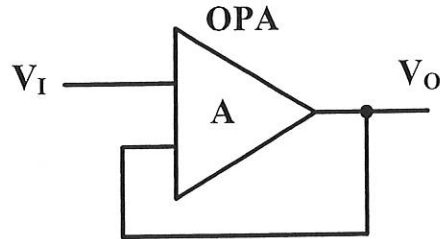


Fig. 3

23. (4%) Regarding a PN diode, which of the following statements is incorrect?
- A. As the forward bias voltage increases, the diffusion capacitance increases.
 - B. The saturation current I_S of a PN diode increases as the temperature rises.
 - C. The saturation current I_S is generally a constant and is independent of the junction area.
 - D. Breakdown phenomena mainly occur in the depletion region.

Part II. Multiple-choice questions (Multiple answers).

Each question is worth points only if all correct answers are selected; otherwise, the question will receive zero points.

24. (2%) Which techniques can reduce static (leakage) power?
- A. Power gating
 - B. High-threshold voltage transistors
 - C. Clock gating
 - D. Lowering temperature
25. (2%) Which statements about CMOS power scaling are correct?
- A. Dynamic power scales quadratically with V_{DD}
 - B. Static power is independent of technology scaling
 - C. Leakage power becomes dominant in advanced nodes
 - D. Lowering V_{DD} always improves performance
26. (2%) Which statements about static CMOS logic gates are correct?
- A. They provide full rail-to-rail output swing
 - B. They consume zero power during switching
 - C. They have good noise margins
 - D. They use complementary PMOS and NMOS networks
27. (2%) Which techniques improve performance of CMOS logic gates?
- A. Increasing transistor width
 - B. Reducing load capacitance
 - C. Lowering supply voltage
 - D. Reducing logic depth
28. (2%) Which issues become more severe as CMOS logic gates scale to advanced technology nodes?
- A. Leakage current
 - B. Process variation
 - C. Noise margin
 - D. Interconnect delay
29. (2%) Which factors increase the propagation delay of a CMOS logic gate?
- A. Larger load capacitance
 - B. More transistors in series
 - C. Higher supply voltage
 - D. Smaller transistor width
30. (2%) Which statements about CMOS propagation delay are correct?

國立中山大學 115 學年度碩士班考試入學招生考試試題

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※本科目依簡章規定「可以」使用計算機(廠牌、功能不拘)(混合題) 共 6 頁第 5 頁

- A. Delay limits maximum clock frequency
 B. Delay is independent of temperature
 C. Higher temperature increases delay
 D. Delay affects setup time constraints
31. (2%) Which statements about CMOS noise margin are correct?
 A. Larger noise margin improves reliability
 B. Noise margin depends on load capacitance
 C. Noise margin can be obtained from the VTC
 D. Noise margin affects tolerance to interference
32. (2%) Which issues become more critical for noise margin in advanced CMOS technologies?
 A. Reduced supply voltage
 B. Increased process variation
 C. Improved output swing
 D. Device mismatch
33. (2%) Which statements regarding noise margin and system design are correct?
 A. Noise margin affects timing only
 B. Noise margin impacts functional correctness
 C. Larger noise margin improves tolerance to crosstalk
 D. Noise margin is unrelated to reliability

Part III. Essay Questions

You must show all calculation and derivation steps for each question. Answers without supporting work may receive zero points.

34. (9%) The op amp (OPA) in the circuit of Fig. 4 has an open-loop gain of 10^5 and a single-pole rolloff with $\omega_{3dB} = 10$ rad/s.
 (a) Sketch a Bode plot for the loop gain. (3%)
 (b) Find the frequency at which $|A\beta| = 1$, and find the corresponding phase margin. (3%)
 (c) Find the closed-loop transfer function, including its zero and poles. (3%)

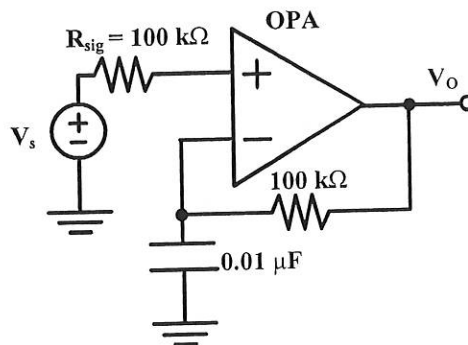


Fig. 4

35. (9%) Fig. 5 shows a circuit suitable for op-amp applications. For all transistors $\beta = 100$, $V_{BE} = 0.7$ V, and $r_o = \infty$.
 (a) For inputs grounded and output held at 0 V (by negative feedback) find the collector currents of all transistors. (3%)
 (b) Calculate the gain of the amplifier with a load of 10 k Ω . (3%)
 (c) With loads as in (b) calculate the value of the capacitor C_1 required for a 3-dB frequency of 1 KHz. (3%)

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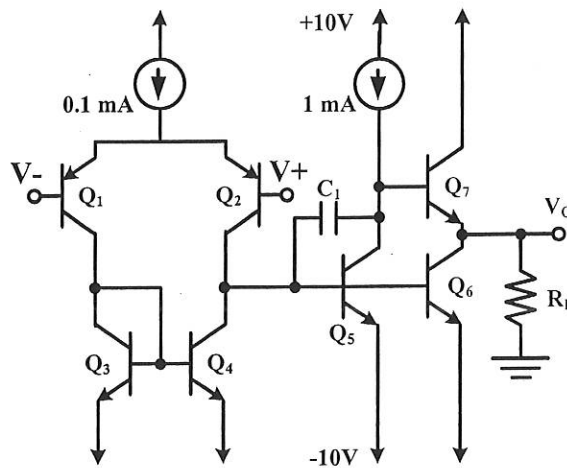


Fig. 5

