國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：電子學(甲組)【電機系碩士班甲組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

科目名稱：電子學(甲組)【電機系碩士班甲組】　　題號：431009

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）(問答申論題)　　共1頁第1頁

1. (10%) (a) Describe the Early effect in MOSFETs and explain its occurrence mechanism (5%). (b) Define depletion-mode MOSFETs and highlight the distinctions from enhancement-mode MOSFETs (5%).

2. (20%) A full-wave bridge-rectifier circuit with a 1-kΩ load operates from a 120 V (rms) 60 Hz household supply through a 12-to-1 step down transformer having a single secondary winding. It uses four diodes as shown in Figure 1, each of which can be modeled to have a 0.7 V drop for any current.
(a) What is the peak value of the rectified voltage across the load (5%)?
(b) For what fraction of a cycle does each diode conduct (5%)?
(c) What is the peak inverse voltage of each diode (5%)?
(d) If the ripple voltage is to be smaller than 0.3 V, determine the required value of the filter capacitor in parallel with the load resistance (5%).

3. (20%) The NMOS and PMOS transistors in the circuit of Figure 2 are matched with $k_n=k_p=1$ mA/V^2 and $V_{tn}=|V_{tp}|=1$ V. Please find i_{DN} and i_{DP} for (a) $v_I=5$ V (10%) and (b) $v_I=2.5$ V (10%).

4. (30%) Figure 3 shows a three-stage amplifier in which the stages are directly coupled. For our purposes here, we shall assume that the capacitors are large enough to act as perfect short circuits at all signal frequencies of interest. Thermal voltage V_T is 25 mV.
(a) Find the dc collector current in each of the three transistors. Assume $|V_{BE}|\approx 0.7$ V (forward bias), $\beta=100$, and neglect the Early effect (15%).
(b) Find the input resistance R_{in} and the output resistance R_{out} (10%).
(c) Evaluate the voltage gain v_o/v_i (5%).

5. (20%) The transistors in the circuit of Figure 4 have $\beta=100$, Early voltage $V_A=100$ V, and $C_\mu=0.2$ pF. At a bias current of 0.1 mA, $f_T=200$ MHz. Thermal voltage V_T is 25 mV. Find an estimate of the upper 3-dB frequency f_H by the method of open-circuit time constants (list each time constant).

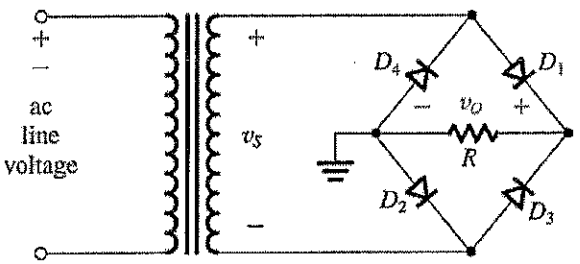

Figure 1

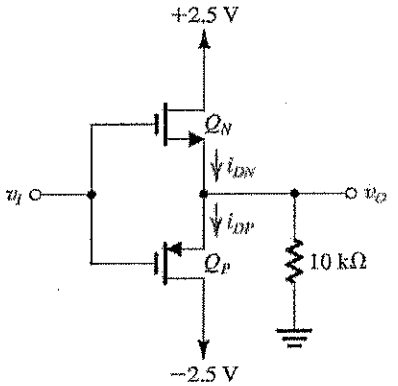

Figure 2

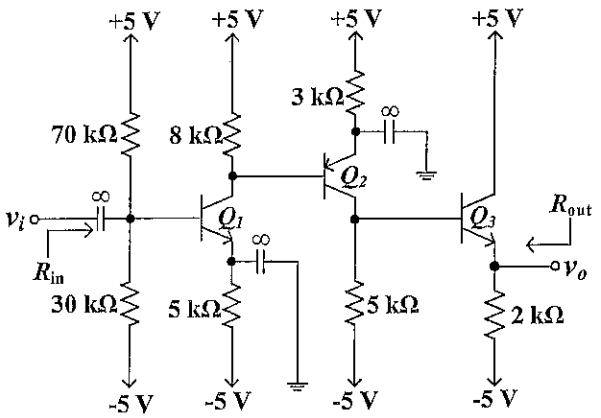

Figure 3

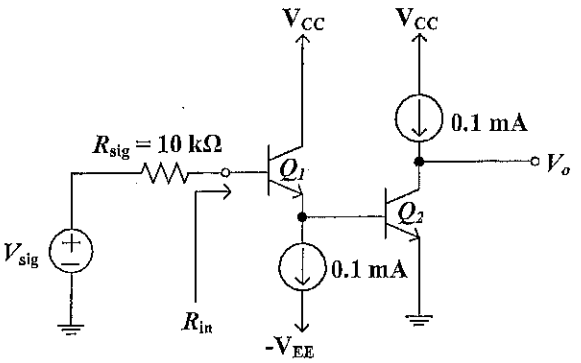

Figure 4

試題請隨卷繳回，請留意背面是否有題

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：半導體概論【電機系碩士班甲組】

—作答注意事項—

考試時間：100分鐘

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國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

科目名稱：半導體概論【電機系碩士班甲組】　　題號：431012

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

Please note that all calculation answers must include the unit and calculation process.

Dielectric permittivity of Si: $\varepsilon_{Si} = 11.7 \times 8.85 \times 10^{-14}$ F/cm

Dielectric permittivity of SiO_2: $\varepsilon_{SiO2} = 3.9 \times 8.85 \times 10^{-14}$ F/cm

Energy bandgap of Si: $E_g = 1.12$ eV.

Charge $q = 1.6 \times 10^{-19}$ C.

Thermal energy $kT = 25.9$ meV at $T = 300$ K

Electron affinity of Si: $q\chi = 4.01$ eV.

Intrinsic concentration of Si at $T = 300$ K: $n_i = 1.5 \times 10^{10}$ cm^{-3}.

1. (10%) Please explain what a degenerate semiconductor is and the freeze-out effect.
2. (5%) How to achieve ohmic contact for a metal/semiconductor junction? Please provide two methods.
3. (20%) The values of effective density of states function in the conduction band N_c and valence band N_v for a semiconductor at $T = 300$ K are 3×10^{19} cm^{-3} and 2×10^{19} cm^{-3}, respectively.
   (a) Calculate the intrinsic concentration of the semiconductor at $T = 300$ K and 400K, respectively. Assume the bandgap energy of the semiconductor is 1 eV and does not vary over this temperature range. (10%)
   (b) Based on the previous question, if this semiconductor is doped with both donor $N_d = 10^{13}$ cm^{-3} and acceptor $N_a = 3 \times 10^{13}$ cm^{-3}. What are the carrier concentrations of electrons and holes at $T = 400$ K? (10%)
4. (15%) A semiconductor Hall device at $T = 300$ K has following geometry as shown in Fig. 1: $d = 10^{-2}$ cm, $W = 0.1$ cm, and $L = 1$ cm. The following parameters are measured: $I_x = 5$ mA, $V_x = 10$ V, $V_H = -5$ mV, and $B_z = 0.1$ tesla. Determine (a) conductivity type (please explain whether this semiconductor is of n-type or p-type) (5%), (b) majority concentration (5%), and (c) majority carrier mobility. (5%)
5. (30%) Consider a uniformly doped silicon pn junction at $T = 300$ K. At zero bias, 20 percent of the total space charge region is in the n-region. The built-in potential barrier is $V_{bi} = 0.75$ V.
   (a) Determine dopant concentration and depletion width in p-region and n-region, respectively. (i.e. determine N_a, N_d, x_n, x_p) (20%)
   (b) Determine the maximum electric field (E_{max}) and junction capacitance density C_j. (10%)
6. (20%) A MOS-Capacitor with metal work function 4.6 eV, gate insulator SiO_2 with thickness 15 nm, silicon substrate with boron doped (p-type) to 1×10^{17} cm^{-3} at T= 300 K. Please find out (a) the ideal flat-band voltage V_{FB} (5%), (b) flat-band capacitance density (5%), (c) ideal threshold voltage V_{TH} (5%), and (d) saturated depletion width (5%). The V_{TH} means the MOC-Cap is biased into strong inversion.

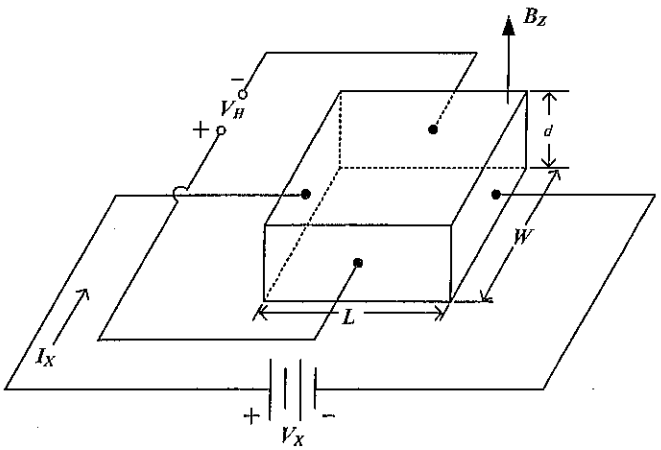

Figure 1

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學甲【電機系碩士班甲組、戊組選考、己組、庚組、通訊所碩士班乙組選考、電波聯合碩士班選考】

—作答注意事項—

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※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）

下面1-8題為單選題，總分40分。每題答對5分，答錯或未作答者以0分計。總分低於0分者以0分計算。

1. Use the Fourier series analysis equation to calculate the coefficients a_k (when $k \neq 0$) for the continuous-time periodic signal

$x(t) = \begin{cases} 1.5, & 0 \le t < 1 \\ -1.5, & 1 \le t < 2 \end{cases}$ with fundamental frequency $\omega_0 = \pi$.

(A) $\frac{3}{2k\pi} e^{-jk\pi/2} sin(\frac{k\pi}{2})$　　(B) $\frac{3}{2k\pi} e^{-jk\pi} sin(k\pi)$

(C) $\frac{3}{k\pi} e^{-jk\pi} sin(k\pi)$　　(D) $\frac{3}{k\pi} e^{-jk\pi/2} sin(\frac{k\pi}{2})$

2. Let $x_1(t)$ be a continuous-time periodic signal with fundamental frequency ω_1 and Fourier coefficients a_k. Given that $x_2(t) = x_1(1-t) + x_1(t-1)$. Find a relationship between the Fourier series coefficients b_k of $x_2(t)$ and the coefficients a_k.

(A) $b_k = e^{-jk\omega_1}(a_k + a_{-k})$　　(B) $b_k = e^{-jk\omega_1}(a_k - a_{-k})$

(C) $b_k = jk\omega_1(a_k + a_{-k})$　　(D) $b_k = jk\omega_1(a_k - a_{-k})$

3. What is the Fourier transform of $e^{-2|t-1|}$?

(A) $2e^{-j\omega}/(4+\omega^2)$ (B) $4e^{-j\omega}/(4+\omega^2)$ (C) $4e^{-j\omega}/(2+j\omega)$ (D) $2e^{-j\omega}/(2+j\omega)$

4. Use the Fourier transform synthesis equation to determine the inverse Fourier transforms of $X(j\omega) = 2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)$.

(A) $1 + \pi cos(4\pi t)$ (B) $1 + sin(4\pi t)$ (C) $1 + cos(4\pi t)$ (D) $1 + \pi sin(4\pi t)$

5. Use the Fourier transform synthesis equation to determine the inverse Fourier transforms of

$$X(j\omega) = \begin{cases} 2, & 0 \le \omega \le 2 \\ -2, & -2 \le \omega < 0 \\ 0, & |\omega| > 2 \end{cases}$$

(A) $(4jcos^2 t)/\pi t$　(B) $(4sin^2 t)/\pi t$　(C) $(4jsin^2 t)/\pi t$　(D) $(4cos^2 t)/\pi t$

6. What is the Laplace transform of $e^{-5t}u(t-1)$?

(A) $e^{(5+s)}/(s+5)$　(B) $e^{-(5+s)}/(s+5)$ (C) $e^{-(5+s)}/(s-5)$　(D) $e^{(5+s)}/(s-5)$

7. Determine the inverse Laplace transform of

$$X(s) = \frac{2(s+2)}{s^2+7s+12}, \ \mathcal{R}e\{s\} > -3.$$

(A) $4e^{-4t}u(-t) - 2e^{-3t}u(-t)$ (B) $4e^{4t}u(t) - 2e^{3t}u(t)$

(C) $4e^{4t}u(-t) - 2e^{3t}u(-t)$ (D) $4e^{-4t}u(t) - 2e^{-3t}u(t)$

8. One corner of a rectangular parallelepiped is at $(1, 1, 1)$, and three incident sides extend from this point to $(-2, 1, 6)$, $(3, 5, 7)$ and $(0, 1, 6)$. Please identify the volume of this solid.
(A) 20 (B) 40 (C) 18 (D) 42

下面 9-20 題為複選題，每題 5 分，總分 60 分，每題有五個選項，其中至少有一個是正確答案，答錯 1 個選項者，得 3 分，答錯 2 個選項者，得 1 分，答錯多於 2 個選項或未作答者，該題以 0 分計算。

9. Consider the wave equation $a^2u_{xx} = u_{tt}$ for a string tied to the x-axis at $x = 0$ and at $x = \pi$. When the string starts to vibrate, the motion takes place in the xu-plane. Let $u(x, t)$ denote the vertical displacement from the x-axis for $t > 0$ and a^2 be a real constant. The initial displacement is $f(x)$, $0 < x < \pi$ and the string is released from rest. Given the product solution $u(x, t) = X(x)T(t)$, which of the following is/are correct?
(A) The wave equation of this string is elliptic.
(B) All the boundary conditions belong to Dirichlet conditions.
(C) The initial condition yields $T'(0) = 0$.
(D) This is referred to as a boundary-value problem.
(E) One of the boundary conditions yields $X(0) = \pi$.

10. Assume the product solution to the wave equation in Question 9 is found as $u(x, t) = A \cdot \cos(B \cdot at) \cdot \sin(Cx)$. If the initial displacement $f(x)$ is a sinusoidal wave with the amplitude of $1/100$ and the frequency of $3/2\pi$, which of the following is/are correct?
(A) There exists a trivial solution.
(B) $B =$ arbitratry integer.
(C) $C = 3$.
(D) There is no motion at $x = 2\pi/3$ and all other x points vibrate vertically over time.
(E) The vertical displacement is reversed $(-f(x))$ when $t = k\pi/a, k = 1, 2, 3...$

11. Consider the differential equation $P'(t) = aP(t) - bP(t)^2$with the initial condition of $P(0) = p_0$, where a and b are positive real constants. Which of the following is INCORRECT?
(A) This is a linear differential equation.
(B) This is a Bernoulli's equation.
(C) None of the critical points are attractors.
(D) When $t \to \infty$, $P(t) = a/b$.
(E) The solution interval is $(-\infty, \infty)$.

12. Given $X' = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} X$, which of the following is correct?
(A) There are two distinct eigenvalues.
(B) The stationary point is a repeller.
(C) All solutions will converge onto the origin.
(D) One of the eigenvalues is $\lambda_2 = -1$.
(E) One of the solution vectors is $K_1 = \binom{1}{1} e^{-4t}$.

13. Define the matrix $\mathbf{A}$ as $\mathbf{pq}^T$, where $\mathbf{p}$ and $\mathbf{q}$ are nonzero vectors in $\mathbb{R}^{n\times 1}$. Which of the following statements are true?
(A) $\text{rank}(\mathbf{A})$ may be 2.
(B) $\mathbf{q}$ is an eigenvector of $\mathbf{A}$.
(C) $\mathbf{q}^T\mathbf{p}$ is an eigenvalue of $\mathbf{A}$.
(D) The linear equation $\mathbf{Ax} = \mathbf{0}$ has infinitely many solutions.
(E) The linear equation $\mathbf{Ax} = \mathbf{b}$ with $\mathbf{b} = \mathbf{q}$ has infinitely many solutions.

14. Define the matrix $\mathbf{M}$ as $\mathbf{I} - 2\mathbf{uu}^T$, where $\mathbf{u}$ is a unit vector in $\mathbb{R}^{n\times 1}$, meaning $\mathbf{u}^T\mathbf{u} = 1$. Which of the following statements are true?
(A) $\mathbf{M} = \mathbf{M}^T$.
(B) $\mathbf{M}^T\mathbf{M} = \mathbf{I}$.
(C) $\mathbf{M} = \mathbf{M}^{-1}$.
(D) $\det(\mathbf{M}) = 1$.
(E) $\mathbf{M}$ has eigenvalues that are complex with non-zero real components.

15. Let $N(\mathbf{A})$ denote the null space of $\mathbf{A}$, and $R(\mathbf{A})$ denote the range space of $\mathbf{A}$. Suppose $\mathbf{u} \in R(\mathbf{A})$, $\mathbf{v} \in N(\mathbf{A}^T)$, and both $\mathbf{u}$ and $\mathbf{v}$ are non-zero. Define $\|\mathbf{x}\| = \sqrt{\langle \mathbf{x}, \mathbf{x} \rangle}$. Suppose

$$\mathbf{x}_1 = \mathbf{u} + 2\mathbf{v},\ \mathbf{x}_2 = 2\mathbf{u} - \mathbf{v},\ \|\mathbf{x}_1\|^2 = 29,\ \|\mathbf{x}_2\|^2 = 41$$

Which of the following statements are true?
(A) $\|\mathbf{u}\| = \sqrt{5}$.
(B) $\|\mathbf{u}\| = 3$.
(C) $\|\mathbf{v}\| = 7$.
(D) $\|\mathbf{v}\| = \sqrt{5}$.
(E) $\|\mathbf{u}\| + \|\mathbf{v}\| = 8$.

16. Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be an orthogonal basis for an inner product space. Suppose $\|\mathbf{u}_1\| = 2$, $\|\mathbf{u}_2\| = 3$, $\|\mathbf{u}_3\| = 5$. If $\mathbf{x} = c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3\mathbf{u}_3$ is a vector with the properties $\langle \mathbf{x},\ \mathbf{u}_1 \rangle = 12$, $\|\mathbf{x}\|^2 = 145$, and the orthogonal projection of $\mathbf{x}$ onto $\mathbf{u}_3$ is $-2\mathbf{u}_3$, then which of the following statements are true?
(A) $c_1 = 12$, (B) $c_2 = 2$, (C) $c_3 = -2$, (D) $\langle \mathbf{x},\ \mathbf{u}_2 \rangle = 18$, (E) $\langle \mathbf{x},\ \mathbf{u}_3 \rangle = 50$

17. Let $L: V \to W$ be a linear transformation, and $E = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ and $F = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ be ordered bases for V and W, respectively. Suppose $\mathbf{A}$ is the matrix representing L relative to E and F, and

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 7 \\ 3 & 4 & 8 \\ 5 & 6 & 9 \end{bmatrix}.$$

If $L(\mathbf{v}_1 + 2\mathbf{v}_2) = c_1\mathbf{w}_1 + c_2\mathbf{w}_2 + c_3\mathbf{w}_3$, then which of the following statements are true?

(A) $c_1 = 7$, (B) $c_2 = 11$, (C) $c_3 = 17$, (D) $c_1 + c_2 + c_3 = 34$, (E) $c_1 + c_2 + c_3 = 40$

18. Let

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \\ -1 & 3 & 2 \\ 2 & 0 & 3 \end{bmatrix}.$$

Which of the following vectors are in the column space of $\mathbf{A}\mathbf{A}^T$?

(A) $[0, 2, 3, 7]^T$
(B) $[3, 0, 2, 4]^T$
(C) $[3, 1, 0, 4]^T$
(D) $[5, 4, -1, 2]^T$
(E) $[-1, 2, 4, 5]^T$

19. Define $\mathrm{i} = \sqrt{-1}$. Suppose $\int_C f(z)dz = a + bi$, where $f(z) = (z + 2)/z$ and C is the semicircle $z = 2e^{i\theta} (0 \le \theta \le \pi)$. Which of the following statements are true?

(A) $a = -4$, (B) $a = 2$, (C) $b = -2\pi$, (D) $b = 0$, (E) $b = 2\pi$

20. Define $\mathrm{i} = \sqrt{-1}$. Suppose $\int_C f(z)dz = a + bi$, where $f(z) = 1/(z^2 + 4)$ and C is the positively oriented circle $|z - i| = 2$. Which of the following statements are true?

(A) $a = \pi/8$, (B) $a = \pi/4$, (C) $a = \pi/2$, (D) $b = \pi/8$, (E) $b = \pi/4$

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學乙【電機系碩士班乙組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學乙【電機系碩士班乙組】　　題號：431001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）　　

下面 1-5 題為單選題，總分 10 分。每題答對 2 分，答錯扣 1 分，未作答者以 0 分計。總分低於 0 分者以 0 分計算。

1. $\cos^2 x, \sin^2 x, \pi$ are linearly independent.
   (A) True　(B) False

2. $\cosh 2x, e^x, e^{-x}, e^{2x}$ are linearly independent.
   (A) True　(B) False

3. $x = 0$ is an ordinary point of the differential equation $\sqrt{x}\frac{d^2y}{dx^2} + 5x^{\frac{3}{2}}\frac{dy}{dx} + x\sqrt{y} = 0$.
   (A) True　(B) False

4. Consider the differential equation $y' + 2xy^2 = 0$, if the initial condition $y(1/2) = -4$ is provided, a unique solution must exist.
   (A) True　(B) False

5. Following question 4, if the initial condition $y(-2) = 1/2$. The largest solution interval is
   (A) $(-\infty, \infty)$　(B) $\left(-\infty, -\sqrt{2}\right)$　(C) $(-\infty, 0)$　(D) $\left(-\infty, -1/\sqrt{2}\right)$

下面 6-23 題為複選題，每題 5 分，總分 90 分，每題有五個選項，其中至少有一個是正確答案，答錯 1 個選項者，得 3 分，答錯 2 個選項者，得 1 分，答錯多於 2 個選項或未作答者，該題以 0 分計算。

6. Given $X' = \begin{pmatrix} -6 & 5 \\ -5 & 4 \end{pmatrix} X$, which of the following is correct?
   (A) There are two distinct eigenvalues.
   (B) The stationary point is an attractor.
   (C) All solutions will converge onto the origin.
   (D) One of the eigenvalues is $\lambda_2 = 1$.
   (E) One of the solution vectors is $K_1 = \begin{pmatrix} 0 \\ 1/5 \end{pmatrix} te^{-t}$.

7. Consider the wave equation $a^2 u_{xx} = u_{tt}$ for a string tied to the x-axis at $x = 0$ and at $x = \pi$. When the string starts to vibrate, the motion takes place in the xu-plane. Let $u(x,t)$ denote the vertical displacement from the x-axis for $t > 0$ and a^2 be a real constant. The initial displacement is $f(x)$, $0 < x < \pi$ and the string is released from rest. Given the product solution $u(x,t) = X(x)T(t)$, which of the following is/are correct?
   (A) The wave equation of this string is elliptic.
   (B) All the boundary conditions belong to Dirichlet conditions.
   (C) This is referred to as a boundary-value problem.
   (D) The initial condition yields $T'(0) = 0$.
   (E) One of the boundary conditions yields $X(0) = \pi$.

8. Assume the product solution to the wave equation in Question 7 is found as $u(x,t) = A \cdot \cos(B \cdot at) \cdot \sin(Cx)$. If the initial displacement $f(x)$ is a sinusoidal wave with the amplitude of $1/100$ and the frequency of $3/2\pi$, which of the following is/are correct?
   (A) There exists a trivial solution.
   (B) $B =$ arbitratry integer.
   (C) $C = 3$.
   (D) There is no motion at $x = 2\pi/3$ and all other x points vibrate vertically over time.
   (E) The vertical displacement is reversed $(-f(x))$ when $t = k\pi/a, k = 1,2,3...$

9. Consider $x'' = 1 - x^2$ with the initial conditions of $x(0) = 1$ and $x'(0) = 3$. If the approximate solution is in the form of $x(t) = A + Bt + Ct^2 + Dt^3 + Et^4 + Ft^5$, which of the following is/are correct?
   (A) The differential equation is an autonomous equation.
   (B) $B + E = 11/4$.
   (C) $C + D = -7/2$.
   (D) The combination of $A + B + C = 2$.
   (E) The combination of $D + E + F = 7/5$.

10. Find the solution of the following equations. If t is the independent variable, which of the following is/are WRONG?

$$y_1' = -2y_1 - y_2 - 5y_3$$
$$y_2' = 25y_1 - 7y_2$$
$$y_3' = y_2 + 3y_3$$

   (A) There are three distinct eigenvalues.
   (B) There are three linearly independent solution vectors.
   (C) The number of the corresponding eigenvectors is less than 3.
   (D) One of the solution vectors is shown below where $a_2 + b_2 + c_2 + d = -6$.

$$\begin{bmatrix} a_1t + a_2 \\ b_1t + b_2 \\ c_1t + c_2 \end{bmatrix} e^{dt}.$$

   (E) One of the solution vectors is shown below where $\alpha + \beta + \gamma + \delta = -5$.

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} e^{\delta t}.$$

11. Consider the differential equation $\frac{d^2y}{dx^2} - \frac{4}{x}\frac{dy}{dx} + \frac{6}{x^2}y = f(x)$. Which of the following is/are correct?
   (A) Consider $f(x) = 0$. The solution interval is $(-\infty, \infty)$.
   (B) Consider $f(x) = 0$ and the initial conditions of $y(-2) = 8$ and $y'(-2) = 0$. The solution interval is $(-\infty, 0]$.
   (C) Consider $f(x) = 0$ and the initial conditions of $y(-2) = 8$ and $y'(-2) = 0$. The solution $y(x) = 6x^2 + 2x^3$.
   (D) Consider $f(x) = \frac{\ln(x^2)}{x^2}$. The solution $y(x) = c_1x^2 + c_2x^3 + \frac{1}{3}\ln(x) + \frac{5}{18}$.
   (E) Consider $f(x) = \frac{\ln(x^2)}{x^2}$. The solution $y(x) = c_1e^{2x} + c_2e^{3x} + \frac{1}{3}x + \frac{5}{18}$.

12. Consider $y' = y^2 - 6y + 9$. Which of the following is/are correct?
(A) A critical point is an attractor.
(B) There is one equilibrium solution.
(C) Given the initial condition of $y(0) = 3/2$, when $x \to \infty$, $y(x) \to \infty$.
(D) Given the initial condition of $y(0) = 6$, when $x \to \infty$, $y(x) \to 3$.
(E) Given the initial condition of $y(0) = 6$, the solution interval is $(-\infty, 1/3)$.

13. Consider the following differential equation. If the solution is in the form of $A \cdot e^x \sin(y) + B \cdot e^x \cos(y) + Cx^2 + Dy^2 + Ex + Fy + G$, which of the following is/are correct?

$$e^x \sin(y) - 2x + [e^x \cos(y) + 1]y' = 0$$

(A) $A + B = 0$.
(B) $C + D = 0$.
(C) $C + F = 0$.
(D) $A + B + C = 0$.
(E) $D + E + F = 0$.

14. Define the matrix $\mathbf{A}$ as $\mathbf{pq}^T$, where $\mathbf{p}$ and $\mathbf{q}$ are nonzero vectors in $\mathbb{R}^{n\times 1}$. Which of the following statements are true?
(A) $\text{rank}(\mathbf{A})$ may be 2.
(B) $\mathbf{q}$ is an eigenvector of $\mathbf{A}$.
(C) $\mathbf{q}^T\mathbf{p}$ is an eigenvalue of $\mathbf{A}$.
(D) The linear equation $\mathbf{Ax} = \mathbf{0}$ has infinitely many solutions.
(E) The linear equation $\mathbf{Ax} = \mathbf{b}$ with $\mathbf{b} = \mathbf{q}$ has infinitely many solutions.

15. Define the matrix $\mathbf{M}$ as $\mathbf{I} - 2\mathbf{uu}^T$, where $\mathbf{u}$ is a unit vector in $\mathbb{R}^{n\times 1}$, meaning $\mathbf{u}^T\mathbf{u} = 1$. Which of the following statements are true?
(A) $\mathbf{M} = \mathbf{M}^T$.
(B) $\mathbf{M}^T\mathbf{M} = \mathbf{I}$.
(C) $\mathbf{M} = \mathbf{M}^{-1}$.
(D) $\det(\mathbf{M}) = 1$.
(E) $\mathbf{M}$ has eigenvalues that are complex with non-zero real components.

16. Let $N(\mathbf{A})$ denote the null space of $\mathbf{A}$, and $R(\mathbf{A})$ denote the range space of $\mathbf{A}$. Suppose $\mathbf{u} \in R(\mathbf{A})$, $\mathbf{v} \in N(\mathbf{A}^T)$, and both $\mathbf{u}$ and $\mathbf{v}$ are non-zero. Define $\|\mathbf{x}\| = \sqrt{\langle \mathbf{x}, \mathbf{x} \rangle}$. Suppose

$$\mathbf{x}_1 = \mathbf{u} + 2\mathbf{v},\ \mathbf{x}_2 = 2\mathbf{u} - \mathbf{v},\ \|\mathbf{x}_1\|^2 = 29,\ \|\mathbf{x}_2\|^2 = 41$$

Which of the following statements are true?
(A) $\|\mathbf{u}\| = \sqrt{5}$.
(B) $\|\mathbf{u}\| = 3$.
(C) $\|\mathbf{v}\| = 7$.
(D) $\|\mathbf{v}\| = \sqrt{5}$.
(E) $\|\mathbf{u}\| + \|\mathbf{v}\| = 8$.

17. Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be an orthogonal basis for an inner product space. Suppose $\|\mathbf{u}_1\| = 2$, $\|\mathbf{u}_2\| = 3$, $\|\mathbf{u}_3\| = 5$. If $\mathbf{x} = c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3\mathbf{u}_3$ is a vector with the properties $\langle \mathbf{x}, \mathbf{u}_1 \rangle = 12$, $\|\mathbf{x}\|^2 = 145$, and the orthogonal projection of $\mathbf{x}$ onto $\mathbf{u}_3$ is $-2\mathbf{u}_3$, then which of the following statements are true?
(A) $c_1 = 12$, (B) $c_2 = 2$, (C) $c_3 = -2$, (D) $\langle \mathbf{x}, \mathbf{u}_2 \rangle = 18$, (E) $\langle \mathbf{x}, \mathbf{u}_3 \rangle = 50$

18. Let $L: V \to W$ be a linear transformation, and $E = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ and $F = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ be ordered bases for V and W, respectively. Suppose $\mathbf{A}$ is the matrix representing L relative to E and F, and

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 7 \\ 3 & 4 & 8 \\ 5 & 6 & 9 \end{bmatrix}.$$

If $L(\mathbf{v}_1 + 2\mathbf{v}_2) = c_1\mathbf{w}_1 + c_2\mathbf{w}_2 + c_3\mathbf{w}_3$, then which of the following statements are true?
(A) $c_1 = 7$, (B) $c_2 = 11$, (C) $c_3 = 17$, (D) $c_1 + c_2 + c_3 = 34$, (E) $c_1 + c_2 + c_3 = 40$

19. Let

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \\ -1 & 3 & 2 \\ 2 & 0 & 3 \end{bmatrix}.$$

Which of the following vectors are in the column space of $\mathbf{A}\mathbf{A}^T$?
(A) $[0, 2, 3, 7]^T$
(B) $[3, 0, 2, 4]^T$
(C) $[3, 1, 0, 4]^T$
(D) $[5, 4, -1, 2]^T$
(E) $[-1, 2, 4, 5]^T$

20. Let $\mathbf{A} \in \mathbb{R}^{3\times3}$, $\mathbf{A} = -\mathbf{A}^T$, and $\mathbf{A}$ is non-zero. Which of the following statements are true?
(A) $\text{trace}(\mathbf{A}) = 0$
(B) $\text{rank}(\mathbf{A}) = 2$.
(C) $\text{rank}(\mathbf{A})$ may be 3.
(D) All eigenvalues of $\mathbf{A}$ are real.
(E) $\mathbf{A}$ is a singular matrix.

21. Let $\mathbf{A} \in \mathbb{R}^{3\times3}$ have eigenvalues $\lambda_1, \lambda_1, \lambda_1$, indicating that all three eigenvalues are the same. Suppose the dimension of $N(\mathbf{A} - \lambda_1\mathbf{I})$ is 1, where $N(\mathbf{A})$ denotes the null space of $\mathbf{A}$. Which of the following statements are true?
(A) $\mathbf{A}$ is diagonalizable.
(B) $\mathbf{A}$ is not symmetric.
(C) $\text{rank}(A) \geq 2$.
(D) λ_1 must be a real number.
(E) Let $\mathbf{x}_1$ be an eigenvector of $\mathbf{A}$. Then, there exists $\mathbf{x}_2$ such that $\mathbf{A}\mathbf{x}_2 = \mathbf{x}_1 + \lambda_1\mathbf{x}_2$.

22. Let $\mathbf{A} \in \mathbb{R}^{m\times n}$ and $\mathbf{A}$ is non-zero. Let $\dim(S)$ denote the dimension of a subspace S. Which of the following statements are true?
(A) $\mathbf{AA}^T$ is diagonalizable.
(B) All eigenvalues of $\mathbf{AA}^T$ are positive.
(C) $\dim(N(\mathbf{AA}^T)) = \dim(N(\mathbf{A}^T\mathbf{A}))$.
(D) If $\mathbf{AA}^T\mathbf{x} = \mathbf{0}$, then $\mathbf{A}^T\mathbf{x} = \mathbf{0}$.
(E) $\mathbf{AA}^T$ can have eigenvalues that are complex, meaning their real parts are non-zero.

23. Let $\mathbb{S}^{2\times2}$ be the set of all 2×2 symmetric matrices. Then $\mathbb{S}^{2\times2}$ is a subspace of $\mathbb{R}^{2\times2}$. Let $L: \mathbb{S}^{2\times2} \to \mathbb{S}^{2\times2}$ be a linear operator defined as $L(\mathbf{P}) = \mathbf{B}^T\mathbf{P} + \mathbf{PB}$, and $E = \{\mathbf{S}_1, \mathbf{S}_2, \mathbf{S}_3\}$ is an ordered basis of $\mathbb{S}^{2\times2}$, where

$$\mathbf{S}_1 = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}, \mathbf{S}_2 = \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}, \mathbf{S}_3 = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}, \mathbf{B} = \begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}.$$

Let $\mathbf{A}$ be the matrix representation of L with respect to E. Denote the last two rows of $\mathbf{A}$ by $[a_{21}, a_{22}, a_{23}]$ and $[a_{31}, a_{32}, a_{33}]$, respectively. Which of the following statements are true?
(A) $a_{21} = 1$ (B) $a_{22} = 6$ (C) $a_{31} = 0$ (D) $a_{32} = 4$ (E) $a_{33} = 4$

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：控制系統【電機系碩士班乙組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

科目名稱：控制系統【電機系碩士班乙組】　　題號：431008

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

Problem 1. Consider the state equations (in Jordan Canonical Form) as follows:

$$\dot{x}=\begin{bmatrix}3&0&0&0&0&0&0\\0&3&0&0&0&0&0\\0&0&3&1&0&0&0\\0&0&0&3&0&0&0\\0&0&0&0&2&1&0\\0&0&0&0&0&2&1\\0&0&0&0&0&0&2\end{bmatrix}x+\begin{bmatrix}1&0&a\\0&1&2\\0&0&0\\3&0&1\\1&3&2\\1&1&0\\3&3&0\end{bmatrix}u,\ y=\begin{bmatrix}1&0&b&0&1&2&0\\0&1&1&0&0&0&1\\2&0&1&0&1&1&0\end{bmatrix}x.$$

(a) Determine the value of a so that the system is not controllable. (5 points)

(b) Determine the value of b so that the system is not observable. (5 points)

Problem 2. The transfer function of a system is given by

$$\frac{Y(s)}{U(s)}=G(s)=\frac{3}{(s-2)\,(s-4)(s-7)}$$

where U and Y are the Laplace transforms of input u and output y, respectively.

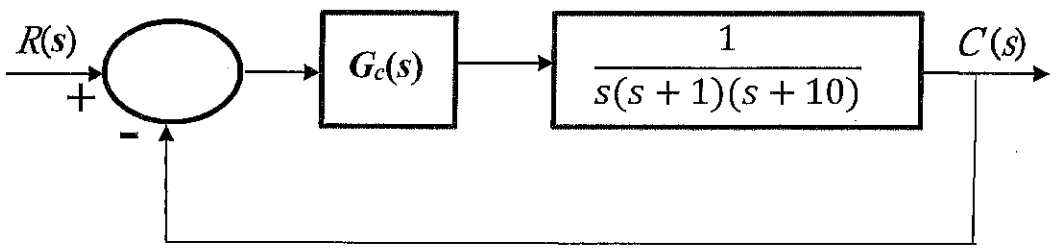
(a) Find the state-space model if $x_1=y$, $x_2=\dot{x}_1$ and $x_3=\dot{x}_2$. (2 points)

(b) Design a state feedback $u=-Kx+v$ with $K=[K_1 \quad K_2 \quad K_3]$ so that the characteristic equation of the closed-loop system is $s^3+5s^2+8s+6=0$. (4 points)

(c) When use the Luenberger type observer: $\dot{\hat{x}}=A\hat{x}+Bu+L(y-C\hat{x})$ to estimate the real state x, design the observe gain $L=[L_1 \quad L_2 \quad L_3]^T$ so that the roots of characteristic polynomial of the error system are located at $s=-3\pm j$ and $s=-2$. (4 points)

(d) Use the dynamic output feedback $u=-K\hat{x}+v$ to compensate the system where K is chosen as in problem (b), and $\hat{x}$ is a solution of $\dot{\hat{x}}=A\hat{x}+Bu+L(y-C\hat{x})$ with L as in problem (c). Find the transfer function of the new closed loop system (from v to y). (5 points)

Problem 3. Consider the control system shown below:



(a) Draw the root-locus for the closed-loop with $G_c(s)$ being K. (5 points)

(b) When $G_c(s)$ is a PD compensator, design the compensator for the following time-domain specifications (using approximation and the dominant poles): (10 points)

i) Damping ratio $\zeta=\sqrt{2}/2$.

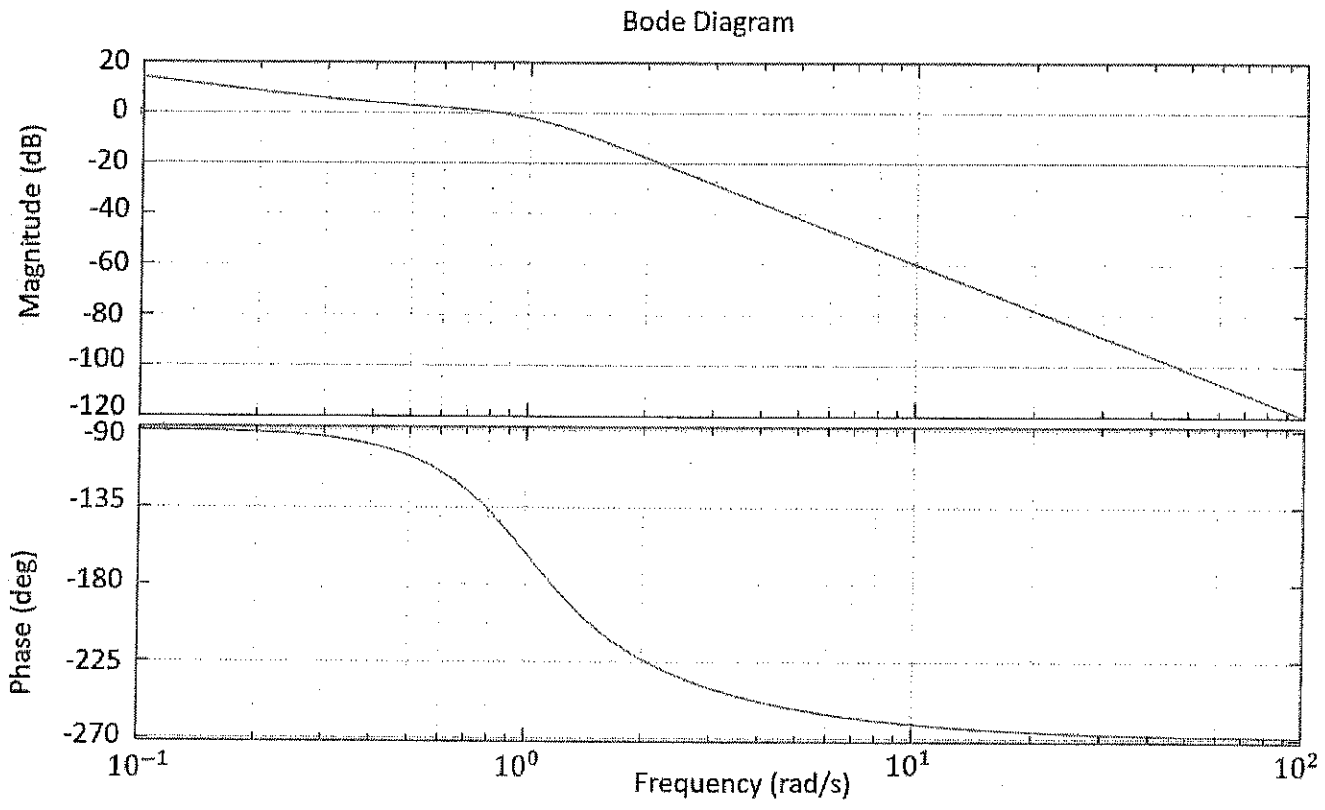
ii) Time constant τ (defined as $1/\zeta\omega_n$) $=1/2$.

Problem 4. A polynomial can be factored as $(s^4 + as^2 + b)(s^6 + cs^4 + ds^2 + e)$ where a, b, c, d and e are constants with a>0 and b>0. Find the condition on a and b so that $s^4 + as^2 + b$ has just two roots on left half-plane. (5 points) In this case, if we know that $s^6 + cs^4 + ds^2 + e$ has two roots on imaginary axis, find the pair (L, I, R) where L, I, and R are the numbers of its roots located on left half-plane, imaginary axis, and right half-plane, respectively. (5 points)

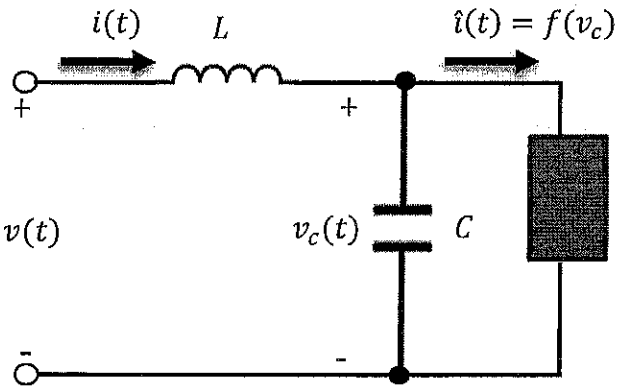
Problem 5. A unit feedback system has the forward-path transfer function $G(s) = \frac{2s^2-s-4K}{s^3+3s^2+2s+6}$. When K=1, find GM. (5 points) Find K>0 such that GM=0. (5 points)

Problem 6. A unit feedback system has the forward-path transfer function $G(s) = \frac{K}{s^2+(a+1)s+a}$. Find the values of K and a to satisfy the following frequency domain specifications: $M_r = 1.05$ and $\omega_r = 12$ rad/sec . (5 points) Calculate the peak percent overshoot of the step response and the bandwidth of the closed-loop system. (5 points)

Problem 7. For a transfer function with the following bode diagram, find PM. (5 points) When the input is $\cos^2(0.5t)$-$\cos(5t)\sin(5t)$, find the steady state output y_{ss}. (5 points)



Problem 8. Consider the following circuit where the block represents a device with the current-voltage characteristic function f. With $x_1 = v_c, x_2 = i$, write down the state space model. (3 points) When $v(t) \equiv 1$ is constant, determine the equilibrium point in case of $f(x)=x^5 - x^3 + 1$. (2 points) Find the linearized system at this equilibrium point. (5 points).



Problem 9. Consider the dynamic equations $\frac{dx(t)}{dt} = Ax(t) + Bu(t), \quad y(t) = Cx(t)$ where

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \; C = [1 \quad 0 \quad a].$$

Can the system be transferred into controllability canonical form (CCF)? (2 points) Find the all possible values of a so that the system is not observable. (3 points) Find the transformation P so that $x(t) = P\bar{x}(t)$ could transform the state equation into the observability canonical form (OCF) when a=1. (5 points)

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：離散數學【電機系碩士班丙組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
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- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

科目名稱：離散數學【電機系碩士班丙組】　　題號：431011

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. 已知 x 為自然數 1 到 100 之和。求 $(100!/4!) \bmod x$。請寫出計算過程（無過程不計分）。並請把答案寫在答案紙上靠左對齊，並在答案前標上 "Ans:"，例如："Ans: 123"。若無標示或未將答案寫在答案紙上靠左對齊，則會扣 5 分。（20 分）

2. 已知 G 為簡單圖（Simple Graph），且 G 有 N 個頂點與 M 個連通部分（Connected Component），求 G 最大邊數（答案需展開整理成標準多項式）。請寫出計算過程（無過程不計分）。並請把答案寫在答案紙上靠左對齊，並在答案前標上 "Ans:"，例如："Ans: 123"。若無標示或未將答案寫在答案紙上靠左對齊，則會扣 5 分。（20 分）

3. 投擲一正常硬幣（含正面與反面）六次，並記錄下來，求出現正面次數至少三次之機率（以最簡分數作答）。請寫出計算過程（無過程不計分）。並請把答案寫在答案紙上靠左對齊，並在答案前標上 "Ans:"，例如："Ans: 123"。若無標示或未將答案寫在答案紙上靠左對齊，則會扣 5 分。（20 分）

4. 八間房排成一直線，有兩塊紅門牌、兩塊綠門牌、兩塊藍門牌、兩塊黃門牌，分配給這八間房，問有幾種分配法滿足相鄰兩房需不同色。請寫出計算過程（無過程不計分）。並請把答案寫在答案紙上靠左對齊，並在答案前標上 "Ans:"，例如："Ans: 123"。若無標示或未將答案寫在答案紙上靠左對齊，則會扣 5 分。（20 分）

5. 已知 $F(n) = 2F(n/2) + n\log n$，求 Big-Theta，即 $\Theta(F(n))$。請寫出計算過程（無過程不計分）。並請把答案寫在答案紙上靠左對齊，並在答案前標上 "Ans:"，例如："Ans: 123"。若無標示或未將答案寫在答案紙上靠左對齊，則會扣 5 分。（20 分）

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：資料結構【電機系碩士班丙組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

科目名稱：資料結構【電機系碩士班丙組】 **題號：431004**

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）

1. 【此題15分】

圖1是用類似C/C++語言所撰寫的函式（function），名為maxHeapify，功能如下：假設max-heap裡頭有 n 個點，每個點的內容皆為正整數，儲存在陣列A裡頭A[0]至A[n-1]的地方。假設以A[i]的左小孩為root的subtree已經是max-heap、以A[i]的右小孩為root的subtree也已經是max-heap，但A[i]可能違反max-heap order property；在這種情況下呼叫函式maxHeapify(A, i, n)，可以使得以A[i]為root的subtree成為max-heap。

現在，給定陣列B，裡頭有 n 個元素，皆為正整數，儲存在B[0]至B[n-1]的地方；陣列B裡頭的這 n 個元素沒有任何規律。限定必須呼叫maxHeapify函式，並以recursive的方式撰寫buildMaxHeap函式，使得當我們呼叫buildMaxHeap(B, 0, n)時，便能將具有 n 個元素的陣列B調整成為root為B[0]的max-heap。

註：(1) 限定使用C、C++、或pseudo-code撰寫buildMaxHeap函式；函式裡頭如果有區域變數，就必須註解說明每一個區域變數所代表的含意。(2) 如果buildMaxHeap函式不是以recursive方式撰寫，或者沒有呼叫maxHeapify函式，此題以0分計算。

```
void maxHeapify(unsigned int A[], int i, int n) {
   int largest;
   /* A[largest] 將會是A[i], A[leftChild], A[rightChild]
      這三個裡頭最大的元素  */
   int leftChild; // A[leftChild]是A[i]的左小孩
   int rightChild;  // A[rightChild]是A[i]的右小孩
   while (true) {
      largest = i;
      leftChild = 2 * i + 1;
      rightChild = 2 * i + 2;
      if (leftChild < n and A[leftChild] > A[largest])
         largest = leftChild;
      if (rightChild < n and A[rightChild] > A[largest])
         largest = rightChild;
      if (largest == i)
         break;
      else {
         swap(A[i], A[largest]);
         // 交換A[i]和A[largest]的內容
         i = largest;
      }
   }
}
```

圖1：maxHeapify函式

國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

科目名稱：資料結構【電機系碩士班丙組】 **題號：431004**

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）

2. 【此題 45 分；第①小題 15 分，第②小題 15 分，第③小題 15 分】

① 假設演算法的執行時間為 $T(n)=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}=\sum_{i=1}^{n}\frac{1}{i}=\Theta(f(n))$，其中 $n\geq 1$ 為演算法的 input size。求出 $f(n)$。注意：$f(n)$ 必須是最簡（simplest）型式。

② 假設演算法 A 和演算法 B 都能解決問題 Q，其中演算法 A 的執行時間為 $n^{\log n}$，演算法 B 的執行時間為 $\log(n^n)$，n 為 input size。假設 n 的值很大，並且我們用時間複雜度來判斷演算法的好壞，那麼我們該選用哪一個演算法來解決問題 Q 呢？

③ 參考圖 2 的函式 `F1`。假設陣列 `A` 的大小為 n，函式 `F2(A, 0, n-1)` 的執行時間為 $\Theta(n)$。`F1(A, 0, n-1)` 的執行時間為 $\Theta(f(n))$。推導出函數 $f(n)$；$f(n)$ 必須是最簡型式。

註：每一小題都必須寫出推導過程；若直接猜答案，沒有推導過程，該小題以 0 分計算。

```
void F1(int A[], int first, int last) {
   if (first < last) {
      int middle = (first + last) / 2;
      F1(A, first, middle);
      F1(A, middle + 1, last);
      F2(A, first, last);
   }
}
```

圖 2：用 C/C++語言所撰寫的函式 `F1`

3. 【此題 10 分】給定 A＝3、B＝2、C＝5、D＝4、E＝6，求出下列後序（postfix）運算式的計算結果。註：此題必須要有推導過程；若直接猜答案，此題以 0 分計算。

C D E - + A B C + * -

4. 【此題 10 分】假設陣列 A 裡的元素皆為正整數，且最多能容納 m 個元素，但目前只儲存了 n 個元素，其中 $n<m$。假設陣列 A 裡的元素已經排序，那麼給定正整數 k，我們可以使用 binary search 在 $O(\log n)$ 的時間內判定陣列 A 裡頭是否有元素 k。然而，在已排序的陣列裡頭，即使不考慮搜尋時間，插入或刪除一個元素都必須耗時 $O(n)$。我們知道，在 doubly linked list 裡頭，在不考慮搜尋時間的情況下，插入或刪除一個元素都只耗時 $O(1)$。給定正整數 k，如果我們在元素皆為正整數且已經排序的 doubly linked list L 裡頭使用 binary search，能否在 $O(\log n)$ 的時間判定 L 裡頭是否有元素 k？先回答你的答案，然後才解說你的立論根據；若只是猜答案，未提供任何解說，此題以 0 分計算。

5. 【此題20分；第①小題10分，第②小題10分】

Graph是由點（node）和邊（edge）所構成的集合。圖3為一undirected graph的adjacency-lists，裡頭的數字為node的ID（編號）。

① 寫出執行depth-first search，從node 0開始拜訪所有nodes所依序產生的node編號。

② 寫出執行breadth-first search，從node 0開始拜訪所有nodes所依序產生的node編號。

注意：當一個node有多個鄰居（neighboring nodes）時，不能按任意次序拜訪，必須按adjacency-list裡頭的次序拜訪。

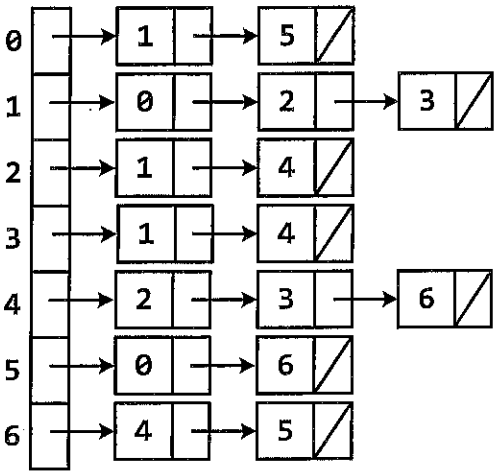

圖3：undirected graph的adjacency-lists，其中數字表示node的編號、「/」表示NULL

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：電路學【電機系碩士班丁組】

－作答注意事項－

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
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- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

科目名稱：電路學【電機系碩士班丁組】　　題號：431006

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. In Fig. 1, please **prove** that the maximum power can be transferred to R_L is $V_{Th}^2 / 4R_{Th}$. And this happens when $R_{Th} = R_L$. (10%)

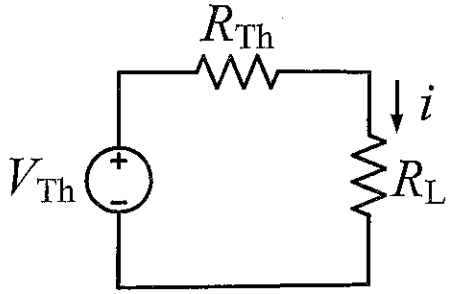
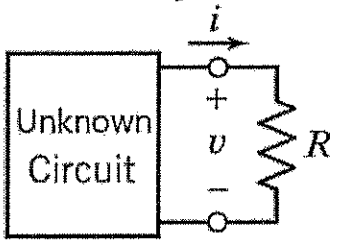


Fig. 1

2. In Fig. 2, by varying R, different current values of i are listed in the table. (10%)
   (a) What value of R is required to cause $i = 2$ mA? (5%)
   (b) Given $R > 0$, what is the maximum possible value of i ? (5%)



R	i
2 kΩ	4 mA
4 kΩ	3 mA

Fig. 2

3. For the circuit in Fig. 3, element A is a nonlinear resistor with the feature of $i = v^2$. Please determine the values of v and I respectively. (10%)

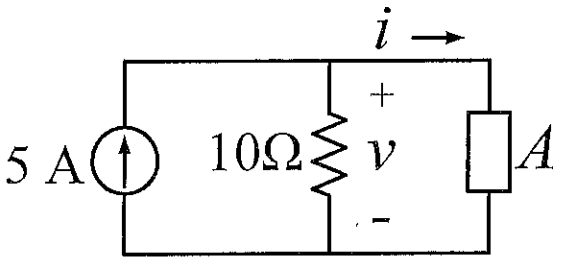


Fig. 3

4. As in Fig. 4, assuming that the switch has been closed for a long time, determine $i_L(t)$ for $t > 0$. (10%)

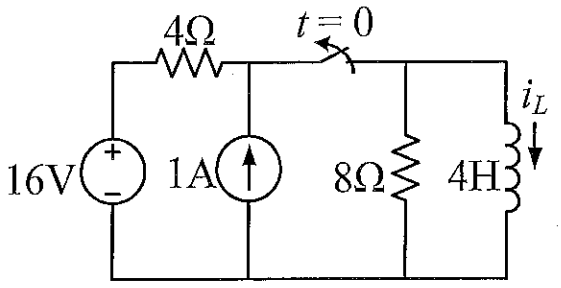


Fig. 4

5. As in Fig. 5, the switch has been open for a long time, find the output voltage $v_o(t)$ for $t > 0$. (10%)

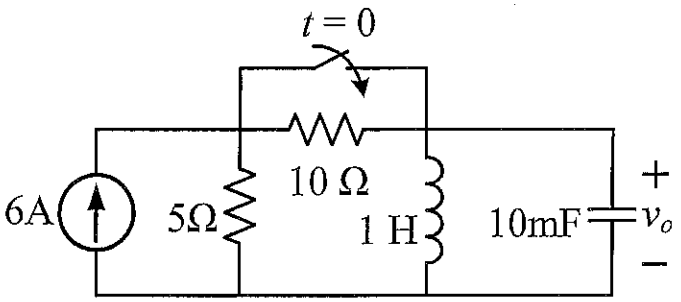


Fig. 5

6. For the circuit in Fig. 6, find the input impedance $\mathbf{Z}_{in}$. (10%)

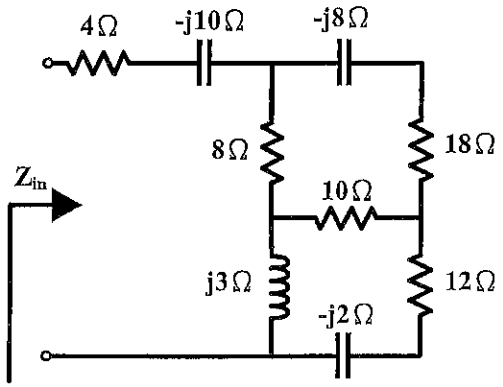


Fig. 6

7. Find average power consumed on 100Ω resistor in Fig. 7 if $i_s = 20sin(50t + 30°)$ A. (10%)

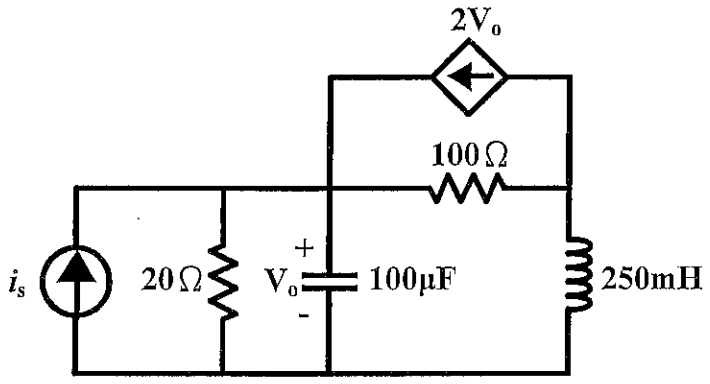


Fig. 7

8. Calculate the phase shift of the circuit between V_o and V_{in} in Fig. 8. (10%)

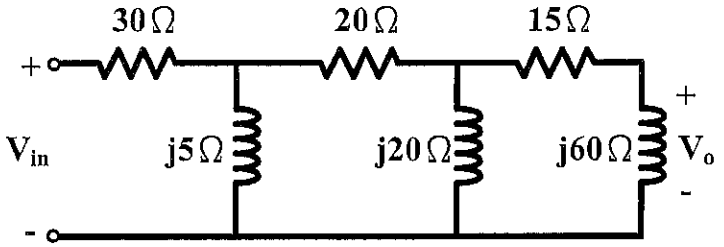


Fig. 8

9. Find the Thevenin equivalent circuit at terminals *a-b* in the circuit of Fig. 9. (10%)

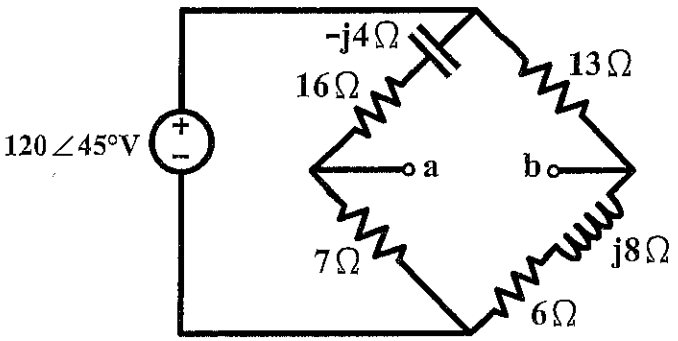


Fig. 9

10. A three-phase transmission line has an impedance of 1 + j2 Ω per phase. If it supplies a load with 4200Vrms line voltage at 1 MVA 0.75 power factor (lagging), find the power loss in the line and the line voltage at the source end. (10%)

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：電磁學【電機系碩士班戊組、通訊所碩士班乙組、電波聯合碩士班】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
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- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

科目名稱：電磁學【電機系碩士班戊組、通訊所碩士班乙組、電波聯合碩士班】題號：482004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　

1. (25%) The radius of the inner conducting sphere and the inner radius of the outer spherical conductor are R_i and R_o, respectively. The voltage between these two concentric spherical conductors is V. The space between the conductors is filled with a dielectric medium with the permittivity ε. Determine the stored electrostatic energy.

2. (25%) The magnetic flux density vector is $\mathbf{B} = (5kx+4)a_x - (3ky+10y)a_y + (8kz)a_z$ in free space. Please determine the value of the constant *k*.

3. (10%) Write the frequency-domain Maxwell's equations with time-varying source. Define phase velocity and group velocity.

4. (15%) The magnetic field intensity of a linearly polarized uniform plane wave propagating in the +y-direction in seawater [$\varepsilon_r = 80$, $\mu_r = 1$, $\sigma = 4$ (S/m)] is

$$\vec{H} = \hat{a}_x 0.1 \sin\left[10^{10}\pi t - \frac{\pi}{3}\right] \quad \text{(A/m)}$$

at $y = 0$.

a) Determine the attenuation constant, the phase constant, the intrinsic impedance, the phase velocity, the wavelength, and the skin depth. (5%)

b) Find the location at which the amplitude of $\vec{H}$ is 0.01 (A/m). (5%)

c) Write the expressions for $\vec{E}(y, t)$ and $\vec{H}(y, t)$ at $y = 0.5$ (m) as function of *t*. (5%)

5. (10%) For the case of oblique incidence of a uniform plane wave with perpendicular polarization on a perfectly conducting plane boundary as shown in Fig. 1, write (a) the instantaneous expressions

$$\vec{E}_1(x,z;t) \text{ and } \vec{H}_1(x,z;t)$$

For the total field in medium 1, using a cosine reference, (5%) and (b) the time-average Poynting vector. (5%)

6. (5%) A standard air-filled S-band rectangular waveguide has dimensions $a = 7.21$ (cm) and $b = 3.40$ (cm). What mode types can be used to transmit electromagnetic waves having the 5-cm wavelengths?

7. (10%) Find the input impedance of the lossless transmission line shown in Fig. 2.

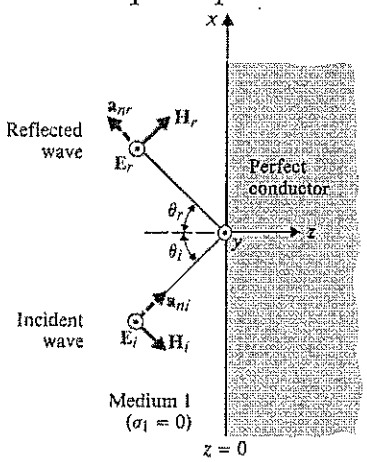

Fig. 1

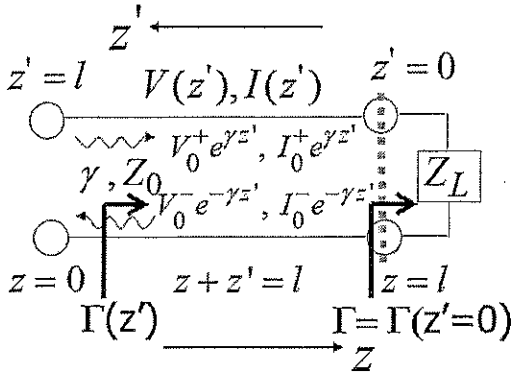

Fig. 2

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

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—作答注意事項—

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國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

科目名稱：電子學【電機系碩士班戊組選考、通訊所碩士班乙組選考、電波聯合碩士班選考】題號：482003

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　

1. (15%) A third-order low-pass filter has transmission zeros at ω = 2 rad/s and at ω = ∞. Its natural modes are at s = −1 and s = −0.5 ± j0.8. The dc gain is unity. Find the transfer function $T(s)$. (15%*1)

2. (30%) For the common-base circuit in Fig. 1, assuming the bias current to be about 1 mA, β = 100, C_μ = 0.5 pF, r_e = 25 Ω, and f_T = 1000 MHz:
   (a) Estimate the midband gain V_o/V_s.
   (b) Use the short-circuit time-constants method to estimate the lower 3-dB frequency, f_L. (*Hint:* In determining the resistance seen by C_1, the effect of the 47-kΩ resistor must be taken into account.)
   (c) Find the high-frequency poles, and estimate the upper 3-dB frequency, f_H. (10%*3)

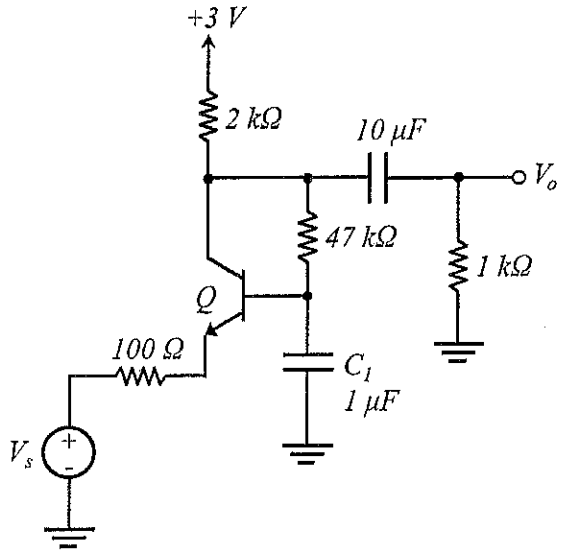


Fig. 1

3. (20%) For the emitter-follower circuit shown in Fig. 2 the BJT used is specified with a β value of 100, find :
   (a) I_E, V_E, and V_B. (10%)
   (b) the input resistance R_i. (5%)
   (c) the voltage gain v_o/v_s. (5%)

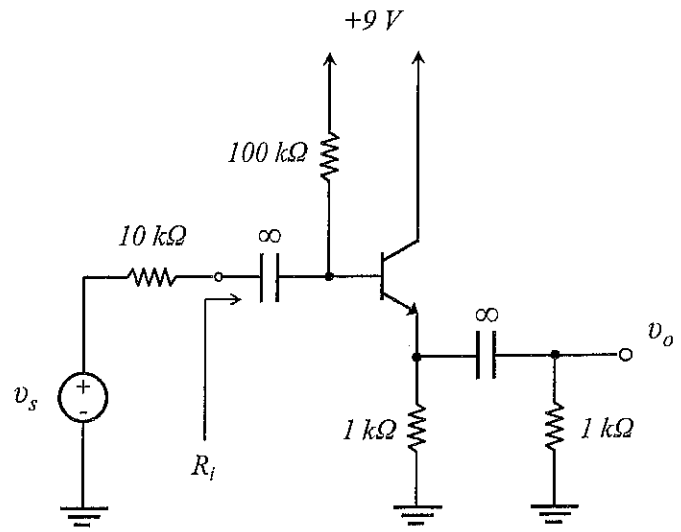
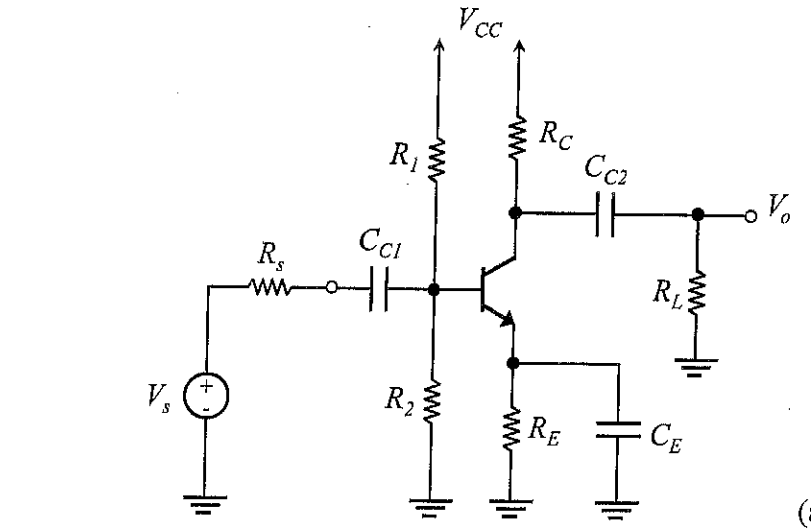
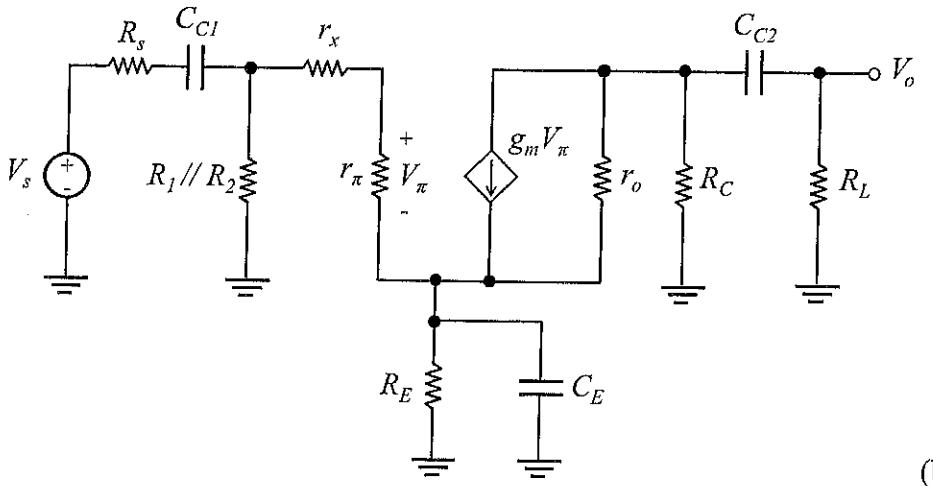


Fig. 2

4. (35%) Consider the common-emitter amplifier of Fig. 3 under the following conditions: R_s = 5 kΩ, R_1 = 33 kΩ, R_2 = 22 kΩ, R_E = 3.9 kΩ, R_C = 4.7 kΩ, R_L = 5.6 kΩ, V_{CC} = 5 V. The dc emitter current can be shown to be $I_E \approx 0.33$ mA, at which $\beta_0 = 120$, r_o = 300 kΩ, and r_x = 50 Ω.
   (a) Find the input resistance, R_{in}. *(Hint:* $R_{in} = R_1 \,//\, R_2 \,//\, (r_x + r_\pi)$*)* (10%)
   (b) Find the midband gain, A_M. (10%)
   (c) For $C_{C1} = C_{C2}$ = 5 μF and C_E = 20 μF, estimate the low-frequency 3-dB frequency. Also find the frequency of the zero introduced by C_E. (15%)



(a)



(b)

Fig. 3. (a) Common-emitter amplifier stage; (b) Equivalent circuit for the amplifier of Fig. 3(a) in the low-frequency band.

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

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國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

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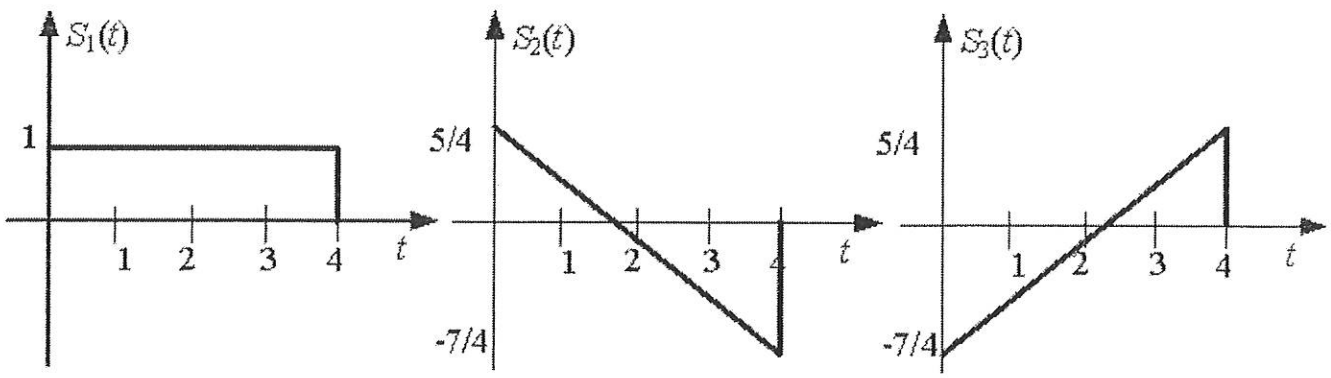
※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　

1. (20%) We are given the complex baseband signal $x_z(t) = \text{sinc}(t-1) + j2\text{sinc}(t)$.
   (A) (5%) Calculate the real and imaginary parts of the Fourier transform $X_z(f)$. To express the transforms, please use the function $\text{rect}(t)$, which is defined as a rectangle of unit height and spanning the interval [-1/2,1/2].
   (B) (5%) Plot the real and imaginary parts of $X_z(f)$.
   (C) (10%) Plot the real and imaginary parts of the Fourier transform of the bandpass signal obtained by upconverting $x_z(t)$ to the carrier frequency of 10 Hz.

2. (20%) Let $x(t) = m(t) + \cos(\omega_c t)$. Let W be the bandwidth of $m(t)$. Assume that the average value of $m(t)$ is zero and that the maximum value of $|m(t)|$ is M. Also assume that the square-law device is defined by $y(t) = 4x(t) + 2x^2(t)$.
   (A) (5%) Write the equation for $y(t)$.
   (B) (10%) Describe the filter with input signal $y(t)$ that produces an AM signal for $g(t)$, where $g(t)$ represents the output of the filter.
   (C) (5%) Specify the requirement of M to ensure no distortion when using envelope demodulation.

3. (10%) A transmitter uses a carrier frequency of 1000 Hz, with the unmodulated carrier represented as $A_c \cos(2\pi f_c t)$. Determine both the phase and frequency deviation for each of the following transmitter outputs:
   (A) (5%) $x_c(t) = \cos[2\pi(1000)t + 40\sin(5t^2)]$
   (B) (5%) $x_c(t) = \cos[2\pi(600)t]$

4. (10%) Consider the following three signals:



   (A) (6%) Use Gram-Schmidt procedure to find the set of basis functions from the three signals and determine the dimensionality of the set.

(B) (2%) Find the signal-space representation of the three signals based on the basis functions obtained in (a). (Represent the signals in terms of vectors)

(C) (2%) Determine the minimum distance between any pair of waveforms.

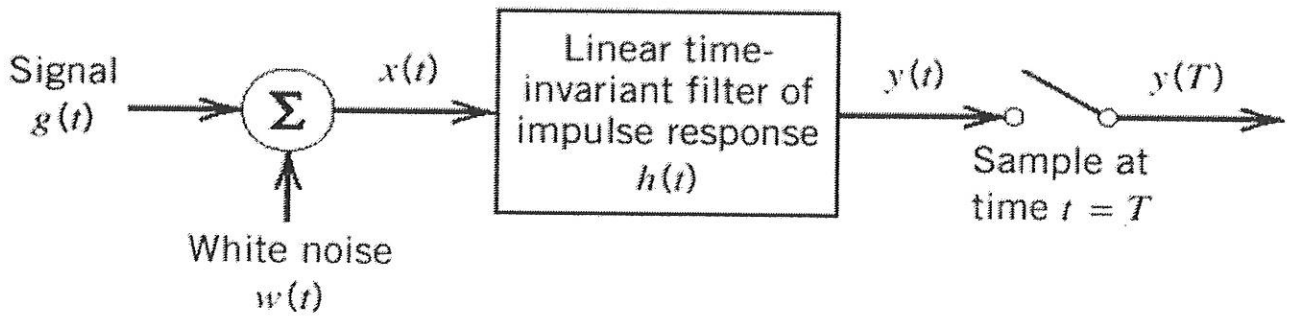
5. (20%) Let $x(t)$ denote a real valued WSS random process with an autocorrelation function $R_x(\tau)$ and $y(t) = x(t)\cos(2\pi f_c t + \theta), \theta \sim U(0,2\pi)$.

(A) (5%) Find $R_y(\tau)$.

(B) (5%) $y(t)$ is passed through a low-pass filter with a frequency-domain response $H(f) = \begin{cases} 1, & |f| < f_c \\ 0, & |f| > f_c \end{cases}$ and $\tilde{y}(t)$ denotes the filter output. Find $E[y(t)^2]$.

(C) (10%) If we let $\theta = \frac{\pi}{4}$ and $r(t) = s(t)\cos(2\pi f_c t) + y(t)$, please show how to demodulate $s(t)$ based on $r(t)$ in detail.

6. (10%) Let $x(t) = g(t) + w(t),\ 0 \le t \le T$, be the received noisy signal, where $g(t)$ denotes the transmitted pulse that represents a binary symbol 0 or 1 and $w(t)$ denotes an additive white noise process with zero mean and power spectral density (PSD) $\frac{N_0}{2}$. Since the filter is linear, the result output can be express as $y(t) = g_o(t) + n(t)$, where $g_o(t)$ denotes the response to $g(t)$ and $n(t)$ denotes the response to $w(t)$. We know that the peak pulse signal to noise ratio of the match filter is $\eta = \frac{|g_0(T)|^2}{E[n^2(t)]}$. Please show that $\eta \le \frac{2}{N_0}\int_{-\infty}^{\infty} |G(f)|^2\, df$.



7. (10%) Consider two discrete random variables X and Y with the joint distribution:

$P(x,y)$	$X=-1$	$X=0$	$X=1$
$Y=2$	0.1	0.15	0.15
$Y=4$	0.05	0.2	0.15
$Y=6$	0.05	0.05	0.1

國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

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(A) (2%) Find the entropy $H(X,Y)$.

(B) (2%) Find the entropy $H(X)$.

(C) (4%) Find the entropy $H(X|Y)$.

(D) (2%) Find the mutual information $I(X;Y)$.

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：計算機結構【電機系碩士班己組】

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考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

科目名稱：計算機結構【電機系碩士班己組】　　題號：431007

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. **[20%]** In hardware, SYS-1 system has a CPU with a base CPI of X, where X is larger than zero, assuming all references hit in the primary cache (L1) and a CPU clock rate of 2GHz. Assume that the main memory access time is 160 ns, including all the cache miss handling. Suppose the miss rate per instruction at L1 is 5%. Based on the construction of SYS-1 system, there are two systems shown as follows.

   (i) SYS-2 system includes SYS-1 and an additional secondary cache (L2). The access time of L2 is 28 ns for either a hit or a miss. Suppose the miss rate per instruction at L2 is Y, where 0 < Y < 1.

   (ii) SYS-3 system includes SYS-2 and an additional tertiary cache (L3). The access time of L3 is 44 ns for either a hit or a miss. Suppose the miss rate per instruction at L3 is 0.1%.

   We know that SYS-2 system is faster than SYS-1 system by 2 times. In addition, SYS-3 system is faster than SYS-2 system by 1.25 times. Please answer the following questions.

   (a) **(4%)** As regards SYS-1 system, please calculate the memory-stall cycles per instruction.

   (b) **(6%)** Please calculate X.

   (c) **(6%)** Please calculate Y.

   (d) **(4%)** According to the result calculated in (b), the program has three different instruction categories, such as A, B, and C. We know that CPI for A, B, and C is 3, 5, and 6, respectively. The instruction count of A and B is 48 and 30, respectively. Please calculate the instruction count of C.

2. **[10%]** A program, SVM, has the total execution time of 500 ns and can be well-divided into P1 - P4 parts. P1 - P3 can be modified to increase or decrease the individual execution time by the certain approaches. But P4 still keeps identical without any influence. The execution time of P1 - P4 is denoted as T1 - T4, respectively. Assume that T1 - T4 originally are 144 ns, 108 ns, 120 ns, and 128 ns, respectively. There are three different design scenarios as follows.

   ✧ *Design scenario 1: If T1 and T2 are improved by X and Z times, respectively, T3 is increased to 1.34 times accordingly. The corresponding total execution time is decreased to 476 ns.*

   ✧ *Design scenario 2: If T1 and T3 are improved by Y and Z times, respectively, T2 is increased to 1.5 times accordingly. The corresponding total execution time is decreased to 490 ns.*

   ✧ *Design scenario 3: If T2 and T3 are improved by Y and X times, respectively, T1 is increased to 1.375 times accordingly. The corresponding total execution time is increased to 512 ns.*

   Which one is the possible solution of (X, Y, Z)? _____ (***single choice***)

(A) (1.25, 1.5, 1.4)	(B) (1.25, 1.2, 1.75)	(C) (1.4, 1.2, 1.25)	(D) (1.25, 1.5, 1.6)
(E) (1.5, 1.2, 1.25)	(F) (1.4, 1.2, 1.5)	(G) (1.75, 1.4, 1.5)	(H) (1.25, 1.2, 1.25)
(I) (1.25, 1.5, 1.25)	(J) (1.4, 1.75, 1.5)	(K) (1.25, 1.2, 1.5)	(L) (1.4, 1.6, 1.5)

3. **[25%]** Assume that X1 – X7 and Y1 – Y7 are well-represented as the 32-bit single-precision format in IEEE-754 standard as shown in **TABLE I**. We know that the relationship is calculated as follows, $W1 = X1 \times Y1, W2 = X2 \times Y2, \ldots\ldots, W7 = X7 \times Y7$. Please answer the following questions.

(a) **(2%)** If $10^n \leq X5 \leq 10^{n+1}$, where $n \in Z$, please find out the value of n.

(b) **(2%)** If $-10^m \leq Y2 \leq -10^{m-1}$, where $m \in Z$, please find out the value of m.

(c) **(21%)** Please sort these seven numbers (W1 – W7) in descending order.

TABLE I

	Bit 31																								Bit 0
X1	1	0	0	0	0	0	1	1	0	1	1	0	0	1	1	0	0	0	1	0	0	0	0	0	0
Y1	0	1	1	0	0	0	1	0	1	0	1	1	0	1	0	0	1	1	1	1	1	0	0	0	0
X2	0	0	0	1	0	0	1	1	1	0	1	0	1	1	1	0	1	1	0	0	1	0	0	0	0
Y2	1	1	0	1	0	0	0	1	0	0	0	1	0	0	1	0	0	0	1	1	0	0	0	0	0
X3	0	0	1	1	0	1	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0
Y3	1	0	1	0	1	1	1	0	1	0	1	0	1	0	0	1	1	1	0	0	0	0	0	0	0
X4	1	1	0	1	1	1	0	0	1	1	0	1	1	0	0	0	1	0	0	1	0	0	0	0	0
Y4	0	0	0	0	1	0	0	1	0	0	0	1	1	1	1	1	1	0	1	0	1	0	0	0	0
X5	0	0	0	1	1	1	1	0	0	0	1	0	1	1	0	0	0	1	1	0	0	0	0	0	0
Y5	1	1	0	0	0	1	0	1	1	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0
X6	1	1	0	0	1	1	0	0	1	1	1	0	1	1	1	1	0	0	0	1	1	0	0	0	0
Y6	1	0	0	1	0	1	1	0	0	0	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0
X7	0	0	0	1	1	1	1	0	1	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
Y7	0	1	0	0	0	1	0	0	0	0	0	1	1	1	1	0	1	0	1	0	1	0	0	0	0

4. **[20%]** The following mathematical equation shows the relationship among different elements in the array, A[ ]. After executing the MIPS codes listed in **Figure 1**, the value of A[i+S] can be computed based on A[i] – A[i+3], where i and S are two positive integers. C0 – C3 denote four integer coefficients. We know that the base address of A[ ] is stored in **$s0**. Please answer the following questions.

(a) **(8%)** Please calculate C0, C1, C2, and C3.

(b) **(4%)** Please calculate i and S.

(c) **(3%)** Please calculate the value of A[i+S] while (A[i], A[i+1], A[i+2], A[i+3]) = (-8, 10, -11, 7).

(d) **(5%)** In hardware implementation, we can utilize 5-stage pipelined MIPS CPU to execute this program. If considering the forwarding (bypass) technique to fix or relax the possible data hazard, what is the minimum number of total clock cycles to complete this program?

$$A[i+S] = C0*A[i] + C1*A[i+1] + C2*A[i+2] + C3*A[i+3] - 5$$

```
0x200   lw  $a0, 128 ($s0)
        lw  $a1, 132 ($s0)
        sub $t0, $a0, $a1
        lw  $a2, 136 ($s0)
0x210   add $t1, $a2, $a2
        sub $t1, $t1, $a1
        add $t2, $t1, $t1
        add $t2, $t2, $t0
0x220   sll $t2, $t2, 3
        sub $t2, $t2, $a0
        lw  $a3, 140 ($s0)
        sll $t0, $a3, 2
0x230   sub $t0, $t0, $a3
        add $t2, $t2, $t0
        sub $t1, $t2, $t1
        addi $t2, $t1, -5
0x240   sw  $t2, 264 ($s0)
```

Figure 1

5. **[15%]** Assume that a dedicated memory system has the parameters as shown below. Please answer the following questions.
   - ✧ *36-bit physical address*
   - ✧ *Cache (virtually-addressed but physically-tagged): direct-mapped, 16KB, 128-bit block size*
   - ✧ *TLB: fully associative, 64 entries*
   - ✧ *Virtual Memory: 42-bit virtual address, 32KB page*

   (a) **(5%)** How many bits are needed in each tag field of this cache?

   (b) **(5%)** How many bits are needed in each tag field of the TLB?

   (c) **(5%)** If each entry of the TLB has three additional status bits, such as Valid bit, Used bit, and Dirty bit, please calculate the total size of the TLB in terms of bytes.

6. **[10%] Figure 2** shows a simple program based on the MIPS codes. Assume that **$s3** = 25, **$s4** = 31, **$s5** = 19, and **$s6** = 21. If **$s0** > 0, **$s1** > 0, and **$s2** > 0, please find out all the possible values of **$s0**.

```
0x1000  div $s0, $s1
        mfhi $s5
        mflo $s3
        div $s0, $s2
0x1010  mfhi $s6
        mflo $s4
```

Figure 2