國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學甲【電機系碩士班甲組、己組、庚組、電波聯合選考：電機系碩士班戊組、通訊所碩士班乙組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學甲【電機系碩士班甲組、己組、庚組、電波聯合選考：電機系碩士班戊組、通訊所碩士班乙組】題號：431002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）

下面 1-15 題為單選題，總分 45 分。每題答對 3 分，答錯扣 4 分，未作答者以 0 分計。總分低於 0 分者以 0 分計算。

1. Consider the autonomous differential equation $y' = (2/\pi)y - \sin y$. Which of the following is INCORRECT?
   (A) There are three critical points.
   (B) One of critical point is semi-stable.
   (C) Two of critical points are unstable.
   (D) One of the critical points is 0.

2. If $y = e^{3x}\cos x$ is the solution to $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + ky = 0$, what is the value of k?
   (A) 3 (B) -2 (C) 10 (D) 8

3. The differential equation $e^x\frac{dy}{dx} + 3y = x^2y$ is linear and separable.
   (A) True (B) False

4. The improved Euler's method is what type of Runge-Kutta method?
   (A) First order (B) Second order (C) Third order (D) Fourth order

5. Consider $y(x)$ is the solution to the initial-value problem $x^2y'' - 2xy' + 2y = 0$ where $x > 0$, $y(1) = 4$, and $y'(1) = 9$, use Euler's method to compute $y(1.2)$. Given $h = 0.1$, which of the following is correct?
   (A) The general solution is $y = C_1x - C_2x^2$, where $C_1 + C_2 = 6$.
   (B) The general solution is $y = C_1x + C_2x^2$, where $C_1 + C_2 = 6$.
   (C) $y(1.2) = 5.9$.
   (D) $y(1.2) = 6$.

6. Given the three vectors (1, 0, 3, 1), (0, 1, -6, -1) and (0, 2, 1, 0) in R^4, they are linearly dependent.
   (A) True (B) False

7. Provided the system below, the rank is
$$X_1 - X_3 + 2X_4 + X_5 + 6X_6 = -3$$
$$X_2 + 2X_3 + 3X_4 + 2X_5 + 4X_6 = 1$$
$$X_1 - 4X_2 + 3X_3 + X_4 + 2X_6 = 0$$
   (A) 1 (B) 2 (C) 3 (D) 4

8. Which one of the following is correct regarding Fourier series?
   (A) $e^{-|x|}$ is odd function.
   (B) f' must be continuous on the interval $[a, b]$ to ensure that the Fourier series of f on $[a, b]$ converges to f.
   (C) $f(x) = |x|$ is continuous on $[-\pi, \pi]$.
   (D) The Fourier series of $f(x) = x^2 + 1$, where $0 < x < 3$, converges to 0 at $x = 0$.

9. Expand $f(x) = 2x^2 - 1, -1 < x < 1$ in a Fourier series and yield $f(x) = A + \sum \frac{B}{n^2\pi^2} C$. Which of the following is correct?
(A) $A = -2/3$　(B) $B = 4$　(C) $C = (-1)^n \cos n\pi x$　(D) None of the above

10. If $y_1(x) = x$ is one of the solutions of the following differential equation, what is the other linear independent solution $y_2(x)$?
$$y'' - \frac{2x}{1+x^2}y' + \frac{2}{1+x^2}y = 0$$
(A) $y_2(x) = 2x^2 + 1$　(B) $y_2(x) = \frac{x^2-1}{x}$　(C) $y_2(x) = \frac{1}{x} - 1$　(D) $y_2(x) = x^2 - 1$

11. Use the Laplace transform to solve the following initial-value problem. If the solution is $y = A + Be^{-t} + Ce^{3t} + De^{4t}$, which of the following is true?
$$y'' - 4y' = 6e^{3t} - 3e^{-t}, y(0) = 1,\ y'(0) = -1$$
(A) $A + B + C + D = 1$.
(B) $B = -2$
(C) $A + B + D = 2$
(D) All of the above

12. The Laplace transform of a function f is denoted by $\mathcal{L}\{f\}$. If $\mathcal{L}\{f(t)\} = F(s)$ and $\mathcal{L}\{g(t)\} = G(s)$, then $\mathcal{L}^{-1}\{F(s)G(s)\} = f(t)g(t)$.
(A) True　(B) False

13. If $\mathcal{L}\{f(t)\}$ represents the Laplace transform of a function $f(t)$. Let $f(t) = \begin{cases} 3 & , if\ 0 \le t \le 2 \\ 5 - t & , if\ t > 2 \end{cases}$, then $\mathcal{L}\{f(t)\}$ is
(A) $\frac{3}{s^2} + \frac{e^{2s}}{s^2}$　(B) $\frac{3}{s} + \frac{e^{-2s}}{s^2}$　(C) $\frac{3}{s} - \frac{e^{-2s}}{s^2}$　(D) $\frac{3}{s^2} - \frac{e^{-2s}}{s^2}$

14. Provided the differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which of the following is true?
(A) first order, linear, non-homogeneous
(B) second order, nonlinear
(C) second order, linear, non-homogeneous
(D) second order, linear, homogeneous

15. The Fourier transform of a function f is denoted by $\Im\{f\}$. Suppose $\Im\{f(t)\} = F(\omega), \Im\{g(t)\} = g(\omega)$, which of the following is INCORRECT?
(A) $\int_{-\infty}^{\infty} f(\tau)g(t-\tau)\,d\tau = \Im^{-1}\{F(\omega)G(\omega)\}$
(B) $\int_{-\infty}^{\infty} f(t-\tau)g(\tau)\,d\tau = \Im^{-1}\{F(\omega)G(\omega)\}$
(C) $\Im\{f(t-\tau)\} = F(\omega)e^{-i\omega\tau}$
(D) None of the above

下面 16-21 題為複選題，每題 5 分，總分 30 分，每題有五個選項，其中至少有一個是正確答案，答錯 1 個選項者，得 3 分，答錯 2 個選項者，得 1 分，答錯多於 2 個選項或未作答者，該題以零分計算。

16. Let $\mathbf{A}$ and $\mathbf{B}$ be matrices in $\mathbb{R}^{n\times n}$. Which of the following statements are true?
(A) $\det(-\mathbf{A}) = -\det(\mathbf{A})$.
(B) If $\mathbf{A}\mathbf{A}^T = \mathbf{I}$, then $\det(\mathbf{A}) = 1$.
(C) If $\mathbf{A}\mathbf{A}^T = \mathbf{I}$, then $\text{trace}(\mathbf{A}) = n$.
(D) If two rows of $\mathbf{A}$ are equal, then $\det(\mathbf{A}) = 0$.
(E) If $\det(\mathbf{A}) = \det(\mathbf{B})$, then $\mathbf{A}$ and $\mathbf{B}$ have the same rank.

17. Let $\mathbf{A} \in \mathbb{R}^{3\times 3}$ and its eigenvalues are λ_1, λ_1, and λ_2, where λ_1 and λ_2 are distinct eigenvalues. Suppose the dimension of $N(\mathbf{A} - \lambda_1\mathbf{I})$ is 1, where $N(\mathbf{A})$ denotes the null space of $\mathbf{A}$. Which of the following statements are true?
(A) λ_1 must be a real number (not a complex number).
(B) λ_2 must be a real number (not a complex number).
(C) The dimension of $N(\mathbf{A} - \lambda_2\mathbf{I})$ equals 1.
(D) $\mathbf{A}$ is diagonalizable.
(E) $\mathbf{A}$ has two linearly independent eigenvectors corresponding to λ_1.

18. Let $\mathbf{A} \in \mathbb{R}^{m\times n}$. Consider the linear equation $\mathbf{A}\mathbf{x} = \mathbf{b}$ or the homogeneous linear equation $\mathbf{A}\mathbf{x} = \mathbf{0}$. Which of the following statements are true?
(A) If $\text{rank}(\mathbf{A}) = m$, then $\mathbf{A}\mathbf{x} = \mathbf{b}$ has at least one solution for any $\mathbf{b} \in \mathbb{R}^m$.
(B) If $\text{rank}(\mathbf{A}) = m$, then $\mathbf{A}\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$.
(C) If $\text{rank}(\mathbf{A}) = n$, then $\mathbf{A}\mathbf{x} = \mathbf{b}$ has at most one solution for any $\mathbf{b} \in \mathbb{R}^m$.
(D) If $\text{rank}(\mathbf{A}) = n$ and $m > n$, then $\mathbf{A}\mathbf{x} = \mathbf{0}$ has infinitely many solutions.
(E) If $\text{rank}(\mathbf{A}) = m$ and $n > m$, then $\mathbf{A}\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

19. Let $\mathbf{A}$ and $\mathbf{B}$ be square matrices. Suppose that $\mathbf{A}$ is similar to $\mathbf{B}$, that is, $\mathbf{B} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ for some nonsingular matrix $\mathbf{P}$. Which of the following statements are true?
(A) If $\mathbf{x}$ is an eigenvector of $\mathbf{B}$, then $\mathbf{x}$ is also an eigenvector of $\mathbf{A}$.
(B) If $\mathbf{y}$ is in the column space of $\mathbf{B}$, then $\mathbf{y}$ is also in the column space of $\mathbf{A}$.
(C) $\text{trace}(\mathbf{A}) = \text{trace}(\mathbf{B})$.
(D) $\mathbf{A} - \mathbf{I}$ is similar to $\mathbf{B} - \mathbf{I}$.
(E) $\mathbf{A}^5$ is similar to $\mathbf{B}^5$.

20. Let $\mathbf{A} \in \mathbb{R}^{m\times n}$, $R(\mathbf{A})$ denotes the column space of $\mathbf{A}$, $N(\mathbf{A})$ denotes the null space of $\mathbf{A}$, and $\dim(S)$ denotes the dimension of a subspace S. Which of the following statements are true?
(A) If $\mathbf{y} \in R(\mathbf{A})$, then $\mathbf{y} \in R(\mathbf{A}\mathbf{A}^T)$.
(B) If $\mathbf{x} \in N(\mathbf{A})$, then $\mathbf{x} \in N(\mathbf{A}\mathbf{A}^T)$.
(C) $\text{rank}(\mathbf{A}) + \dim\big(N(\mathbf{A})\big) = \text{rank}(\mathbf{A}^T) + \dim(N(\mathbf{A}^T))$.
(D) It is possible for a matrix $\mathbf{A}$ to have $[2, 1, -1]^T$ in $N(\mathbf{A})$ and $[1, -2, 3]^T$ in $R(\mathbf{A}^T)$.
(E) Let $\mathbf{y} \in \mathbb{R}^m$. If $\mathbf{y} = \mathbf{u}_1 + \mathbf{v}_1 = \mathbf{u}_2 + \mathbf{v}_2$, where $\mathbf{u}_1, \mathbf{u}_2 \in R(\mathbf{A})$ and $\mathbf{v}_1, \mathbf{v}_2 \in N(\mathbf{A}^T)$, then $\mathbf{u}_1 = \mathbf{u}_2$ and $\mathbf{v}_1 = \mathbf{v}_2$.

21. Let

$$\mathbf{A}=\begin{bmatrix}4 & 1 & 3 & 2\\ 1 & 4 & 3 & 3\\ -1 & 11 & 6 & 7\end{bmatrix}.$$

Which of the following vectors are in the column space of **A**?

(A) $[3,1,2]^T$
(B) $[1,0,-1]^T$
(C) $[0,1,3]^T$
(D) $[2,1,1]^T$
(E) $[4,2,-1]^T$

以下第22題到第23題需要詳明推導計算過程。如推導計算過程錯誤，將酌扣分數或不給分。

22. (10分) 求出以下複平面上之路徑積分值，z為複數。

$\int_C \frac{z^5}{1-z^3}dz$，其中$C$為沿著$\{z:|z|=2\}$正向旋轉一周之封閉路徑。

23. (15分) 利用餘值（residues）求取以下瑕積分，其中參數$a>0$。

$$\int_0^\infty \frac{\cos ax}{x^2+1}dx$$

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碩士班暨碩士在職專班招生考試試題

科目名稱：電子學(甲組)【電機系碩士班甲組】

—作答注意事項—

考試時間：100分鐘

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國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：電子學(甲組)【電機系碩士班甲組】　　題號：431009

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）(問答申論題)　　

Please note that all calculation answers must include the unit and calculation process.

1. (25pt) The amplifier of Fig. 1 consists of two identical common-emitter amplifiers connected in cascade. For V_{CC} = 15 V, R_{B1} = 100 kΩ, R_{B2} = 47 kΩ, R_E = 3.9 kΩ, R_C = 6.8 kΩ, β = 100, and thermal voltage V_T = 25 mV, (a) determine the dc collector current of Q_1. (5pt) (b) Find R_{in}, v_{b1}/v_{sig} and v_{b2}/v_{b1} for R_{sig} = 5 kΩ. (15pt) (c) For R_L = 2 kΩ, find v_o/v_{b2}. (5pt)
2. (45pt) The two-stage CMOS operational amplifier in Fig. 2 is fabricated in a 0.18-μm technology having $k_n' = 4k_p' = 400\ \mu A/V^2$, $V_{tn} = -V_{tp} = 0.4$ V, and V_{DD} = 0.9 V, V_{SS} = –0.9 V, I_{REF} = 200 μA. (a) With A and B grounded, perform a dc design that will result in each of Q_1, Q_2, Q_3, and Q_4 conducting a drain current of 100 μA and each of Q_6 and Q_7 a drain current of 200 μA. Design so that all transistors operate at 0.2 V overdrive voltages. Specify the W/L ratio required for each MOSFET. (20pt) (b) What is the dc voltage at the output (ideally)? (5pt) (c) Find the input common-mode range. (5pt) (d) Find the allowable range of the output voltage. (5pt) (e) With $v_A = v_{id}/2$ and $v_B = -v_{id}/2$, find the voltage gain v_o/v_{id}. Assume an Early voltage V_A of each MOSFET is 6 V. (10pt)
3. (30pt) A common-source amplifier as shown in Fig. 3 has $C_{C1} = C_{C2} = C_{C3}$ = 1 μF, $R_G = R_{G1} \parallel R_{G2}$ = 10 MΩ, R_{sig} = 100 kΩ, g_m = 2 mA/V, $R_D = R_S = R_L$ = 10 kΩ. Please analyze the low-frequency response of the amplifier to find midband gain v_o/v_{sig} (5pt), pole frequency f_{p1}(5pt), f_{p2}(5pt), f_{p3}(5pt), and zero frequency f_z. (10pt)

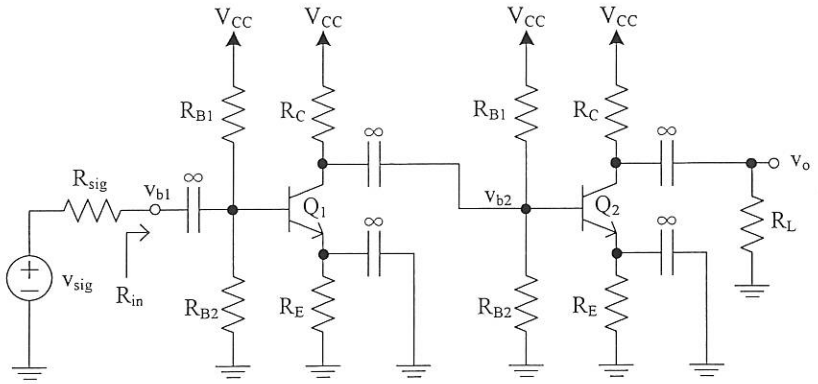

Fig. 1

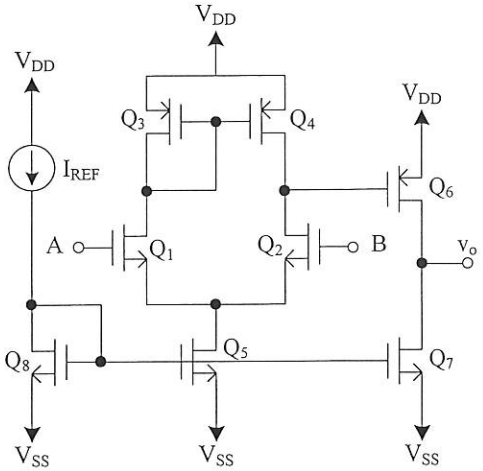

Fig. 2

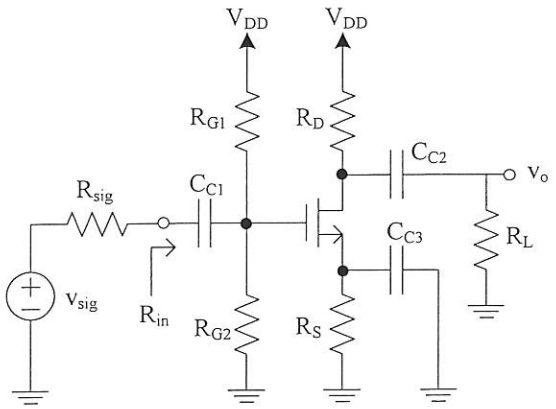

Fig. 3

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碩士班暨碩士在職專班招生考試試題

科目名稱：半導體概論【電機系碩士班甲組】

—作答注意事項—

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國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：半導體概論【電機系碩士班甲組】　　題號：431012

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

Dielectric constant: Si = 11.7 ; SiO_2 = 3.9

1. Consider a silicon p-n junction at T = 300K with doping concentrations of $N_A = 5 \times 10^{15}$ cm^{-3} and $N_D = 5 \times 10^{16}$ cm^{-3}. Calculate the depletion layer width in each side and the peak electric field at the junction. (20%)

2. An n-channel silicon MOSFET with a doping of $N_A = 5 \times 10^{15}$ cm^{-3}. Oxide thickness is 18 nm. Initial flat-band voltage is -1.25 V. Determine the ion implantation dose such that a threshold voltage of $V_T = +0.4$ V is obtained. (20%)

3. The values of effective density of states in conduction and valance bands are $N_C = 2.8 \times 10^{19}$ cm^{-3} and $N_V = 1.04 \times 10^{19}$ cm^{-3} at 300K. Both N_C and N_V vary as $T^{3/2}$. Assume the bandgap energy of silicon is 1.12 eV and does not vary over this temperature range. Calculate the intrinsic carrier concentration in silicon at T = 250K and at T = 400K. (20%)

4. A silicon p-n junction with doping concentrations of $N_A = 10^{16}$ cm^{-3} and $N_D = 10^{15}$ cm^{-3} at T = 300K. The applied reverse bias $V_R = -5$ V. Calculate the junction capacitance if the cross-sectional area of the device is 10^{-4} cm^2. (20%)

5. An n-channel MOSFET with source and drain doping concentrations of $N_D = 10^{19}$ cm^{-3} and a channel region doping of $N_A = 10^{16}$ cm^{-3}. Assume the channel length L = 1.2 μm. Source and body are at ground potential. Calculate the theoretical punch-through voltage. (20%)

國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學乙【電機系碩士班乙組】

－作答注意事項－

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學乙【電機系碩士班乙組】　　題號：431001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　

下面 **1-10** 題為複選題，每題 **5** 分，總分 **50** 分，每題有五個選項，其中至少有一個是正確答案，答錯 **1** 個選項者，得 **3** 分，答錯 **2** 個選項者，得 **1** 分，答錯多於 **2** 個選項或未作答者，該題以零分計算。

1. Let $\mathbf{A}$ and $\mathbf{B}$ be matrices in $\mathbb{R}^{n\times n}$. Which of the following statements are true?
   (A) $\det(-\mathbf{A}) = -\det(\mathbf{A})$.
   (B) If $\mathbf{A}\mathbf{A}^T = \mathbf{I}$, then $\det(\mathbf{A}) = 1$.
   (C) If $\mathbf{A}\mathbf{A}^T = \mathbf{I}$, then $\text{trace}(\mathbf{A}) = n$.
   (D) If two rows of $\mathbf{A}$ are equal, then $\det(\mathbf{A}) = 0$.
   (E) If $\det(\mathbf{A}) = \det(\mathbf{B})$, then $\mathbf{A}$ and $\mathbf{B}$ have the same rank.

2. Let $\mathbf{A} \in \mathbb{R}^{3\times 3}$ and its eigenvalues are λ_1, λ_1, and λ_2, where λ_1 and λ_2 are distinct eigenvalues. Suppose the dimension of $N(\mathbf{A} - \lambda_1\mathbf{I})$ is 1, where $N(\mathbf{A})$ denotes the null space of $\mathbf{A}$. Which of the following statements are true?
   (A) λ_1 must be a real number (not a complex number).
   (B) λ_2 must be a real number (not a complex number).
   (C) The dimension of $N(\mathbf{A} - \lambda_2\mathbf{I})$ equals 1.
   (D) $\mathbf{A}$ is diagonalizable.
   (E) $\mathbf{A}$ has two linearly independent eigenvectors corresponding to λ_1.

3. Let $\mathbf{A} \in \mathbb{R}^{m\times n}$. Consider the linear equation $\mathbf{A}\mathbf{x} = \mathbf{b}$ or the homogeneous linear equation $\mathbf{A}\mathbf{x} = \mathbf{0}$. Which of the following statements are true?
   (A) If $\text{rank}(\mathbf{A}) = m$, then $\mathbf{A}\mathbf{x} = \mathbf{b}$ has at least one solution for any $\mathbf{b} \in \mathbb{R}^m$.
   (B) If $\text{rank}(\mathbf{A}) = m$, then $\mathbf{A}\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$.
   (C) If $\text{rank}(\mathbf{A}) = n$, then $\mathbf{A}\mathbf{x} = \mathbf{b}$ has at most one solution for any $\mathbf{b} \in \mathbb{R}^m$.
   (D) If $\text{rank}(\mathbf{A}) = n$ and $m > n$, then $\mathbf{A}\mathbf{x} = \mathbf{0}$ has infinitely many solutions.
   (E) If $\text{rank}(\mathbf{A}) = m$ and $n > m$, then $\mathbf{A}\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

4. Let $\mathbf{A}$ and $\mathbf{B}$ be square matrices. Suppose that $\mathbf{A}$ is similar to $\mathbf{B}$, that is, $\mathbf{B} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ for some nonsingular matrix $\mathbf{P}$. Which of the following statements are true?
   (A) If $\mathbf{x}$ is an eigenvector of $\mathbf{B}$, then $\mathbf{x}$ is also an eigenvector of $\mathbf{A}$.
   (B) If $\mathbf{y}$ is in the column space of $\mathbf{B}$, then $\mathbf{y}$ is also in the column space of $\mathbf{A}$.
   (C) $\text{trace}(\mathbf{A}) = \text{trace}(\mathbf{B})$.
   (D) $\mathbf{A} - \mathbf{I}$ is similar to $\mathbf{B} - \mathbf{I}$.
   (E) $\mathbf{A}^5$ is similar to $\mathbf{B}^5$.

5. Let $\mathbf{A} \in \mathbb{R}^{m\times n}$, $R(\mathbf{A})$ denotes the column space of $\mathbf{A}$, $N(\mathbf{A})$ denotes the null space of $\mathbf{A}$, and $\dim(S)$ denotes the dimension of a subspace S. Which of the following statements are true?
   (A) If $\mathbf{y} \in R(\mathbf{A})$, then $\mathbf{y} \in R(\mathbf{A}\mathbf{A}^T)$.
   (B) If $\mathbf{x} \in N(\mathbf{A})$, then $\mathbf{x} \in N(\mathbf{A}\mathbf{A}^T)$.
   (C) $\text{rank}(\mathbf{A}) + \dim\big(N(\mathbf{A})\big) = \text{rank}(\mathbf{A}^T) + \dim(N(\mathbf{A}^T))$.
   (D) It is possible for a matrix $\mathbf{A}$ to have $[2, 1, -1]^T$ in $N(\mathbf{A})$ and $[1, -2, 3]^T$ in $R(\mathbf{A}^T)$.
   (E) Let $\mathbf{y} \in \mathbb{R}^m$. If $\mathbf{y} = \mathbf{u}_1 + \mathbf{v}_1 = \mathbf{u}_2 + \mathbf{v}_2$, where $\mathbf{u}_1, \mathbf{u}_2 \in R(\mathbf{A})$ and $\mathbf{v}_1, \mathbf{v}_2 \in N(\mathbf{A}^T)$, then $\mathbf{u}_1 = \mathbf{u}_2$ and $\mathbf{v}_1 = \mathbf{v}_2$.

6. Let

$$\mathbf{A}=\begin{bmatrix}4 & 1 & 3 & 2\\ 1 & 4 & 3 & 3\\ -1 & 11 & 6 & 7\end{bmatrix}.$$

Which of the following vectors are in the column space of $\mathbf{A}$?

(A) $[3,1,2]^T$
(B) $[1,0,-1]^T$
(C) $[0,1,3]^T$
(D) $[2,1,1]^T$
(E) $[4,2,-1]^T$

7. Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors in an inner product space V, and $\mathbf{p}$ is the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$. Define $\|\mathbf{v}\|=\sqrt{\langle\mathbf{v},\mathbf{v}\rangle}$. Which of the following statements are true?

(A) $\|\mathbf{u}\|\geq\|\mathbf{p}\|$
(B) $\|\mathbf{u}-\mathbf{p}\|\geq\|\mathbf{p}\|$
(C) $\|\mathbf{u}-\mathbf{p}\|+\|\mathbf{p}\|\geq\|\mathbf{u}\|$
(D) $\mathbf{p}=\mathbf{u}$ implies $\mathbf{v}=\mathbf{u}$
(E) $\|\mathbf{p}\|^2+\|\mathbf{u}-\mathbf{p}\|^2=\|\mathbf{u}\|^2$

8. Suppose that vectors $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^n$ satisfy $\mathbf{Rx}=\mathbf{y}$ for some $\mathbf{R}\in\mathbb{R}^{n\times n}$ with $\mathbf{R}^T\mathbf{R}=\mathbf{I}$. Which of the following options are possible?

(A) $\mathbf{x}=[2,1,2]^T$, $\mathbf{y}=[3,0,0]^T$.
(B) $\mathbf{x}=[2,1,1]^T$, $\mathbf{y}=[3,1,0]^T$.
(C) $\mathbf{x}=[1,2,3]^T$, $\mathbf{y}=[0,0,6]^T$.
(D) $\mathbf{x}=[3,4,2]^T$, $\mathbf{y}=[2,0,5]^T$.
(E) $\mathbf{x}=[2,3,5]^T$, $\mathbf{y}=[1,5,4]^T$.

9. Let L be the operator on P_3 (the set of polynomials of degree less than 3) defined by

$$L\big(p(x)\big)=xp'(x)+p''(x),$$

where $p'(x)$ denotes the derivative of $p(x)$ with respect to x. Find the matrix $\mathbf{A}$ representing L with respect to $E=\{1+x,-1+x^2,1+x^2\}$. Denote the last two rows of $\mathbf{A}$ by $[a_{21},a_{22},a_{23}]$ and $[a_{31},a_{32},a_{33}]$, respectively. Which of the following statements are true?

(A) $a_{21}=0$ (B) $a_{22}=0$ (C) $a_{31}=0.5$ (D) $a_{32}=1$ (E) $a_{33}=2$

10. Let $\mathbb{S}^{2\times2}$ be the set of all 2×2 symmetric matrices. Then $\mathbb{S}^{2\times2}$ is a subspace of $\mathbb{R}^{2\times2}$. Suppose that $E=\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\}$ and $F=\{\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3\}$ are two bases of $\mathbb{S}^{2\times2}$, where

$$\mathbf{v}_1=\begin{bmatrix}1 & 3\\ 3 & -7\end{bmatrix},\ \mathbf{v}_2=\begin{bmatrix}4 & 2\\ 2 & 7\end{bmatrix},\ \mathbf{v}_3=\begin{bmatrix}5 & 1\\ 1 & 17\end{bmatrix},\ \mathbf{w}_1=\begin{bmatrix}2 & 0\\ 0 & 4\end{bmatrix},\ \mathbf{w}_2=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix},\ \mathbf{w}_3=\begin{bmatrix}0 & 0\\ 0 & -3\end{bmatrix}.$$

Let $\mathbf{S}$ be the transition matrix from E to F. Denote the first two rows of $\mathbf{S}$ by $[s_{11},s_{12},s_{13}]$ and $[s_{21},s_{22},s_{23}]$, respectively. Which of the following statements are true?

(A) $s_{11}=1$ (B) $s_{12}=1$ (C) $s_{13}=3$ (D) $s_{21}=3$ (E) $s_{22}=1$

國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學乙【電機系碩士班乙組】　　題號：431001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　

以下11到20題為簡答題，每題3分，總分30分。答錯或未作答者該題以0分計。簡答題只需簡短寫出答案，無須提供詳細的推導或證明。請將答案寫在答案卷上，並明確標示題號。

以下第11題到14題，考慮函數 $f:[0,\infty)\mapsto\mathbb{R}$（$\mathbb{R}$代表實數所成之集合）及Laplace轉換 $\mathbb{L}:f\mapsto F$，定義為：

$$F(s)=\int_0^\infty f(\tau)\mathrm{e}^{-s\tau}d\tau:=\mathbb{L}(f(t))$$

11. 令 $F(s)=\mathbb{L}(f(t))$，$G(s)=\mathbb{L}(g(t))$。請問 $\mathbb{L}(3f(t)+g(t))$ 等於哪個函數？

12. 令 $F(s)=\mathbb{L}(f(t))$，$G(s)=\mathbb{L}(g(t))$。請問 $\mathbb{L}(f(t)g(t))$ 等於哪個函數？

13. 令 $F(s)=\mathbb{L}(f(t))$，且 $f(0)=0$。請問 $\mathrm{e}^{3s}F(s)+s^2F(s)$ 是哪個函數的Laplace轉換？

14. 令 $F(s)=\mathbb{L}(f(t))$。請問 $\mathbb{L}(t*f(t))$ 是哪個函數的Laplace轉換？在這裡，$t*f(t)$ 代表函數 $g(t)=t$ 與函數 $f(t)$ 的convolution。

以下第15題到20題，考慮線性微分方程式：$\ddot{x}(t)+a\dot{x}(t)+bx(t)=u(t)$，$x(0)=x_1$, $\dot{x}(0)=x_2$。

15. 令 $u(t)\equiv 0$。請問當參數 a 與 b 滿足甚麼條件的情況下，給予任何非零初值 (x_1,x_2)，方程式的解 $x(t)$ 都不會發散？

16. 令 $u(t)\equiv 0$。請問當參數 a 與 b 滿足甚麼條件的情況下，給予任何非零初值 (x_1,x_2)，方程式的解 $x(t)$ 都會發散？

17. 令 $u(t)\equiv 0$。請問當參數 a 與 b 滿足甚麼條件的情況下，給予任何非零初值 (x_1,x_2)，方程式的解 $x(t)$ 都會收斂到0？

18. 令 $u(t)\equiv 0$，$(a,b)=(0,-1)$。請列出所有會讓方程式的解收斂到0的非零初值。

19. 令 $(x_1,x_2)=(0,0)$，$(a,b)=(0,1)$。請列出所有會讓方程式的解發散的 $u(t)$ 函數。

20. 令 $(x_1,x_2)=(0,0)$，$(a,b)=(1,1)$，$u(t)\equiv\sin(\omega t)$。請問 ω 之值為何，會讓方程式之解有最大振幅？

國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學乙【電機系碩士班乙組】　　題號：431001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）(混合題)　　

以下第 **21** 題所有小題都需要詳明推導計算過程。如推導計算過程錯誤，將酌扣分數或不給分。

21. (共 20 分) 考慮聯立微分方程組：

$$\begin{aligned}\dot{x}_1 &= x_3\\ \dot{x}_2 &= x_4\\ \dot{x}_3 &= -\alpha_1 x_1 + \beta x_2\\ \dot{x}_4 &= \beta x_1 - \alpha_2 x_2\end{aligned}$$

其中$\alpha_1, \alpha_2, \beta$ 皆為正數。令 $A = \begin{bmatrix} -\alpha_1 & \beta \\ \beta & -\alpha_2 \end{bmatrix}$。

(a). (10 分) 令 x_1 所滿足之微分方程式的特徵方程式的根為 λ。請證明λ^2 為矩陣A 之特徵值。

(b). (10 分) 請推導 $\alpha_1, \alpha_2, \beta$ 須滿足之條件，來讓微分方程組的解在給定任何初值的情況下都不會發散。

【試題到此結束】

國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：控制系統【電機系碩士班乙組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：控制系統【電機系碩士班乙組】　　題號：431008

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

Problem 1 (16%) Find the transfer function $Y_1(s)/R_2(s)$ of the system displayed in Figure 1.

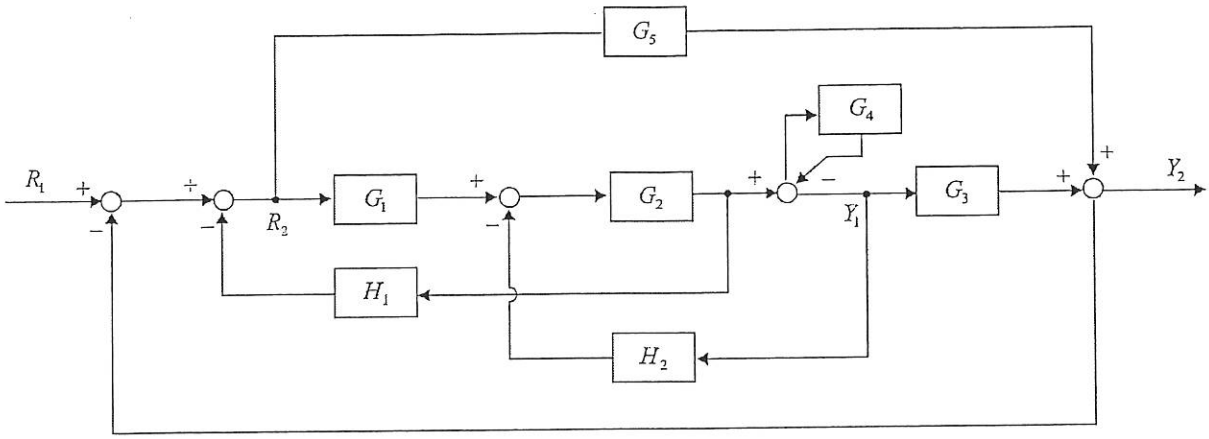

Figure 1: Control system of Problem 1.

Problem 2 (12%) Consider two second order linear time-invariant systems, system A and system B, respectively. Assume that the system A has s_1 and s_1^* (complex conjugate pair) for its poles and system B has s_2 and s_2^* (complex conjugate pair) for its poles. The location of these poles are indicated in Figure 2. Assume that the DC gain of boh systems are 2. Find the settling time for both system A and systm B.

Hint: settling time $= \begin{cases} 3.2/(\zeta\omega_n), & 0<\zeta<0.69 \\ 4.5\zeta/\omega_n, & \zeta>0.69 \end{cases}$

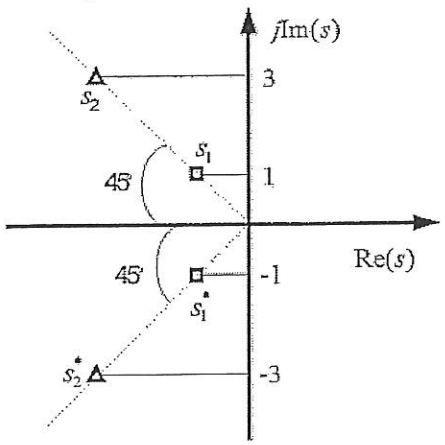

Figure 2: Locations of poles of Problem 2.

Problem 3 (22%) Figure 3 shows a block diagram of a feedback control system.

(a) (8%) Find the range(s) of k_1 and k_2 such that all the closed-loop poles, except those at the origin, are in the left half s-plane.

(b) (14%) Suppose that the closed-loop transfer function of the control system you obtained in part (a) is

$$M(s)=\frac{s+1}{s^4+(2+k_1)s^3+(4+k_1)s^2+k_2 s}.$$

We also assume that the poles of $M(s)$, except one at the origin, are all in the left half s-plane. In addition to the range of k_1 and k_2 you find in part (a), find the range(s) of k_1 and k_2 such that the steady-state error $\lim_{t\to\infty} e(t) \le 1\%$ due to an unit-step input $r(t)$.

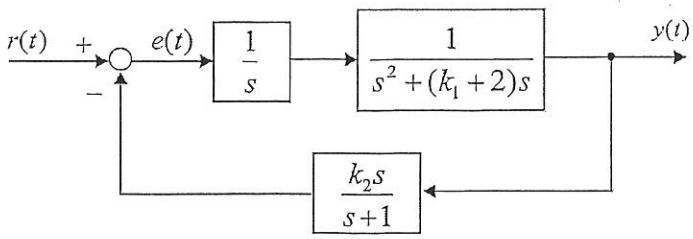

Figure 3: Control system of Problem 3 and 4.

Problem 4 (15%) Consider the feedback system shown in Figure 3 where $k_2 = 3k_1$.

(a) (3%) Find the transfer function $G_c = Y(s)/R(s)$ of the closed-loop system where $R(s)$ and $Y(s)$ are the Laplace transform of the input $r(t)$ and the output $y(t)$, respectively.

(b) (3%) Find the intersection of the asymptotes (centroid).

(c) (4%) Find the intersection of the root loci with the imaginary axis and the corresponding value of k_1.

(d) (5%) Construct the root loci for $k_1 \geq 0$.

Problem 5 (15%) Consider the following block diagram shown in Figure 4.

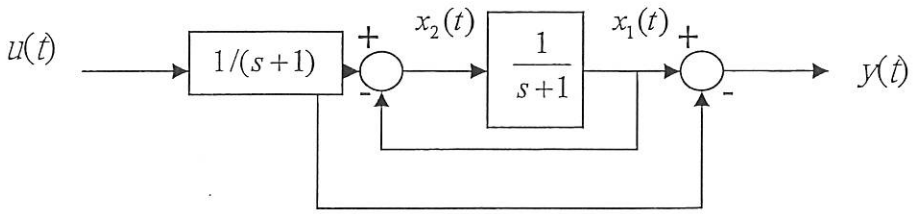

Figure 4: Control system of Problem 5.

(a) (5%) Write down the dynamic equation of this control system in state space model, i.e.,

$$\begin{pmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{pmatrix} = A\begin{pmatrix}x_1(t)\\ x_2(t)\end{pmatrix} + bu(t);\ y = c\begin{pmatrix}x_1(t)\\ x_2(t)\end{pmatrix} + du(t).$$

(b) (5%) Show that this system is controllable, but not observable.

(c) (5%) Re-assign state variables so that the system is controllable and observable, and has the same transfer function.

Problem 6 (20%) Consider the following block diagram shown in Figure 5.

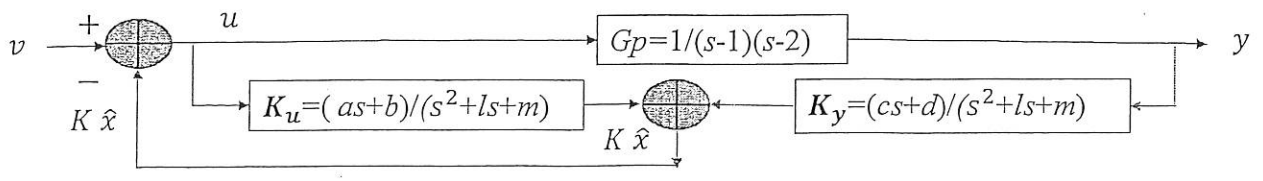

Figure 5: Control system of Problem 6.

(a) (3%) Write down the dynamic equation of the control system with its transfer function $G_p = 1/(s-1)(s-2)$ in state space model, i.e.,

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = A\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + Bu(t);\ y = C\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + Du(t),$$

such that this (state space) model is controllable and observable.

(b) (3%) Find a state feedback gain $K = [k_1\ k_2]$ such that with the state feedback law $u = -K\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$, the system matrix $A - BK$ of the closed-loop system has the eigenvalues (characteristic values) being equal to $-1, -2$.

(c) (3%) Consider the following Luenberger observer:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - C\hat{x}(t) - Du(t)). \qquad (1)$$

With the estimation error $e(t) = x(t) - \hat{x}(t)$, show that $\dot{e}(t) = (A - LC)e(t)$.

(d) (3%) Find an observer gain $L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$ such that $A - LC$ has the eigenvalues (characteristic values) being equal to $-3, -4$.

(e) (5%) Consider the architecture shown in Figure 5 where the dynamic output feedback law $u(t) = v(t) - K\hat{x}(t)$ is used with $\hat{x}(t)$ satisfying the dynamic equation (1) and

$$K_u = K(sI - A + LC)^{-1}(B - LD) = \frac{as + b}{s^2 + ls + m},\ K_y = K(sI - A + LC)^{-1}L == \frac{cs + d}{s^2 + ls + m}.$$

Find the constants a, b, c, d, l, m.

(f) (3%) Find the transfer function of the closed-loop system.

國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：資料結構【電機系碩士班丙組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：資料結構【電機系碩士班丙組】　　題號：431004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. 【每一小題10分，共30分】依序輸入正整數60、35、11、45、8、72、6。

① 請依據上述輸入次序，建造一棵 binary search tree。

② 請依據上述輸入次序，建造一棵 min-heap tree。

③ 請依據上述輸入次序，建造一棵 AVL tree。

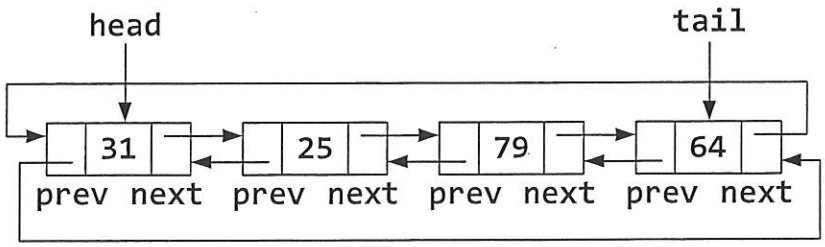
【註】：在每個小題裡，都必須畫出在每增加一個整數之後的樹狀結構，不可直接畫出最後答案；直接畫出最後答案，該小題以0分計算。

2. 【每一小題15分，共30分】承上題，

① 在建造完畢的 min-heap tree 裡頭刪除「6」之後的 min-heap tree 為何？請畫出來。

② 在建造完畢的 AVL tree 裡頭刪除「35」之後的 AVL tree 為何？請畫出來。

3. 【此題15分】下圖為「雙向環狀串列（doubly circular linked list）」的資料結構範例，



其中每個 Node 結構包含三個欄位「prev、data、next」；更具體地說，下方為 Node 結構的宣告：

```
struct Node {
   Node* prev;    // 指向前一個 Node
   int data;
   Node* next;    // 指向下一個 Node
};
```

雙向環狀串列裡頭包含二個指標變數：head 和 tail。指標變數 head 永遠指向串列裡頭的第一個 Node，指標變數 tail 永遠指向串列裡頭的最後一個 Node。請寫出 push_back 函式（function）的程式碼，其宣告為 void push_back(int d)，其功能是在雙向環狀串列裡頭增加一個 Node，其 data 欄位值為 d，並且限定「此一 Node 在加入雙向環狀串列之後，必須成為雙向環狀串列裡頭的最後一個 Node」。【註】：push_back 函式必須考慮雙向環狀串列在一開始，完全沒有任何 Node 的情況，此時 head 和 tail 的初始值皆為 NULL。

國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：資料結構【電機系碩士班丙組】　　題號：431004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

4. 【此題15分】請將後序運算式（postfix expression）

A B * C D * - E F + /

轉為中序運算式（infix expression）

5. 【此題10分】(A)、(B)、(C)、(D)這四個序列（sequence）裡頭，哪些序列不可能是用 inoder 次序拜訪 binary search tree 所產生的結果？請解釋你的理由。沒說明理由，或理由錯誤，本題0分。

(A) 12 35 24 52 40

(B) 64 48 37 26 15

(C) 13 24 36 47 60

(D) 34 17 11 25 43

國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：離散數學【電機系碩士班丙組】

—作答注意事項—

考試時間：100分鐘

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國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：離散數學【電機系碩士班丙組】　　題號：431011

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　共 3 頁第 1 頁

Problem 1: True (T) or False (F). You will receive **mark penalty of −8 points** for each incorrect answer (until you get 0 points in this exam). Do **NOT** justify your answer.

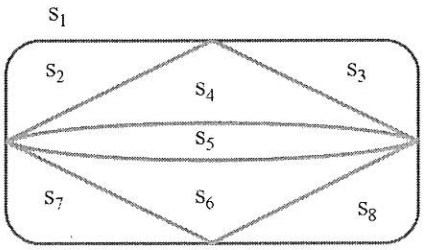
[ ] Observations showed that at least 100 students successfully got correct answers in eight questions, and 200 students successfully answered the rest questions. We concluded that some students got at least 11 questions right. (6 pt)

[ ] There is only one integer solution to $2^x = x^2 + x^3$. (6 pt)

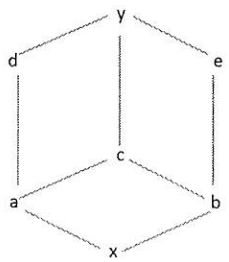
[ ] 5^{4321} is greater than 4^{5321}. (6 pt)

[ ] Both K_5 and $K_{3,3}$ are not planar graphs. (6 pt)

Problem 2: Use the minimal number of colors to color the following map. Use English alphabets to represent different colors, e.g., a, b, and c. Please justify your answer. Otherwise, you get 0 points. (12 pts)

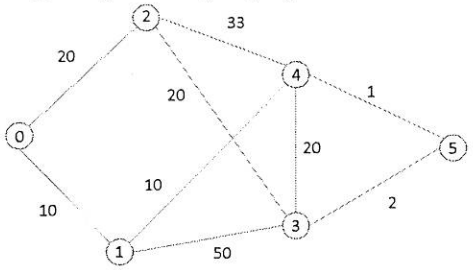


Problem 3: Consider the lattice L in the following figure. Identify whether L_1, L_2, L_3, and L_4 are sublattices of L: L_1 = |x,a,b,y|, L_2 = |x,a,e,y|, L_3 = |a,c,d,y|, and L_4 = |x,c,d,y|. Please justify your answer. Otherwise, you get 0 points. (24 pts)

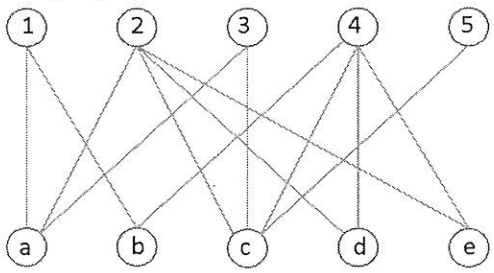


試題請隨卷繳回，請留意背面是否有題

Problem 4: Use Kruskal's algorithm to find the minimum spanning tree of the following graph. Please justify your answer. Otherwise, you get 0 points. (12 pts)



Problem 5: Find the maximal relational mapping in the following bipartite graph. Please justify your answer. Otherwise, you get 0 points. (12 pts)



Problem 6: x and y are prime numbers that satisfy $x^y + y^x = xy$. Please find x and y. Please justify your answer. Otherwise, you get 0 points. (4 pts)

國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：離散數學【電機系碩士班丙組】　　題號：431011

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

Problem 7: There are five baseballs and four basketballs. You have to select three balls from them, but these three balls should contain at least one baseball and one basketball. How many choices do you have? Please justify your answer. Otherwise, you get 0 points. (12 pts)

國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：電路學【電機系碩士班丁組】

—作答注意事項—

考試時間：100分鐘

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國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：電路學【電機系碩士班丁組】　　題號：431006

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. (10 pt) In the circuit of Figure. 1, find the resistance of R_1 and R_2.

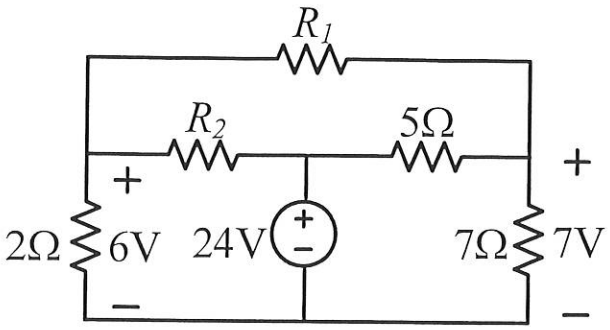


Figure. 1

2. (10 pt) In the circuit of Figure. 2, find the *Thévenin* equivalent circuit.

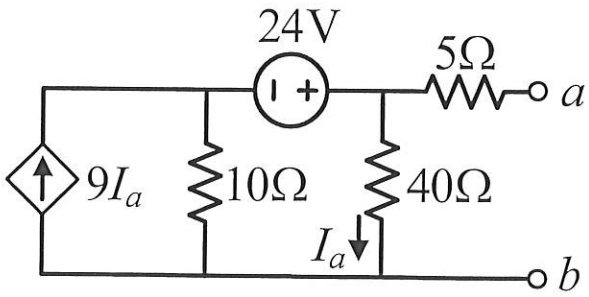


Figure. 2

3. (10 pt) Express the voltage gain v_o/v_s in terms of the resistance R in the circuit of Figure. 3.

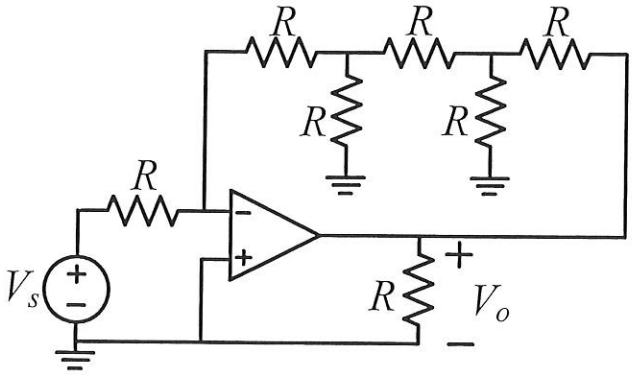


Figure. 3

4. (10 pt) The circuit in Figure. 4 is at steady state before $t = 0$. Determine the current $i(t)$ for $t \geq 0$.

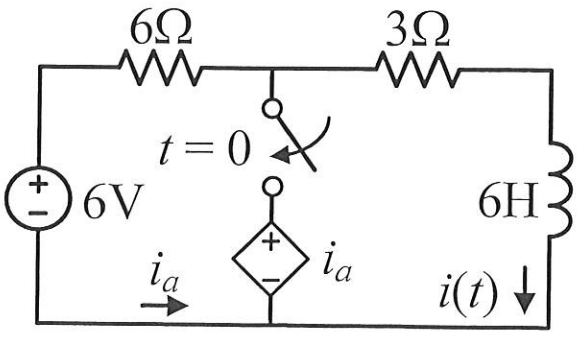
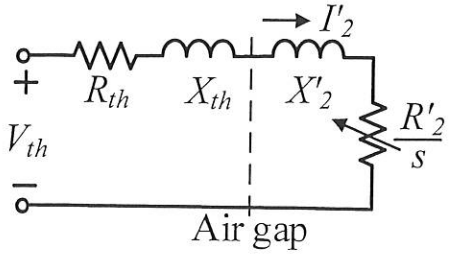


Figure. 4

5. (10 pt) The circuit shown in Figure. 5 is the Thevenin equivalent circuit of an induction motor, where s is slip, and V_{th} is directly proportional to the ac voltage imposed on the motor input.
   (a) (5 pt) Please derive the equation of real power transferred to the rotor.
   (b) (5 pt) Please derive the equation for input power factor at motor starting.



$$s = \frac{\omega_{syn} - \omega_r}{\omega_{syn}}$$

ω_{syn}: synchronous frequency

ω_r: rotor electric frequency

Figure. 5

6. (10 pt) The voltage across a load and the current through it are given by $v(t)= 100\cos(100t)$ V and $i(t)=10\cos(100t-30^o)+3\cos(300t)$ A, respectively. Find:
   (a) (5 pt) RMS value of the current.
   (b) (5 pt) Average power of the load.

7. (10 pt) A 3-phase 3-wire balanced source 380V system supplies a three-phase Y-connection balanced load 10kVA at 0.866 power factor lagging. Find line current and load impedance.

8. (10 pt) A 1200/120V transformer has load 10Ω at the secondary side. If 600Vrms voltage source connects to the primary side, find source current and load power.

9. (20 pt) Short questions：
   (a) (5 pt) Explain operation principle of boost converter.
   (b) (5 pt) Explain operation principle of flyback converter.
   (c) (5 pt) Explain ESR effect of the capacitor.
   (b) (5 pt) Explain reverse recovery issue of PiN diode.

國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組、電波聯合選考：電機系碩士班戊組、通訊所碩士班乙組】

－作答注意事項－

考試時間：100分鐘

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- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組、電波聯合選考：電機系碩士班戊組、通訊所碩士班乙組】題號：437002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）

1. (10%) Please explain the following concepts as detail as possible:
   (a) (2%) Describe the conditions for a random process to be wide-sense stationary (WSS).
   (b) (2%) What is an Ergodic process?
   (c) (2%) Describe the Wiener-Khinchin Theorem.
   (d) (2%) What is an Additive White Gaussian Noise?
   (e) (2%) What is a Gaussian Process?

2. (12%) Find the Fourier transform of $x(t) = sinc^3(t)$. (Hint: The answer is a piecewise function which consists of five intervals.)

3. (15%) The characteristic function of a random variable X is defined as the statistical average
$$E\left(e^{jvX}\right) \equiv \psi(jvX) = \int_{-\infty}^{\infty} e^{jvx} p(x)dx.$$
   (a) (10%) Find the characteristic function of a Gaussian random variable.
   (b) (5%) Show that the variable Y, which is defined as the sum of N independent and identically distributed (i.i.d.) Gaussian random variables X_i, $i = 1,2, \ldots, N$, is a Gaussian random variable.

4. (10%) An AM signal $s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$ is considered in the following systems:
   (a) (5%) If $s(t)$ is used as the input to a square-law detector which has a transfer characteristic defined as $v_O(t) = a_1 v_I(t) + a_2 v_I^2(t)$, where a_1 and a_2 are constants, $v_I(t)$ denotes the input, and $v_O(t)$ denotes the output. Find the conditions for which the message signal $m(t)$ can be recovered from $v_O(t)$.
   (b) (5%) Let $r(t)$ denote the recovered signal in (a). Suppose that we use an ideal sampling with a sampling interval of T_s to sample $r(t)$ and obtain the sampled signal $r_\delta(t)$, please find the Fourier transform of $r_\delta(t)$.

5. (15%) Please answer the following questions.
   (a) (5%) For a quaternary communication system, the possible transmitted signals are
$$s_k(t) = A\cos\left(\frac{20\pi}{T}t - \frac{(k-1)}{2}\pi\right), 0 \le t \le T, k = 1,\cdots,4.$$
   Assume $T = 40ms, A = 100mV, P\left(s_k(t)\right) = \frac{1}{4}, \forall k$, and the noise PSD $S_n(f) = 20\ \mu$W/Hz. Please calculate the error probability P_e.
$$\left(Hint:\ P_e = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) - Q^2\left(\sqrt{\frac{E_s}{N_0}}\right)\right)$$
   (b) (5%) If T changes to $1ms$, in order to maintain the same P_e obtained in (a), please calculate the required amplitude value A.
   (c) (5%) Please show the orthonormal basis functions for the signal constellation $s_k(t)$.

6. (20%) The definition of entropy is the expected value of the self information:

$$H(X) \triangleq E[I(x_k)] = \sum_{k=0}^{K-1} p_k \log_2\left(\frac{1}{p_k}\right) = -\sum_{x \in X} P(x)\log_2\big(P(x)\big).$$

Let variables X, Y have the joint probability

$$P(X,Y) = \begin{bmatrix} P(x_1,y_1) & P(x_1,y_2) \\ P(x_2,y_1) & P(x_2,y_2) \end{bmatrix} = \begin{bmatrix} 0.54 & 0.06 \\ 0.06 & 0.34 \end{bmatrix}.$$

Please find the following quantities:

(a) (4%) $P(Y|X)$ and $P(X|Y)$

(b) (4%) $H(X)$ and $H(Y)$

(c) (6%) Calculate $H(X|Y)$ and describe the physical meaning of $H(X|Y)$.

(d) (6%) Calculate $I(X;Y)$ and describe the physical meaning of $I(X;Y)$.

7. (18%) Consider the encoder for a binary (3,1,2) convolutional code shown in Fig. 1. There are one input message u, two registers and three outputs v_1, v_2 and v_3.

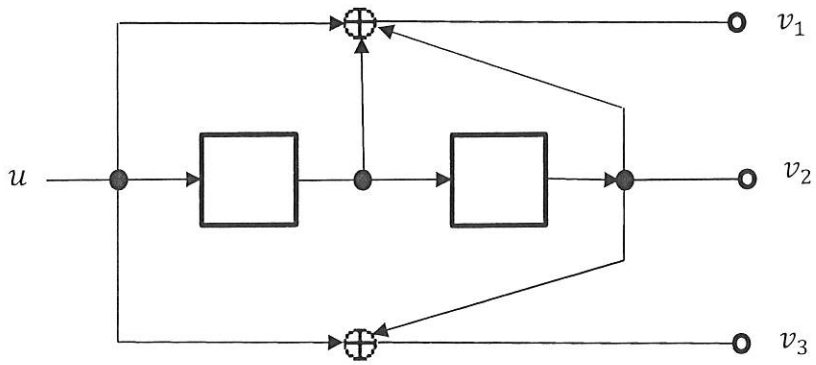

Figure 1

(a) (3%) Find the codeword $\mathbf{v}$ corresponding to the information sequence $\mathbf{u} = (1\ 1\ 1\ 0\ 1\ 0\ 0)$.

(b) (5%) Draw the state diagram of this encoder.

(c) (10%) Please use Viterbi algorithm to decode the received sequence (110 010 111 100 101 001), assuming that a binary symmetric channel with a crossover probability $p < 1/2$ is considered.

國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：電子學【電波聯合碩士班選考、通訊所碩士班乙組選考、電機系碩士班戊組選考】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
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- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
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- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：電子學【電波聯合碩士班選考、通訊所碩士班乙組選考、電機系碩士班戊組選考】題號：482003

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）

1. (30%) For the common-base circuit in Fig. 1, assuming the bias current to be about 1 mA, β = 100, C_μ = 0.8 pF, r_e = 25 Ω, and f_T = 800 MHz:
   (a) Estimate the midband gain V_o/V_s. (10%)
   (b) Use the short-circuit time-constants method to estimate the lower 3-dB frequency, f_L. (10%) (*Hint:* In determining the resistance seen by C_1, the effect of the 47-kΩ resistor must be taken into account.)
   (c) Find the high-frequency poles, and estimate the upper 3-dB frequency, f_H. (10%)

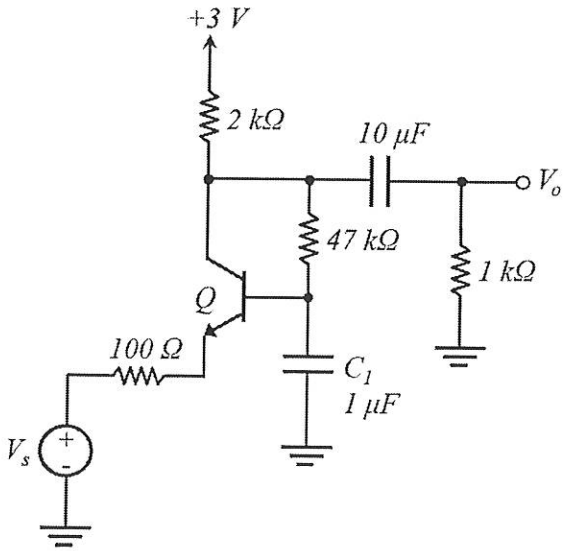

Fig. 1

2. (25%) The current-steering circuit of Fig. 2 is fabricated in a CMOS technology for which k_n' = 90 μA/V², k_P' = 30 μA/V², V_{tn} = 0.8 V, and V_{tp} = -0.9 V. If all devices have L = 2 μm, design the circuit so that I_{REF} = 20 μA, I_2 = 100 μA, and I_5 = 40 μA. Use the minimum width of 2 μm for as many of the devices as possible. (a) Give the required width for each transistor and the value of R required. (10%) (b) What is the highest voltage possible at the drain of Q_2? (5%) (c) What is the lowest voltage possible at the drain of Q_5? If $|V_{Ap}|$ = 16 L, where L is in μm and V_{Ap} is in volts, (5%) (d) find the output resistance of the current source Q_2. (5%)

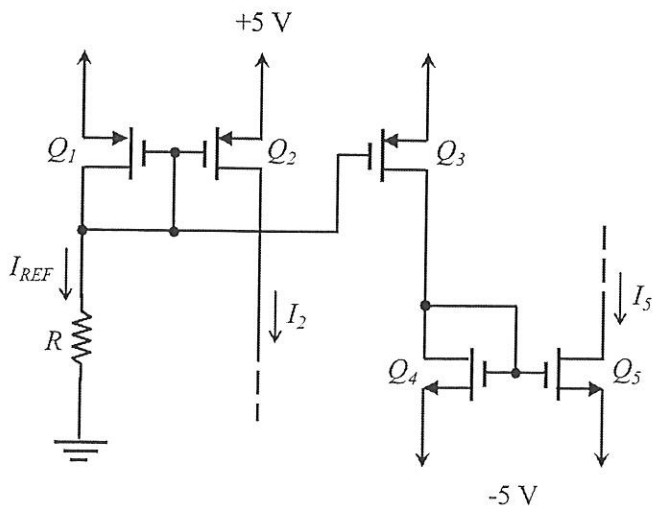

Fig. 2

試題請隨卷繳回，請留意背面是否有題

3. (15%) A second-order filter has its poles at $s = -(1/8) \pm j(\sqrt{63}/8)$. The transmission is zero at $\omega = 5$ rad/s and is unity at dc ($\omega = 0$). Find the transfer function.

4. (30%) For the emitter-follower circuit shown in Fig. 3 the BJT used is specified to have β values in the range of 20 to 200. For the two extreme values of $\beta = 20$ and $\beta = 200$, find :
   (a) I_E, V_E, and V_B. (10%)
   (b) the input resistance R_i. (10%)
   (c) the voltage gain v_o/v_s. (10%)

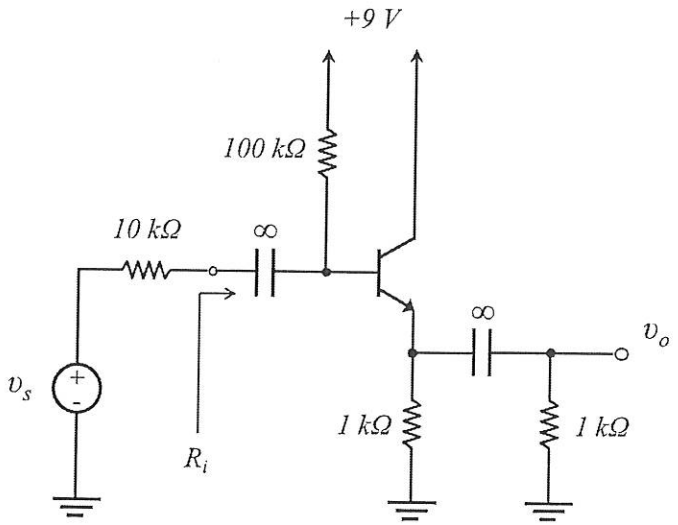

Fig. 3

國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：電磁學【電波聯合碩士班、通訊所碩士班乙組、電機系碩士班戊組】

－作答注意事項－

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
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國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：電磁學【電波聯合碩士班、通訊所碩士班乙組、電機系碩士班戊組】題號：482004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　

1. (3%) (a) At any point (x_0, y_0, z_0) in the domain of a scalar function $V(x, y, z)$, we take a path a_ℓ along $V = c_i$, where c_i is a constant, or take a path a_n along ∇V. Tell me about the main characteristic (主要特徵) of these two paths a_ℓ and a_n, and also what is $a_\ell \cdot a_n$, the dot product of a_ℓ and a_n?
   (3%) (b) $() = \nabla V \cdot (a_\ell)\, d\ell$, where V is a scalar function, $d\ell$為任意方向 a_ℓ 之小路徑。() 裏應填什麼？
   (3%) (c) 利用 Divergence theorem for ∇·E 寫下 E 和 Q (真空中有一 charge Q)的關係。
   (3%) (d) 利用 Stokes' theorem for ∇×B 寫下 B 和 I (真空中有一 current I) 的關係。
   (3%) (e) 在運算 Divergence ∇·A 或 Curl ∇ × A時 (A 為一向量場) ，我們選擇的體積或面積在大小和形狀各有何限制？

2. (5%) Using the *Method of Image*, write down the potential distribution, $V(x, y, z)$, for a point P(x, y, z) in the space, Fig. 1, the dielectric constant of the space is ε_0. Q is a positive point charge of Q 庫侖 *Coul.*

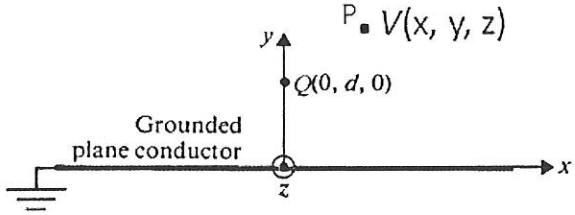

Fig. 1. A point charge Q distance d above the Ground.

3. (3%) (a) 在 dielectric constant 為 ε_r (=1+X_e) dielectric 之內部電場為 E (V/m)，請問 Polarization vector P 為何？
   (3%) (b) 在 relative permeability 為 μ_r (=1+X_m)的一 ferromagnetic material 外面線圈通電流，在其內部產生磁場 H (A/m)，請問 Magnetization vector M 為何？
   (4%) 銅的導電性很好，(c) 它的 permittivityε 和 permeability μ 各為何？請簡單提供你的理由。

4. For a coaxial transmission line shown in Fig. 2, the capacitance per unit length is $c = \frac{2\pi\varepsilon_0}{ln\frac{b}{a}} \left[\frac{\mathrm{F}}{\mathrm{m}}\right]$, and the inductance per unit length is $\ell = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi} \ln\frac{b}{a} \left[\frac{\mathrm{H}}{\mathrm{m}}\right]$. At high frequencies, the internal inductance drops off (that is, approaching 0, and you should know which term is the internal inductance).
   (2%) (a) Find the characteristic impedance of the coaxial line $Z_c=(l/c)^{0.5}$ at high frequencies 請務必寫 Z_c 之單位。
   (2%) (b) 請問在地 (Ground, 即半徑 b 粗體部分) 之外的 magnetic flux density B 值為何？

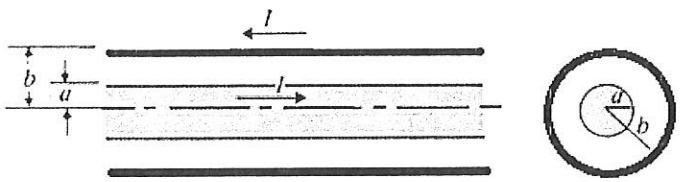

Fig. 2. Coaxial cable side and cross-sectional views, 粗體線代表地 Ground

The capacitance of a line charge of radius a over a ground 0, as shown in Left of Fig. 3 $C = \frac{2\pi\varepsilon_0}{ln\frac{2h}{a}} \left[\frac{\mathrm{F}}{\mathrm{m}}\right]$.
(3%) (c) Find the external inductance L for such a transmission system in air using a quasi-TEM property $L \cdot C = \mu_0 \cdot \varepsilon_0$.
(3%) (d) Obtain the internal inductance from the inductance formula l, also the external inductance found in c), write down the per unit length internal & external inductance for the conductor system shown in the Right of Fig. 3.

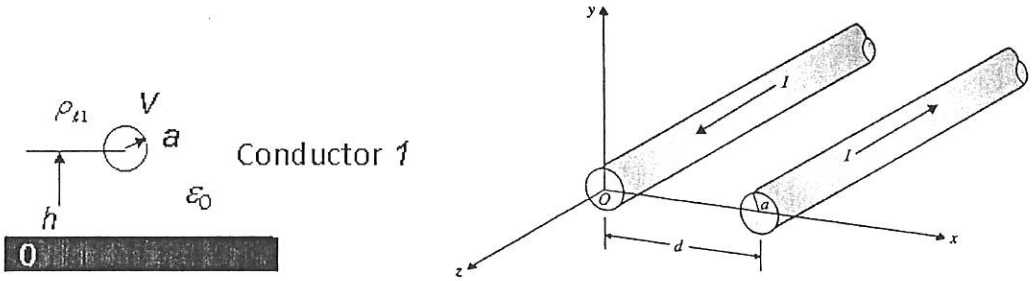

Fig. 3. A single conductor above a Ground (Left); A two-conductor system with currents flow in opposite direction (Right); the two conductors both with radius a and d distance apart.

5. (5%) (a) A position vector $\mathrm{R} = a_x(x-x') + a_y(y-y') + a_z(z-z')$, $R = [(x-x')^2+(y-y')^2+(z-z')^2]^{1/2}$, $a_R = \mathrm{R}/R$, where $P(x, y, z)$ is an observation point, and $P'(x', y', z')$ is a source point. Show that $\nabla'(\frac{1}{R}) = (a_R \frac{1}{R^2})$, $\nabla' f$ is the gradient operator with respect to the source coordinates, that is, $\nabla' f = a_x \frac{\partial f}{\partial x'} + a_y \frac{\partial f}{\partial y'} + a_z \frac{\partial f}{\partial z'}$.
(5%) (b) As shown in Fig. 4, determine B at $P(0, 0, z^*)$?

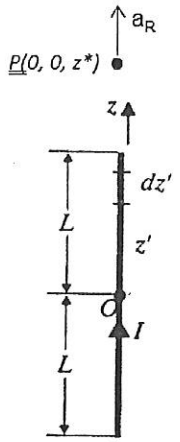

Fig. 4.

6. (10%) (a) Derive the electromagnetic wave equation in free space.
(5%) (b) Explain the traveling-wave factor.

7. A uniform plane wave ($\mathbf{E}_i$, $\mathbf{H}_i$) of an angular frequency ω is incident from air (medium 1) on a very large, perfectly conducting wall (medium 2) at an angle of incidence θ_i with perpendicular polarization. Find
(10%) (a) **E** and **H** in medium 1.
(5%) (b) **E** and **H** in t medium 2.
(5%) (c) the current induced on the wall surface, and
(5%) (d) the time-average Poynting vector in medium 1.

8. (10%) As shown in Fig. 5 with $Z_1 = \sqrt{Z_0 Z_L}$ and $l = \lambda/4$, please explain how the circuit works to achieve impedance matching between Z_0 at Z_L, and obtain the bandwidth with the maximum Γ of Γ_m.

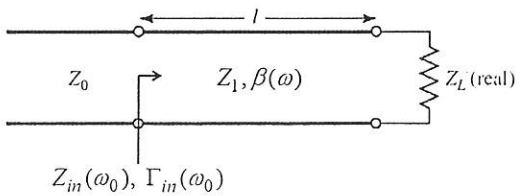

Fig. 5.

國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：計算機結構【電機系碩士班已組】

－作答注意事項－

考試時間：100分鐘

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- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
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- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
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- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：計算機結構【電機系碩士班己組】　　題號：431007

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　

1. **[20%]** (1) **(12%)** Write MIPS assembly codes that implement a **sort** function on an array *v* with the size of *n*. Note that you need to add comments to each line of your written assembly codes (2) **(8%)** Please also describe your ideas on implementing the sort function using a high-level programming language such as C. Note that your written high-level programming codes should support your assembly codes.

2. **[20%]** Explain the following terms.
   (a) **(4%)** Forwarding
   (b) **(4%)** Fully associative cache
   (c) **(4%)** IEEE 754 standard
   (d) **(4%)** Branch prediction
   (e) **(4%)** SRAM

3. **[20%]** The following questions are for the five-pipeline-stage MIPS processor design.
   (a) **(10%)** Draw the **data path** of the five-pipeline-stage MIPS processor with hazard detection and forwarding capabilities.
   (b) **(5%)** Illustrate and explain in details how a branch instruction is executed in your drawn data path of the five-pipeline-stage MIPS processor.
   (c) **(5%)** What is data hazard and how this problem can be resolved in the five-pipeline-stage MIPS processor design in (a)? Please design an example assembly program to help illustrate and explain in details.

4. **[20%]** Cache designs are important for enhancing the performance of a processor system. Below is a list of 32-bit memory address references, given as word addresses.
   75, 33, 131, 18, 217, 25, 181, 21, 194, 253, 1, 11
   (a) **(4%)** For each of these references, identify the binary address, the tag, and the index given a direct-mapped cache with 16 one-word blocks. Also list if each reference is a hit or a miss, assuming the cache is initially empty. Please also calculate the overall miss rate accordingly.
   (b) You are asked to optimize a cache design for the given references. There are three possible direct-mapped cache designs, all with a total of 8 words of data: C1 has 1-word blocks, C2 as 2-word blocks, and C3 has 4-word blocks.
      I. **(3%)** In terms of miss rate, which cache design is the best?
      II. **(3%)** If the miss stall time is 25 cycles, and C1 has an access time of 2 cycles, C2 takes 3 cycles and C3 takes 5 cycles, which is the best cache design?
   (c) **(5%)** Show the final cache content for a three-way set associative cache with two-word blocks and a total size of 24 words. Use LRU replacement. For each reference, identify the index bits, the tag bits, the block offset bits and if it is a hit or a miss, assuming the cache is initially empty. Please also calculate the overall miss rate accordingly.
   (d) **(5%)** Show the final cache content for a fully associative cache with one-word blocks and a total size of 8 words. Use LRU replacement. For each reference, identify the index bits, the tag bits, the block offset bits and if it is a hit or a miss, assuming the cache is initially empty. Please also calculate the overall miss rate accordingly.

國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：計算機結構【電機系碩士班己組】　　題號：431007

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

5. **[20%]** Please design a finite-state machine based cache controller. The key characteristics of the target cache are shown below.
   - Direct-mapped cache
   - Write-back using write allocate
   - Block size is 4 words (16 bytes)
   - Cache size is 32 KB, so it holds 2048 blocks
   - 32-byte addresses
   - The cache includes a valid bit and a dirty bit per block

   The signals between the processor and the cache are.
   - 1-bit Read or Write signal
   - 1-bit Valid signal, saying whether there is a cache operation or not
   - 32-bit address
   - 32-bit data from processor to cache
   - 33-bit data from cache to processor
   - 1-bit Ready signal, saying the cache operation is complete

   (a) **(10%)** Please draw the state diagram of your designed finite-state machine based cache controller with all necessary signals in each state and transitions between states.
   (b) **(10%)** Explain in details the functionalities of each state and transitions between states.