國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學乙【電機系碩士班乙組】　　題號：431001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　

第 1 至 4 題請照下列指示作答

- 所有對定義或問答題的做答，都要用該題目所給定的、和線性代數課程中所學的數學符號(包含 $\exists$ 和 $\forall$ 的使用)，完全正確回答才給分。
- 對於集合的數學描述要完整且完全正確才給分。例如用集合 $\{s\in\mathbb{C}\,|\,s=\sigma+j\omega,\ \sigma>0\ and\ \omega\in\mathbb{R}\}$ 表示複數平面的開右半平面。
- 所有需要推導或證明的子題都已用**粗體**標示出來，其他子題都只需用數學符號(而非用文字敘述)明確且精簡地做答，否則不予計分。
- 占分"≤2%"的證明題要完全正確才給分。占分"≥3%"的證明題才會給部分成績。

1. (12%) Let $A\in\mathbb{R}^{m\times n}$ and $\mathbf{b}\in\mathbb{R}^{m}$.

(a) (2%) (i) What is the mathematic relationship of dimensions of two subspaces $R(A)$ and $N(A)$ for any A matrix considered here? (ii) What is the mathematic relationship described in the Fundamental Subspaces Theorem about matrix A?

(b) (2%) Use the Fundamental Subspaces Theorem to **show** that $N(A^TA)\subset N(A)$.

(c) (2%) (i) What is the condition on $R(A)$ or $N(A)$ that is equivalent to the existence of solution to $A\mathbf{x}=\mathbf{b}$? (ii) What is the condition on $R(A)$ or $N(A)$ that is equivalent to the uniqueness of solution to $A\mathbf{x}=\mathbf{b}$, if it is solvable?

(d) (2+4%) When the equation $A\mathbf{x}=\mathbf{b}$ is unsolvable, we may consider the so-called least squares problem to find a set of solutions, having the least squares error, from solving a normal equation. (i) Use the property and condition mentioned in (b)-(c) to explain why the normal equation is always solvable. (ii) Suppose that $rank(A)=k<\min(m,n)$ and let $A=BC$ be a full rank decomposition of A. Use the known matrices B, C, and $\mathbf{b}$ to describe the unique projection vector $\mathbf{p}$ of $\mathbf{b}$ onto $R(A)$ with the least $\|\mathbf{b}-\mathbf{p}\|_2$.

2. (13%) Let V be a vector space.

(a) (1+2%) Let $\boldsymbol{\delta}$ be a unit element of "+", the addition operation of V. (i) Write the equality condition about $\boldsymbol{\delta}$ as shown in the corresponding axiom. (ii) Let $\boldsymbol{\gamma}$ be another unit element of "+". **Show** that $\boldsymbol{\delta}=\boldsymbol{\gamma}$. (須註明使用到的所有向量空間定義中的公設，否則不計分)

(b) (3%) Let X and Y be two subspaces of V. (i) Write the definition of $X+Y$. (ii) Write the definition of $X\oplus Y$. (iii) What is the mathematical relationship between $\dim(X+Y)$ and $\dim(X\oplus Y)$?

(c) (2%) Let $\langle\bullet,\bullet\rangle$ be an inner product defined on V. **Show** that the function $f(\mathbf{v})\coloneqq\sqrt{\langle\mathbf{v},\mathbf{v}\rangle}$ defined $\forall\mathbf{v}\in V$ satisfies the triangular inequality property.

(d) (2+3%) Consider the vector space $\mathbb{R}^{2\times2}$ with the inner product $\langle A,B\rangle\coloneqq trace(A^TB)$ and denote $Y\coloneqq\{A\in\mathbb{R}^{2\times2}\,|\,A=A^T\}$. (i) Describe $Y^{\perp}$ as the span of an orthonormal basis. (ii) What is the matrix, denoted by $P_{Y^{\perp}}$, that represents the orthogonal projecting operation $\Pi_{Y^{\perp}}:\mathbb{R}^{2\times2}\to Y^{\perp}$ along with the subspace Y with respect to the ordered basis $E=\{G,H\}$, where G and H are vectors of the standard bases for Y and $Y^{\perp}$, respectively?

3. (14%) Let L be a mapping from vector space V to vector space W.

(a) (1+2%) (i) Write the definition of superposition principle and (ii) **show** the implication from the combination of additive and homogeneous properties to the superposition principle.

(b) (2%) Write (i) the definition of $Ker(L)$, the kernel of L, and (ii) the definition of $L(V)$, the image of vector space V, respectively.

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(c) (1+2%) Suppose L is linear. (i) Use $Ker(L)$ to describe a mathematic condition that is equivalent to L being one-to-one. (ii) Moreover, **show** the implication from the condition on $Ker(L)$ to L being one-to-one.

(d) (2+4%) Let L be linear with A as its matrix representation with respective to bases $E=[\mathbf{v}_1,\cdots,\mathbf{v}_n]$ and $F=[\mathbf{w}_1,\cdots,\mathbf{w}_m]$ for V and W, respectively. (i) Describe a mathematic condition about A that is equivalent to L being onto. (ii) Moreover, **show** the implication from that condition to L being onto.

4. (11%) Let $A\in\mathbb{C}^{n\times n}$.

(a) (2%) Let $\lambda\in\mathbb{C}$ be an eigenvalue of A with corresponding eigenvector $\mathbf{x}$. **Derive** in details the equation for solving λ. (要求推導出公式、而非背寫出公式卻沒有任何數學的說明)

(b) (2+2%) (i) Let μ be an eigenvalue of matrix A. What is the mathematic notation for describing the number of linearly independent eigenvectors associated with μ? (ii) Let $\{\mu_1,\cdots,\mu_k\}$ be the set of all distinct eigenvalues of matrix A. Use the corresponding notation as in (i) to describe the condition for A being a diagonalizable matrix.

(c) (5%) Suppose $A=B+iC$ is a Hermitian matrix and let $\Omega=\begin{bmatrix} B & C \\ -C & B \end{bmatrix}\in\mathbb{R}^{2n\times 2n}$. **Show** that any eigenvalue λ of Ω is also an eigenvalue of A.

以下第5題到第6題中之所有的提問，需要寫出推導過程或詳細說明理由，答案正確但沒有推導過程或說明不正確，將酌扣分數或不給分。

5. (20%) Consider the following set of differential equations

$$\ddot{x}_1(t) = -2x_1(t) + 2x_2(t)$$
$$\ddot{x}_2(t) = 2x_1(t) + 5x_2(t) + u(t)$$

(a) (15%) Let $u(t)\equiv 0$ and the initial conditions be $x_1(0)=x_2(0)=1$, $\dot{x}_1(0)=\dot{x}_2(0)=0$. Find the solutions of the differential equations.

(b) (5%) Let initial conditions be $x_1(0)=x_2(0)=\dot{x}_1(0)=\dot{x}_2(0)=0$, and $u(t)$ be the unit step function. Does the solutions of the differential equations converge to constant values as time approaches infinity? Justify your answers.

6. (30%) Consider the following set of differential equations

$$\dot{x}_1(t) = (1-\sqrt{x_1(t)^2+x_2(t)^2})x_1(t) - x_2(t)$$
$$\dot{x}_2(t) = (1-\sqrt{x_1(t)^2+x_2(t)^2})x_2(t) + x_1(t)$$

(a) (10%) Express the differential equations using the polar coordinate; i.e. express the equations in terms of (r,θ), where $x_1(t)=r(t)\cos(\theta(t))$ and $x_2(t)=r(t)\sin(\theta(t))$.

(b) (10%) Solve the differential equations for any nonzero initial conditions (x_{10},x_{20}).

(c) (10%) Draw the phase plane portraits for the solutions (x_1,x_2) of the following initial conditions: $(x_{10},x_{20})=(0,0.5)$, $(-1,0)$, and $(1,-1)$.

End of Examination

國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學甲【電機系碩士班甲組、己組、電波領域選考】　　題號：431002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　

第1題到第10題為單選題，每題四分，共計四十分。每題請選出一個最正確選項，答錯倒扣一分。第1題到第10題中，若$z := x + jy$是一個複數，則 x, y 是實數而 j 代表 $\sqrt{-1}$。

1. Given a continuous-time periodic signal

$$f(t) = 4 + 8\cos\left(\frac{2\pi}{3}t\right) - \sin\left(\frac{5\pi}{3}t\right).$$

Let a_k and ω_0 be the coefficients and fundamental frequency of the Fourier series of $f(t)$ respectively. Then which of the following statements is correct?

(A) $a_{-1} + a_5 = -j/2$, $\omega_0 a_0 = 4\pi/3$, $a_0 a_{-1} = 0$.

(B) $a_{-1} + a_2 = 4$, $\omega_0 a_{-5} = -j\pi/6$, $a_0 a_2 = 16$.

(C) $a_{-2} + a_5 = 4 + j/2$, $\omega_0 a_2 = 4\pi/3$, $a_1 a_5 = j/2$.

(D) $a_0 < 5$, $\omega_0 < 2$, $a_5 > 0$.

(E) None of the above statements are correct.

2. Consider a discrete signal $x(n) = \cos(2n\pi/N)$, where integer N is the fundamental period. Let a_k be the coefficients of the discrete-time Fourier series of $x(n)$. Then which of the following statements is correct?

(a) $a_1 = -1/2$, $a_2 = 1/2$.

(b) $a_1 = a_2 = j/2$

(c) $a_{N+1} = 1/2$, $a_{N-1} = -1/2$.

(d) $a_{N+1} = a_{N-1} = 1/2$

(e) None of the above statements are correct.

3. Consider an LTI system whose impulse response is $G(j\omega) = 1/(a + j\omega)$, $a > 0$. Suppose that there is an input signal $X(j\omega) = 1/(a + j\omega)$. Assume that the output signal is $y(t) = \beta e^{\gamma t} u(t)$, where $u(t)$ is a unit-step signal. Then which of the following statements is correct?

(a) $\beta + \gamma = -a$

(b) $\beta\gamma = -a$

(c) $\beta - \gamma = t + a$

(d) $\beta\gamma = at$

(e) None of the above statements are correct.

4. Consider the following three systems, where $x[n]$ or $x(t)$ is the system input, $y[n]$ or $y(t)$ denotes the system output, and

I. $y[n] = x[-n]$, $-\infty < n < \infty$

II. $y(t) = [\sin(2t)]x(t)$

III. $y[n] = \begin{cases} x[n-4], & n \geq 1 \\ 0, & n = 0 \\ x[n-1], & n \leq -1 \end{cases}$

Which of the following statements is correct?

(a) I is time-invariant, II is linear, III is causal.

(b) I is memoryless, II is causal, III is nonlinear.

(c) I is stable, II is linear, III is memoryless.

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(d) I is causal, II is memoryless, III is linear.
(e) None of the above statements are correct.

5. Let $z=1-j\sqrt{3}$. Assume that $z^{1/5}=re^{j\theta}$, where θ denotes the principal argument. Then which of the following statements is correct?
(a) $r=2,\ \theta=\pi/5$
(b) $r=4^{1/5},\ \theta=-\pi/15$
(c) $r=2^{1/5},\ \theta=\pi/15$
(d) $r=2^{1/5},\ \theta=-\pi/5$
(e) None of the above statements are correct.

6. Which one of the following functions, where $z=x+jy$ is a complex variable, is analytic?
(a) $f(z)=\overline{z}^2$, where $\overline{z}=x-jy$.
(b) $f(z)=xy+jx$
(c) $f(z)=x^2-jy^2$
(d) $f(z)=x^2+y^2+j2xy$
(e) None of the above statements are correct.

7. Let z be a complex number. Which of the following statements is correct?
(a) $Log(z_1z_2)=Log(z_1)+Log(z_2)$, where $Log(z)$ is the principal value of the complex logarithm.
(b) $\cos(j)$ is not a real value.
(c) $j^{j2}=e^{-j(4n+1)\pi}$, $n=0,\ \pm1,\ \pm2,\ldots\ldots$
(d) $\overline{z^2+2z-e^z}=\overline{z}^2+2\overline{z}-\exp(\overline{z})$, where $\exp(z)=e^z$.
(e) None of the above statements are correct.

8. Let $f(z)=z/(z^2+9)$, and C be a circle $|z-j2|=4$ in counterclockwise direction. The evaluation of $\oint_C f(z)dz$ is $\alpha+j\beta$. Then which of the following statements is correct?
(a) $\alpha<0,\ \beta>0$
(b) $\alpha>0,\ \beta>0$
(c) $\alpha=0,\ 1<\beta<4$
(d) $\alpha>0,\ -3<\beta<4$
(e) None of the above statements are correct.

9. Let $f(z)=\overline{z}$, and C be the right-hand half of the circle $|z|=2$ form $z=j2$ to $z=-j2$. Compute the value of $\int_C f(z)dz=\alpha+j\beta$. Then which of the following statements is correct?
(a) $\alpha=0,\ -15<\beta<0$
(b) $a>0,\ 0<\beta<15$
(c) $-12<\alpha+\beta<12$
(d) $-2<\alpha\beta<-12$
(e) None of the above statements are correct

10. The Laurent series of $e^{1/z}$ is

$$e^{1/z}=1+\frac{1}{z}+\frac{1}{\alpha z^2}+\frac{1}{\beta z^3}+\ldots\ldots,\ 0<|z|<\infty$$

Which of the following statements is correct?

(a) $\alpha\beta = 1/6$

(b) $\alpha\beta = 1/12$

(c) $\alpha + \beta = 5/6$

(d) $\alpha + \beta = 1/3$

(e) None of the above statements are correct.

以下第11題到第13題中之所有的提問，都不需要寫出推導過程，只要寫出答案即可，答案正確就得分。

11. (10%) Let $A \in \mathbb{R}^{m\times n}$ and $\mathbf{b} \in \mathbb{R}^m$.

(a) (5%) Suppose that $m = n = 3$, $A = \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & 4 \\ -1 & -4 & 2 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$. Find the set of all solutions to the equation $A\mathbf{x} = \mathbf{b}$ if it is consistent. Otherwise, find vector $\mathbf{p}$ to solve $\min_{\mathbf{p}\in R(A)} \|\mathbf{b}-\mathbf{p}\|_2$ and, moreover, compute the value of $\min_{\mathbf{p}\in R(A)} \|\mathbf{b}-\mathbf{p}\|_2$.

(b) (5%) When the equation $A\mathbf{x} = \mathbf{b}$ is unsolvable, we may consider the so-called least squares problem to find a set of solutions, having the least squares error, from solving a normal equation. Suppose that $rank(A) = k < \min(m, n)$ and let $A = BC$ be a full rank decomposition of A. Use the known matrices B, C, and $\mathbf{b}$ to describe the unique projection vector $\mathbf{p}$ of $\mathbf{b}$ onto $R(A)$ with the least $\|\mathbf{b}-\mathbf{p}\|_2$.

12. (10%) Let $f_1 = x+\alpha$ and $f_2 = x-\alpha$, $\alpha \in \mathbb{R}$, be two vectors in the vector space $C[0, 1]$ with inner product $\langle f, g\rangle := \int_0^1 f(x)g(x)dx$.

(a) (4%) Denote the angle between f_1 and f_2 by θ. Find all possible values of α^2 such that $\theta = \pi/4$.

(b) (6%) Now set $\alpha = 1$. Find functions g_1 and g_2 such that $\{g_1, g_2\}$ is an orthonormal set that satisfies $Span(g_1) = Span(f_1)$ and $Span(g_1, g_2) = Span(f_1, f_2)$.

13. (10%) Consider a linear transformation $L: P_2 \to \mathbb{R}^2$ defined by $L(p(x)) := \begin{bmatrix} \int_0^1 p(x)dx \\ \beta\cdot p(0) + \int_1^\beta p(x)dx \end{bmatrix}$, for every $p(x) \in P_2$, with $\beta > 1$.

(a) (4%) Find all possible values of β such that L^{-1}, the inverse of L, does not exist.

(b) (6%) Suppose that L^{-1} exists. Find the matrix representation of L^{-1} corresponding to the ordered bases $\{x+1, x-1\}$ and $\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\}$ for P_2 and $\mathbb{R}^2$, respectively.

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國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學甲【電機系碩士班甲組、己組、電波領域選考】　　題號：431002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　

以下第 14 題到第 15 題中之所有的提問，需要寫出推導過程或詳細說明理由，答案正確但沒有推導過程或說明不正確，將酌扣分數或不給分。

14. (20%) Consider the following set of differential equations

$$\ddot{x}_1(t) = -2x_1(t) + 2x_2(t)$$
$$\ddot{x}_2(t) = 2x_1(t) + 5x_2(t) + u(t)$$

(a) (15%) Let $u(t) \equiv 0$ and the initial conditions be $x_1(0) = x_2(0) = 1$, $\dot{x}_1(0) = \dot{x}_2(0) = 0$. Find the solutions of the differential equations.

(b) (5%) Let initial conditions be $x_1(0) = x_2(0) = \dot{x}_1(0) = \dot{x}_2(0) = 0$, and $u(t)$ be the unit step function. Does the solutions of the differential equations converge to constant values as time approaches infinity? Justify your answers.

15. (10%) Evaluate the following integral

$$\int_0^{\infty} \int_{x^2}^{\infty} x\mathrm{e}^{-y^2} dy dx$$

End of Examination

國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：資料結構【電機系碩士班丙組】　　題號：431004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）　　

Note: There are 20 questions in total. Each one is 5 points. Please choose one answer for each question. No extra points will be deducted for wrong answers.

1. Suppose a node in a linked list A contains two fields, data and link, where link is the pointer to the following node. Let head_ptr be a pointer to the first node of A. Suppose we would like to traverse all the nodes of A by the loop "while (!head_ptr) $\{B;\}$". What is B? (a) head_ptr == NULL; (b) head_ptr = $\star$(head_ptr) $\rightarrow$ link; (c) head_ptr = head_ptr $\rightarrow$ link; (d) $\star$head_ptr = $\star$(head_ptr) $\rightarrow$ link.
2. Suppose we have a tree where the left subtree of its root contains 2500 nodes and the right subtree contains 200 nodes. How many nodes are processed before the root node for the post-order traversal? (a) cannot be determined; (b) 2500; (c) 200; (d) 2700.
3. For any general tree, which of the following statements is false? (a) Pre-order traversal makes sense; (b) Post-order traversal makes sense; (c) In-order traversal makes sense; (d) No traversals make sense.
4. Suppose a binary search tree is built with the following words (inserted in this order): blueberry, peach, orange, banana, pear, cherry, mango. How many comparisons are needed to search for the word mango? (a) 5; (b) 4; (c) 3; (d) 2.
5. What is the prefix expression of $(((3+7)\times(3/5))-(4\times 2))$? Note that each operand is a one-digit number. (a) $-\times+/\times 373542$; (b) $-\times+37/35\times 42$; (c) $373542\times/+\times-$; (d) $-\times/+3735\times 42$.
6. What is the value of the postfix expression $293/\times 4-9+$? Note that each operand is a one-digit number. (a) 11; (b) 13; (c) 15; (d) 17.
7. Consider a complete binary tree with exactly 10000 nodes, implemented with an array. Suppose that a node has its value stored at index 4999 in the array. What index is the value stored at for this node's parent? (a) 1499; (b) 1450; (c) 2499; (d) 2450.
8. Consider a complete binary tree with exactly 10000 nodes, implemented with an array. Suppose that a node has its value stored at index 4999 in the array. What index is the value

stored at for this node's right child? (a) 9998; (b) 9999; (c) 10000; (d) The node has no right child.

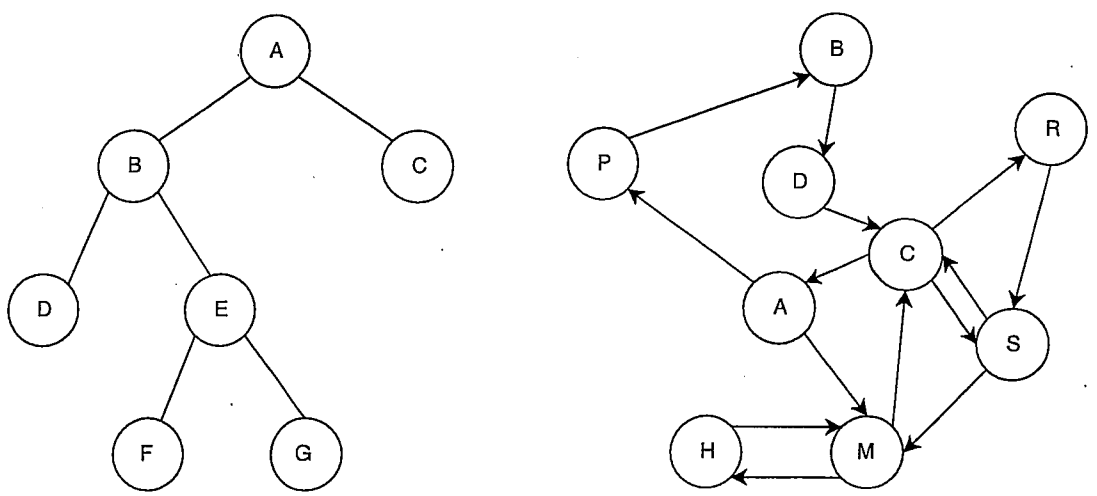
9. Consider the tree in Figure 1(a). What is the order of the nodes processed in the in-order traversal? (a) D-F-B-E-G-A-C; (b) D-B-F-E-G-A-C; (c) D-B-F-G-E-A-C; (d) D-B-F-E-A-G-C.
10. Consider the tree in Figure 1(a). What is the order of the nodes processed in the pre-order traversal? (a) A-B-E-D-F-G-C; (b) A-B-F-E-D-G-C; (c) A-B-D-G-F-E-C; (d) A-B-D-E-F-G-C.
11. Start with an empty max-heap of integers, and enter the ten numbers 1 through 10. Let the max-heap be stored in an array. What index is 8 stored at in the array? (a) 2; (b) 3; (c) 4; (d) 5.
12. Start with an empty max-heap of integers, and enter the ten numbers 1 through 10. Let the max-heap be stored in an array. Remove one entry from the heap. What index is 3 stored at in the array? (a) 5; (b) 4; (c) 3; (d) 2.
13. Suppose you are given an array containing six integers 5, 36, 4, 20, 19, and 9 initially. Starting with 5, use insertionsort to sort the array in increasing order. What is the content of the array after 4 is processed? (a) 4, 5, 20, 36, 19, 9; (b) 5, 36, 4, 9, 19, 20; (c) 4, 5, 19, 36, 20, 9; (d) 4, 5, 36, 20, 19, 9.
14. Suppose you are given an array containing six integers 1, 2, 3, 4, 5, and 6 initially. You want to sort the array in increasing order. Which sorting method is the most efficient to use? (a) selectionsort; (b) insertionsort; (c) mergesort; (d) quicksort.
15. Feature A: The worst-case running time is $O(n \log n)$; Feature B: No additional memory is required. Which of the following sorting methods has both features A and B? (a) heapsort; (b) quicksort; (c) insertionsort; (d) mergesort.
16. Consider the graph in Figure 1(b). Starting with node S, what is the order of the nodes processed by the depth-first search? If two or more nodes can be chosen, choose the node with the smallest label first. (a) S-C-A-P-B-M-H-D-R; (b) S-M-H-C-A-P-B-D-R; (c) S-C-A-M-H-P-B-D-R; (d) S-C-R-A-M-H-P-B-D.

17. Consider the graph in Figure 1(b). Starting with node S, what is the order of the nodes processed by the breadth-first search? If two or more nodes can be chosen, choose the node with the smallest label first. (a) S-C-A-M-R-H-P-B-D; (b) S-C-M-A-H-B-P-R-D; (c) S-C-M-A-R-P-B-H-D; (d) S-C-M-A-R-H-P-B-D.

18. Suppose you are doing a breadth-first search of a graph with n nodes. How large can the queue get during the search? (a) n; (b) $n-1$; (c) $n-2$; (d) 1.

19. An empty hash table has a capacity of 13, and you insert six entries with keys 20, 15, 7, 9, 21, 33, and 48. Using linear probing and the hash function $x\%(13)$, what index 21 is stored at in the table? Note that % is the remainder operator, e.g., (100)%(13)=9. (a) 7; (b) 8; (c) 9; (d) 10.

20. An empty hash table has a capacity of 13, and you insert six entries with keys 20, 15, 7, 9, 21, 33, and 48. Using linear probing and the hash function $x\%(13)$, what index 48 is stored at in the table? Note that % is the remainder operator, e.g., (100)%(13)=9. (a) 10; (b) 11; (c) 12; (d) 9.



(a) A binary tee (b) A graph

Figure 1: Two figures.

國立中山大學107學年度碩士暨碩士專班招生考試試題

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※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）(選擇題)　　

【本頁供計算使用】

國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：電路學【電機系碩士班丁組】　　題號：431006

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（單選題共20題）　共3頁第1頁

每題正確答案得5分，錯誤答案倒扣1分，未作答者，不給分亦不扣分

1. A Lithium Ion battery module is rated at 12V/6Ah. A module is discharged by 0.5C constant current for 0.5 hour. Determine the power supplied by the battery module.
   Ⓐ、144W
   Ⓑ、72W
   Ⓒ、36W
   Ⓓ、18W

2. A capacitor 0.1F is charged from 50V to 100V by a constant current 1A. Determine the energy is stored in the capacitor during this process.
   Ⓐ、500J
   Ⓑ、375J
   Ⓒ、250J
   Ⓓ、125J

3. Current flowing in a resistor 10kΩ is equal to 2mA. Determine power consumption of the resistor.
   Ⓐ、40kW
   Ⓑ、40W
   Ⓒ、40mW
   Ⓓ、40μW

4. A voltage source 10 V is applied to a series RLC circuit with parameters L=4H, C=0.25F, R=10Ω. Determine power consumption.
   Ⓐ、200W
   Ⓑ、100W
   Ⓒ、50W
   Ⓓ、0W

5. Determine reactance of a capacitor C=20nF at 100kHz.
   Ⓐ、0.002Ω
   Ⓑ、0.0125Ω
   Ⓒ、500Ω
   Ⓓ、80Ω

6. Determine time constant of a series RC first-order circuit with parameters C=0.25F and R=40Ω.
   Ⓐ、10s
   Ⓑ、0.1s
   Ⓒ、10rad
   Ⓓ、0.1rad

7. Current 10cos(377t) A flows into a series RL circuit with parameters L=1mH and R=10Ω. Determine average power of the RL circuit.
   Ⓐ、1000W
   Ⓑ、500W
   Ⓒ、250W
   Ⓓ、0W

8. A 10Ω resistive load is applied by 100cos(377t) V. Choose a false statement.
   Ⓐ、instantaneous power is equal to average power
   Ⓑ、average power is equal to apparent power
   Ⓒ、power factor is equal to 1
   Ⓓ、reactive power is equal to 0

9. For an AC circuit, choose a false statement.
   Ⓐ、inductive current lags inductive voltage
   Ⓑ、capacitive current leads capacitive voltage
   Ⓒ、average power of an inductor is zero
   Ⓓ、instantaneous power of an inductor is zero

10. Determine the power factor of a load with impedance 10+j10Ω.
   Ⓐ、0.707 leading
   Ⓑ、0.707 lagging
   Ⓒ、0.866 leading
   Ⓓ、0.866 lagging

11. A delta-connected resistive load 100Ω per phase is connected to a balanced three-phase three-wire 200V circuit. Choose a false statement.
   Ⓐ、power factor is 1
   Ⓑ、reactive power is 0
   Ⓒ、average power is 1200W
   Ⓓ、line current is 2A

12. A Y-connected resistive load 40Ω per phase is connected to a balanced three-phase four-wire voltage source. Phase voltage is 100V. A neutral line with resistance 10Ω is connected between source and load. Determine current in the neutral line.
   Ⓐ、10A
   Ⓑ、2A
   Ⓒ、2.5A
   Ⓓ、0A

13. For an ideal transformer, voltage at the primary side and the secondary side are 500V and 100V, respectively. Secondary side is connected to a 100Ω resistor. Choose a false statement.
   Ⓐ、output power is 100W
   Ⓑ、input power is 100W
   Ⓒ、output current is 1A
   Ⓓ、input current is 1A

14. For a two-winding coupling device in an AC circuit, choose a false statement.
   Ⓐ、coupling coefficient is less than 1
   Ⓑ、mutual inductance is between -1 and 1
   Ⓒ、induced voltage is determined by Faraday's law
   Ⓓ、direction of induced voltage is determined by Lenz's law

國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：電路學【電機系碩士班丁組】　　題號：431006

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（單選題共20題）　

每題正確答案得5分，錯誤答案倒扣1分，未作答者，不給分亦不扣分

15. Determine RMS value of a half-wave rectified sine wave with amplitude 100V.
    Ⓐ、141.4V
    Ⓑ、100V
    Ⓒ、70.7V
    Ⓓ、50V

16. An ideal buck converter is operated in CCM mode. Input voltage is 20V, output voltage is 10V and load resistance is 2Ω. Determine the average value of switch current.
    Ⓐ、2.5 A
    Ⓑ、5 A
    Ⓒ、10 A
    Ⓓ、20 A

17. An ideal boost converter is operated in CCM mode. Input voltage is 10V and output voltage is 50V and load resistance is 10Ω. Determine the average value of diode current.
    Ⓐ、5A
    Ⓑ、4A
    Ⓒ、2A
    Ⓓ、1A

18. Determine the speed of rotation of a synchronous generator with parameters 480V/60Hz/six poles/delta-connection.
    Ⓐ、3600rpm
    Ⓑ、1800rpm
    Ⓒ、1200rpm
    Ⓓ、900rpm

19. A four-pole, 60Hz induction motor supplies 15kW to a load at a speed of 1710 rpm. Determine rotor frequency at this condition.
    Ⓐ、3Hz
    Ⓑ、6Hz
    Ⓒ、63Hz
    Ⓓ、57Hz

20. Consider load flow in an inductive feeder. Choose a true statement.
    Ⓐ、real power flow is primarily determined by voltage difference between two buses
    Ⓑ、reactive power flow is primarily determined by angle difference between two buses
    Ⓒ、injection of reactive power increases voltage
    Ⓓ、consumption of real power increases voltage

國立中山大學 107 學年度碩士暨碩士專班招生考試試題

科目名稱：計算機結構【電機系碩士班己組】　　題號：431007

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. [15%] Complete the calculations of the following two numbers, X=134.0625, Y=-18

(a) [5%] Represent X and Y using IEEE 754 single precision format, respectively

(b) [5%] Add X and Y, and represent the result using IEEE 754 single precision format

(c) [5%] Multiply X and Y, and represent the result using IEEE 754 single precision format

2. [20%] Translate the following C code to the **minimum** MIPS assembly instructions

```
int A[30], B[30];
for (i=1; i < 30; i++) {
A[i] = A[i] - B[A[i-1]]+i;
}
```

At the beginning of this code segment, the only values in registers are the base address of arrays A and B in registers $a1 and $a2. Assume that the values of i is stored in the register $s0

3. [20%] Figure 1 shows a complete datapath with control for MIPS CPU.

(a) [5%] Explain **in detail** how the architecture shown in Figure 1 is used to execute an I-type instruction (e.g., addi $s3, $s3, 1) **step-by-step**.

(b) [5%] Explain **in detail** how the architecture shown in Figure 1 is used to execute a jump instruction (e.g., j L1) **step-by-step**.

(c) [5%] Describe **in detail** how this architecture is used to execute a load instruction (e.g., lw $t0, 32($s3)) **step-by-step**.

(d) [5%] Explain **in detail** how the architecture shown in Figure 1 is used to execute a branch instruction (e.g., beq $t0, $s5, Exit) **step-by-step**.

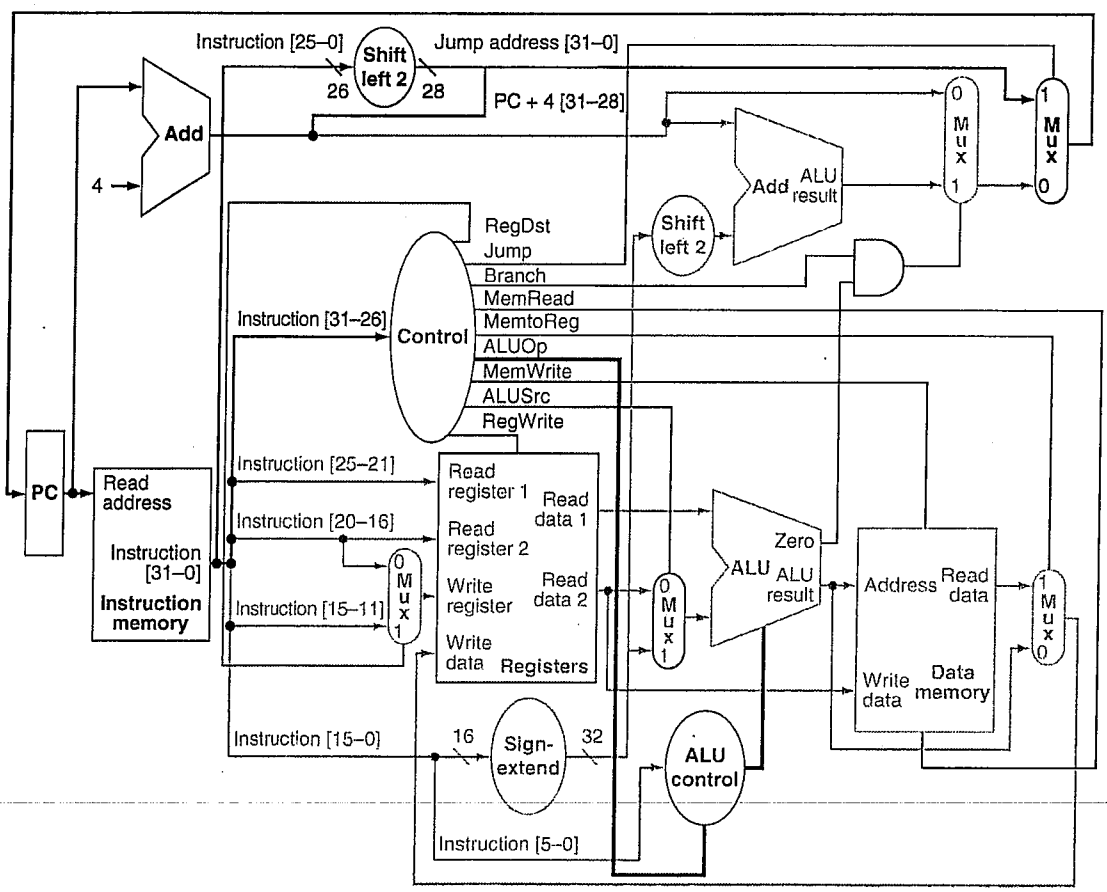

Figure 1: Architecture for Problem 3

背面有題　試題隨卷繳回

4. [20%] Design of a pipelined-CPU

(a) [5%] Explain the functions of the five pipeline stages of the pipelined MIPS CPU, respectively.

(b) [6%] Use examples to explain the three types of hazards of the pipelined MIPS CPU, respectively.

(c) [9%] Explain **in detail** how we can solve the three types of hazards of the pipelined MIPS CPU based on the architecture shown in Figure 2, respectively.

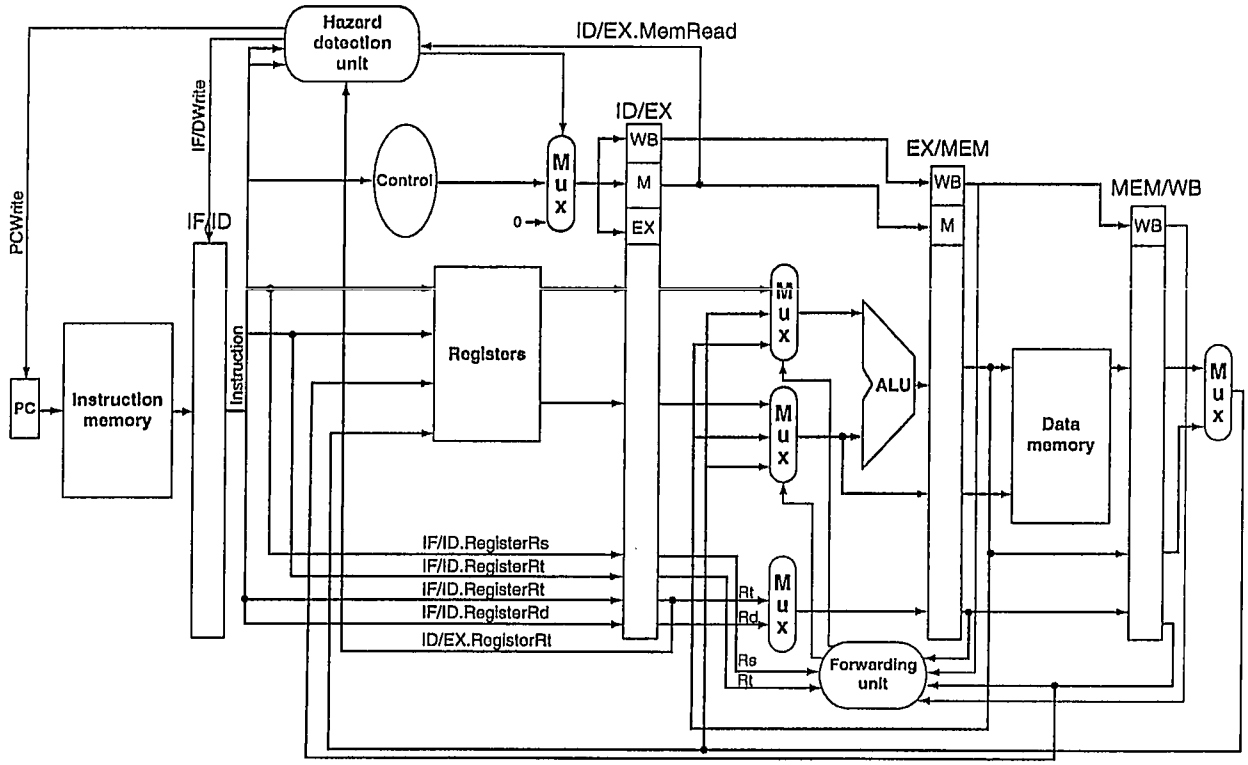

Figure 2: Architecture for Problem 4

5. [25%] Cache memory

(a) [4%] Explain what two localities of memory data are and give examples, respectively.

(b) [2%] Explain how a hierarchical memory system takes advantage of localities.

(c) [4%] What are the advantages and disadvantages of SRAMs and DRAMs, respectively? How are they used in a hierarchical memory system, respectively?

(d) [15%] For a cache with 4 blocks, complete the cache access results for a direct mapped cache, a 2-way set associative cache and a fully associative cache, respectively, shown in the three tables shown below. Please draw these three tables in your answer papers and fill in your answers. Note that in this problem the least recently used block replacement policy is assumed.
請特別注意！請將第三頁表一、表二、 表三的內容繪製在答案卷中並填入答案才允予計分。填寫在題目卷上將不予計分。

國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：計算機結構【電機系碩士班己組】　　題號：431007

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

Table 1: Direct mapped cache

Block address	Cache index	Hit/miss	Cache content after access (Mem[x] where x is the block address)			
			0	1	2	3
4						
2						
4						
1						
3						
5						
3						
1						

Table 2: 2-way set associative cache

Block address	Cache index	Hit/miss	Cache content after access (Mem[x] where x is the block address)			
			Set 0		Set 1	
4						
2						
4						
1						
3						
5						
3						
1						

Table 3: fully associative cache

Block address	Hit/miss	Cache content after access (Mem[x] where x is the block address)			
4					
2					
4					
1					
3					
5					
3					
1					

國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：電子學【電機系碩士班甲組】　　題號：431009

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. (20%) Figure 1 shows a MOSFET circuit with the V_{DC} = 5 V, R_{sig} = 120 kΩ, R_{G1} = 300 kΩ, R_{G2} = 200 kΩ, R_D = 2 kΩ, R_S = 1 kΩ, R_L = 1 kΩ. The threshold voltage V_{TH} and $\mu C_{ox}W/L$ of MOSFET are –1 V and 2 mA/V^2 respectively. Neglect the channel length modulation effect. (a) Please find the DC drain current I_D. (b) For AC analysis, please find the input resistance R_{in}, output resistance R_{out} and overall gain v_o/v_{sig}. (5%*4)
2. (20%) Figure 2 shows a BJT circuit with the V_{DC} = 5 V, R_{sig} = 120 kΩ, R_{B1} = 300 kΩ, R_{B2} = 200 kΩ, R_E = 5 kΩ, R_C = 2 kΩ, R_L = 5 kΩ. The BJT has $|V_{BE}| \approx 0.7$ V, β = 100, thermal voltage V_T = 25 mV. Neglect the Early effect. (a) Please find the DC collector current I_C. (b) For AC analysis, please find the input resistance R_{in}, output resistance R_{out} and overall gain v_o/v_{sig}. (5%*4)
3. (30%) The amplifier shown in Fig. 3 has V_{DC} = 1.5 V, $R_{sig} = R_L$ = 1 kΩ, R_C = 1 kΩ, R_B = 47 kΩ. The BJT has $|V_{BE}| \approx 0.7$ V, β = 100, C_μ = 0.8 pF, f_T = 600 MHz, and thermal voltage V_T = 25 mV. Assume the coupling capacitors to be very large. Neglect the Early effect. (a) Find the collector current of the transistor. (b) Find the overall gain v_o/v_{sig} and the input resistance R_{in} at midband frequency. (c) Find the high frequency response f_H. (5%,5%,10%,10%)
4. (30%) Consider the BiCMOS amplifier shown in Fig. 4 has V_{DC} = 5 V, R_{sig} = 100 kΩ, R_G = 10 MΩ, R_S = 6.8 kΩ, R_C = 3kΩ, R_L= 1 kΩ, C_1 = 0.1 μF, C_2 = 1 μF. The BJT has $|V_{BE}| \approx 0.7$ V, β = 200, C_μ = 0.8 pF, f_T = 600 MHz, and thermal voltage V_T = 25 mV. The MOSFET has V_{TH} = 1 V, $\mu C_{ox}W/L$ = 2 mA/V^2, and $C_{gs} = C_{gd}$ = 1 pF. Neglect the Early effect and channel length modulation effect. (a) Consider the DC bias circuit. Neglect the base current of Q2 in determining the current in Q1. Find the DC drain current in Q1 and collector current in Q2. (b) Consider the circuit at low frequencies. Determine the frequency of the poles due to C1 and C2. (c) Consider the circuit at higher frequencies. Use open-circuit time constants to estimate f_H. (5%,5%,5%,5%,10%)

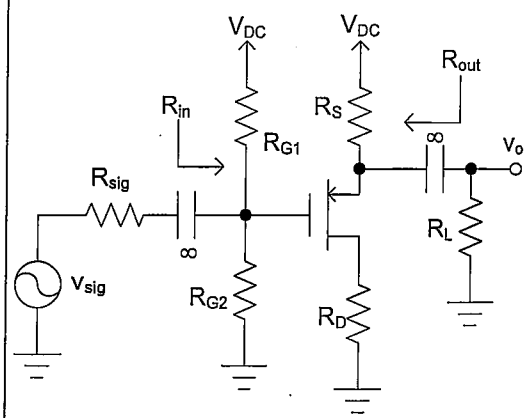

Figure 1

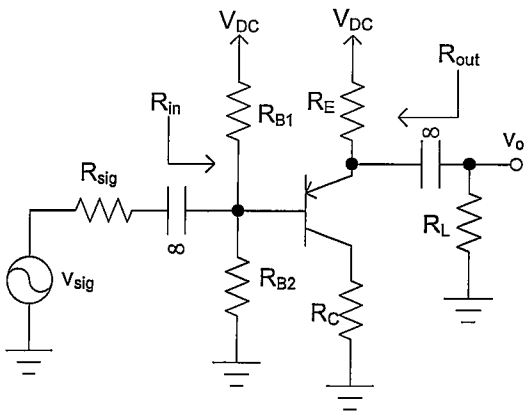

Figure 2

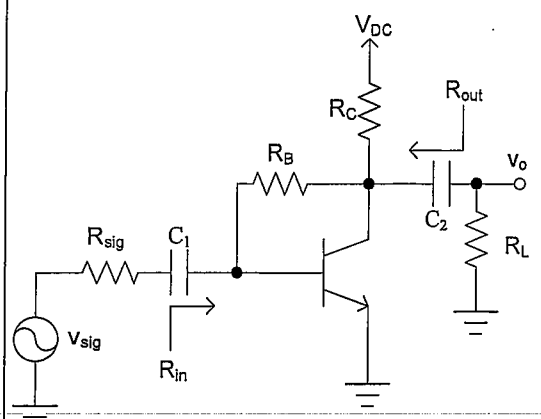

Figure 3

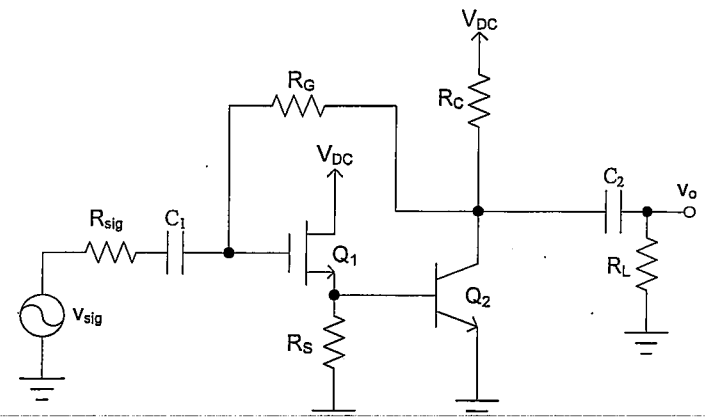

Figure 4

試題隨卷繳回

國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：控制系統【電機系碩士班乙組】　　題號：431008

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

Question 1 (30%)

一個系統的輸入 u 與輸出 y 之間有以下的關係：

$$y(t) = \int_{-\infty}^{\infty} e^{-|t-\tau|} u(\tau) d\tau,$$

請回答以下問題。你必須證明或嚴謹說明你的答案。只有答案但無證明或說明不予計分。

(a) (5%) 請問此系統是否為線性系統?

(b) (5%) 請問此系統是否為非時變系統?

(c) (5%) 請問此系統有無轉移函數? 若有，請問轉移函數為何? 若無，請詳明理由。

(d) (5%) 請問此系統是否為BIBO (Bounded Input Bounded Output) 穩定?

(e) (5%) 請問此系統是否具有因果性 (為 causal system)?

(f) (5%) 請問此系統對哪個頻率的訊號有最大震幅放大的效果?

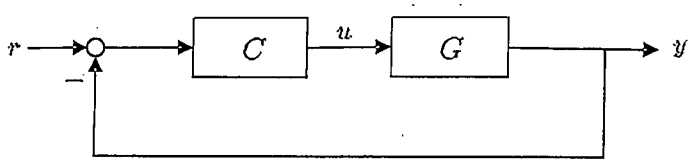

Figure 1: the feedback system for Questions 2 and 3.

Question 2 (40%)

考慮如圖 Figure 1 之回授控制系統，其中G 與 C分別代表受控系統及控制器，且 G 的動態由下列之微分方程式所支配

$$\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = -b\dot{u}(t-\tau) + u(t-\tau)$$

其中 a_0, a_1, b, τ 為常數且 $\tau \geq 0$。請回答以下問題。你必須證明或嚴謹說明你的答案。只有答案但無證明或說明不予計分。

(a) (5%) 當 $b = 0$ 時，請問 a_0 與 a_1 需滿足何種條件方能使 G 為穩定且其步階響應無震盪現象？

(b) (5%) 當 $\tau = 0$且 G 為穩定時，請問 G 之增益邊界 (gain margin) 可否為無窮大？

(c) (5%) 當 $\tau \neq 0$，$b \leq 0$，且 G 為穩定時，請問 G 之增益邊界 (gain margin) 可否為無窮大？

(d) (5%) 當 $\tau \neq 0$ 且 G 為穩定時，請問 G 之相位邊界 (phase margin) 可否為無窮大？

(e) (5%) 假設 $\tau = 0$ 且控制器 C 為比例控制器 (這裡我們僅考慮正增益值)。當 G 為極小相位系統時，請問 C 之增益是否可為任意值而閉迴路系統皆為穩定？

(f) (5%) 假設 $a_0 = a_1 = 0$ 且 $\tau \neq 0$ 。如果控制器 C 為比例控制器 (這裡我們僅考慮正增益值) 的話，請問 b 需滿足何種條件閉迴路系統才有可能穩定？

(g) (5%) 假設 $a_0 = a_1 = b = 0$ 且 $\tau \neq 0$。請問下列哪些控制器有可能讓閉迴路系統穩定？

(1) I 控制器 (2) D 控制器 (3) PI 控制器 (4) PD 控制器 (5) PID 控制器

背面有題　　試題隨卷繳回

(h) (5%) 假設 $b = \tau = 0$，控制器 C 為一階且恰適 (proper)，並且已知閉迴路系統為穩定。請問這些條件下，閉迴路系統是否有可能完美追蹤（即:達成零穩態誤差）下列訊號?
(1) 步階訊號 (2) 斜坡訊號 (3) 拋物線訊號

Question 3 (20%)

再次考慮如圖 Figure 1 之回授控制系統。

(a) (5%) 令 G 的轉移函數為 $\frac{4}{(s+1)(s^2+2s+4)}$，$C$ 為一為比例控制器 (這裡我們僅考慮正增益值)。當輸入為單位步階訊號時，請問系統輸出之穩態誤差之極小值 (infimum) 為何？

(b) (15%) 針對上述系統 G，請重新設計一控制器，確保閉迴路系統之步階響應之穩態誤差小於 0.1%。

Question 4 (10%)

請問「相位落後」補償器（phase-lag compensator）可否用來提升系統之相位邊界（phase margin）；也就是說，加了 phase-lag 補償器後的系統，其相位邊界可否較未補償前的系統之相位邊界為大? 必要時，請輔以適當的波德圖（Bode plot）來說明你的理由。

End of Examination

國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：離散數學【電機系碩士班丙組】　　題號：431011

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）　　

Note: There are 20 questions in total. Each one is 5 points. Please choose one answer for each question. No extra points will be deducted for wrong answers.

1. In how many ways can the letters in AAANSYSU be linearly arranged? (a) 40320; (b) 13440 (c) 6720; (d) 3360.
2. In how many ways can the letters in AAANSYSU be linearly arranged such that all the A's are adjacent? (a) 720; (b) 360; (c) 240; (d) 180.
3. In how many ways can the letters in AAANSYSU be linearly arranged such that N and U are adjacent? (a) 840; (b) 420; (c) 210; (d) 140.
4. What is the probability that a random, linear arrangement of the letters in AAANSYSU starts and ends with the letter S? (a) $\frac{1}{112}$; (b) $\frac{1}{56}$; (c) $\frac{1}{28}$; (d) $\frac{1}{14}$.
5. What is the coefficient of x^7y^3 in the expansion of $(\frac{1}{2}x - 3y)^{10}$? (a) $-\frac{405}{8}$; (b) $-\frac{405}{16}$; (c) $-\frac{405}{32}$; (d) $-\frac{405}{64}$.
6. What is the sum of all the coefficients in the expansion of $(x+y)^{11}$? (a) 256; (b) 512; (c) 1024; (d) 2048.
7. Let p, q be primitive statements for which $p \to q$ is false. Which of the following is true? (a) $p \wedge q$; (b) $\neg p \vee q$; (c) $q \to p$; (d) $\neg q \to \neg p$.
8. Let p, q be arbitrary statements. Which of the following is equivalent to $p \to q$? (a) $p \wedge q$; (b) $p \vee \neg q$; (c) $\neg q \to p$; (d) $\neg q \to \neg p$.
9. Let p, q, and r be arbitrary statements. If $p \to r$ is true and $\neg p \to q$ is true, which of the following is also true? (a) $\neg r \to q$; (b) $r \to q$; (c) $\neg r \vee q$; (d) $r \vee \neg q$.
10. If a set S has 127 proper subsets, what is $|S|$? (a) 128; (b) 254; (c) 256; (d) 512.
11. How many subsets of $\{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$ contain at least one even integer? (a) 1981; (b) 1982; (c) 1983; (d) 1984.
12. What is the value of n, a positive integer, for which $\sum_{i=1}^{n}(i+9) = \sum_{i=1}^{n}(2i-1)$? (a) 17; (b) 18; (c) 19; (d) 20.

試題隨卷繳回

背面有題

13. Let $b_1 = 3$ and $b_{n+1} = b_n + 3n + 2$ for $n \geq 1$. What is b_{10}? (a) 176; (b) 156; (c) 136; (d) 116.

14. If we want to totally make sure at least two of n different people have birthdays that occur on the same day of January, what is the minimum for n? (a) 64; (b) 32; (c) 2; (d) 31.

15. Let $S = \{\text{Alan, John, Jone, Sam, Don, Mary, Tom, Bill}\}$. How many subsets are there which contain both Sam and Tom? (a) 16; (b) 32; (c) 48; (d) 64.

16. A company gets graphics cards from two sources. The first source provides 40% of the cards, and the second source provides 60% of the cards. Past experience has shown that $\frac{1}{8}$ of the cards from the first source are found to be defective, while $\frac{1}{6}$ of the cards from the second source are found to be defective. If a graphics cards is selected and found to be defective, what is the probability it was provided by the first source? (a) $\frac{1}{6}$; (b) $\frac{2}{6}$; (c) $\frac{3}{6}$; (d) $\frac{4}{6}$.

17. The number of bacteria is 100 initially, and the number doubles every four hours. How many bacteria are there after one day? (a) 6400; (b) 3200; (c) 1600; (d) 800.

18. What is x for making the equation $\log_{10} x - \log_{10} 6 = 2$ hold? (a) 1000; (b) 9; (c) 300; (d) 600.

19. Let A, B, C be matrices. Which of the following is true? (a) IF AB and BA are computable, then $AB = BA$; (b) If $AB = AC$, then $B = C$; (c) If $A + B = A + C$, then $B = C$; (d) If A is square, then A is invertible.

20. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 13$, what is the value of $\begin{vmatrix} 2a & 3b & 4c \\ 2d & 3e & 4f \\ 2g & 3h & 4i \end{vmatrix}$? (a) 312; (b) 156; (c) 104; (d) 52.

國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：離散數學【電機系碩士班丙組】　　題號：431011

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）(選擇題)　　

【本頁供計算使用】

國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：離散數學【電機系碩士班丙組】　　題號：431011

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【本頁供計算使用】

國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：半導體概論【電機系碩士班甲組】　　題號：431012

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

Dielectric constant: Si = 11.9 ; SiO_2 =3.9. Bandgap: Si = 1.12 eV ; GaAs = 1.42 eV.

1. A silicon sample has a doping profile with donors $N_D = N_O \exp(-mx)$. If $N_D >> n_i$, find the expression for the built-in electric field at equilibrium. (20%)

2. A Si p-n junction has same doping concentration 2×10^{16} cm^{-3} in each side. The peak electric filed in the junction at breakdown is 2×10^5 V/cm. Calculate the reverse breakdown voltage of this junction at 300K. (20%)

3. Calculate the oxide capacitance, the flatband capacitance, and the high frequency capacitance in inversion of a silicon MOS capacitor with a substrate doping $N_A = 2\times10^{17}$ cm^{-3}, a 25 nm thick silicon dioxide and an aluminum gate (ϕ_M = 4.1 V). (20%)

4. For heterojunctions in the GaAs-AlGaAs system, the direct bandgap difference is accommodated approximately $\frac{2}{3}$ in the conduction band and $\frac{1}{3}$ in the valence band. The bandgap of AlGaAs is 1.85 eV if Al composition is 0.3. Draw the band diagrams of two heterojunctions:
(a) N^+-$Al_{0.3}Ga_{0.7}As$ on n-GaAs (b) P^+-$Al_{0.3}Ga_{0.7}As$ on n-GaAs (20%)

5. In a metal-Si Schottky barrier contact, the barrier height is 0.85 eV and the effective Richardson constant is 110 $A/K^2\text{-}cm^2$. Calculate the ratio of the injected hole current to the electron current at 300K. $D_p = 12$ cm^2/s, $\tau_p = 5\times10^{-7}$ s, and $N_D = 2\times10^{16}$ cm^{-3}. (20%)

試題隨卷繳回

國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：電子學【電波領域聯合】　　題號：482003

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）　　

1. (30%) Figure 1 shows an amplifier made of a single MOSFET that is biased with I_D = 0.5 mA. Assume that all capacitors C_1, C_2 and C_3 are large enough to act like shorted in the frequency band of interest, and the parasitic capacitances of the MOSFET Q and the series gate resistance are negligible. The transistor Q has the device parameters: W/L = 80, $\mu_n C_{ox}$ = 50 μA/V^2, V_{TH} =0.7 V, λ = 0.1. **(a)(20%)** Draw the ac equivalent circuit. Determine the frequency (in rad/s) at which the amplifier achieves the peak gain, and determine this maximum gain. **(b) (10%)** Find the bandwidth of the amplifier (in rad/s). Note: $I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda V_{DS})$ for Q in saturation.

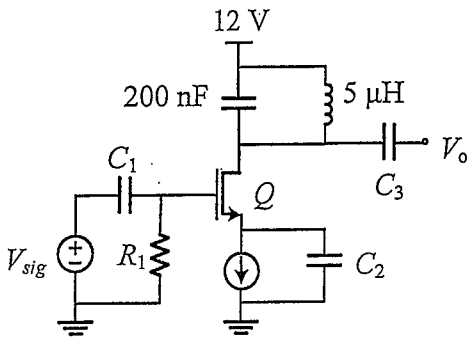


Fig. 1

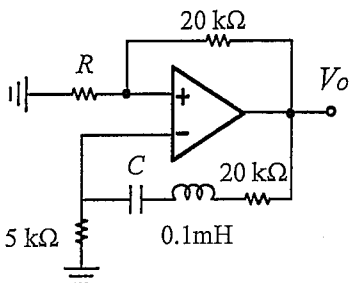


Fig. 2

2. (20%) Use the Barkhausen criterion to determine the values of R and C so that the Wien-bridge circuit in Fig. 2 oscillates at 100 kHz.

3. (30%) (a) (20%) Determine the values of R and C in Fig. 3 so that the average power dissipation on resistor R is maximized. **(b) (10%)** Calculate this maximum power.

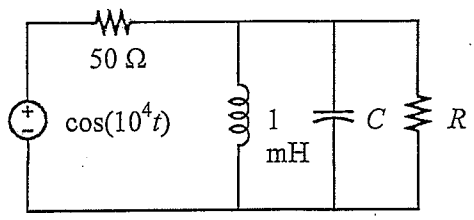


Fig. 3

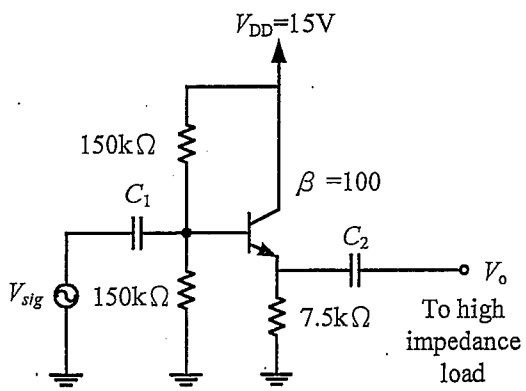


Fig. 4

4. (20%) An emitter follower in Fig. 4 is used to drive a very high impedance. C_1 forms a high-pass filter with the divider resistances and the resistance looking into the base. Choose the value of C_1 so that the resulting cutoff frequency is 1 kHz.

試題隨卷繳回

國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：電磁學【電機系碩士班戊組、通訊所碩士班乙組、電波領域聯合】　題號：482004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　

1. (10%) (a) Explain Gradient, Divergence, and Curl. (b) Curl of a Gradient field, $\nabla \times \nabla V = 0$, Divergence of a Curl, $\nabla \cdot \nabla \times \mathbf{A} = 0$. V is a scalar field, A is a vector field. 說明其物理意義(why?)，或舉例。

2. (10%) For a coaxial transmission line, Fig. 1, the capacitance per unit length is $c' = \frac{2\pi \cdot \varepsilon_0}{\ln \frac{b}{a}} \left[\frac{\mathrm{F}}{\mathrm{m}}\right]$, and the inductance per unit length is $\ell' = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi} \ln \frac{b}{a} \left[\frac{\mathrm{H}}{\mathrm{m}}\right]$. At high frequencies, the internal inductance drops off. Find the characteristic impedance of the coaxial line, $Z_c = \sqrt{\frac{\ell'}{c'}}$, at high frequencies. Please also write down the unit of Z_c, *i.e.*, what is the square root of (H/F)? What is the speed the wave travels in the coaxial cable? You can find it by calculating velocity $= \frac{1}{\sqrt{\ell' \cdot c'}}$.

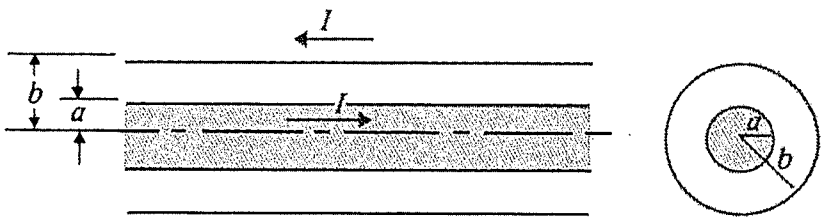


Fig. 1. Coaxial cable

3. (10%) In the following configurations, Fig. 2, assuming both grounds are perfect conductors, current directions are as indicated (the solid arrows); the image current for both cases are shown. Using $a_n \times H = J$, H is the magnetic field intensity on the ground, a_n is the normal vector of the top surface of the grounds, determine the direction of the currents on the top surface of the grounds.

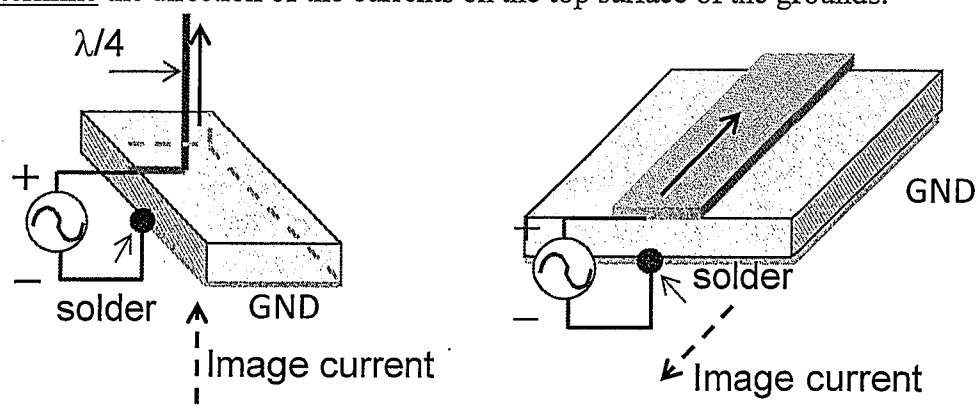


Fig. 2. Current relative to the ground.

Considering both the original current and its induced current on the ground, which one, left or right, is likely to be an effective antenna structure, why?

4. (5%) Using the Method of Image, write down the potential distribution, $V(x, y, z)$, for an observing point $P(x, y, z)$ in the space, Fig. 3. The dielectric constant of the space is ε_0. Q is a positive point

charge of Q *Coul.*

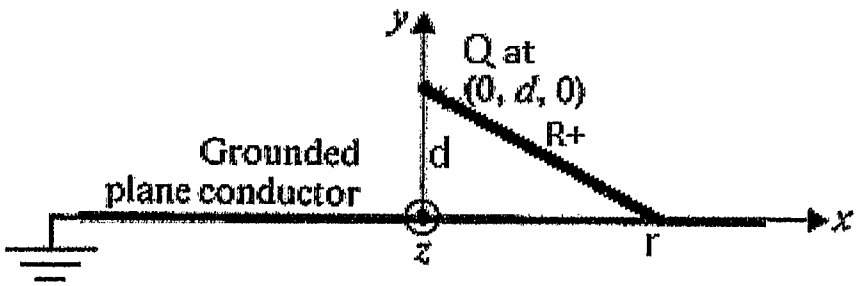
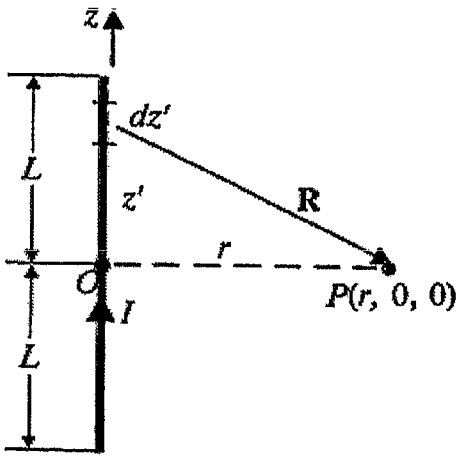


Fig. 3. A charge source Q above a ground.

5. (10%) 下圖 Fig. 4 之 magnetic flux density B can be found as,



$$\mathrm{B} = a_\phi \frac{\mu_0 IL}{2\pi r\sqrt{L^2+r^2}}$$

Fig. 4. An observing point P near a current source I.

簡化上列之 B as a function of r, μ_0, and a_ϕ when $L>>r$. In cylindrical coordinate system,

$\nabla\cdot\mathrm{B}=\frac{1}{r}\frac{\partial}{\partial r}(rB_r)+\frac{1}{r}\frac{\partial B_\phi}{\partial\phi}+\frac{\partial B_z}{\partial z}$. Show that $\nabla\cdot\mathbf{B}=\mathbf{0}$.

6. (5%) What are the permittivity ε and permeability μ of Copper, a very good conductor? Provide your reasoning.

7. (10%) Analyze the performance of a right-hand circularly polarized wave received respectively by linearly or circularly polarized antennae.

8. (15%) As shown in Fig. 5, a waveguide filled with a material whose $\varepsilon_r = 2.25$ has dimensions $a = 2$ cm and $b = 1.4$ cm. If the guide is to transmit 13.5 GHz signals, what possible modes can be used for the transmission? Please respectively calculate the cutoff frequencies of the possible modes.

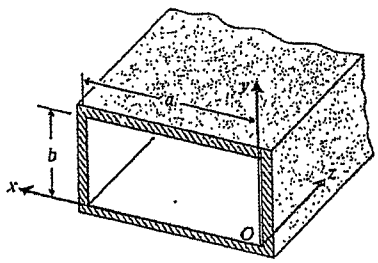


Fig. 5

9. (15%) According to Fig. 6, write the input impedance of the transmission line in differential special cases.
   (a) (6%) Open-circuit termination, and also plot the reactance-line length diagram
   (b) (6%) Short-circuit termination, and also plot the reactance-line length diagram
   (c) (3%) Quarter-wave section

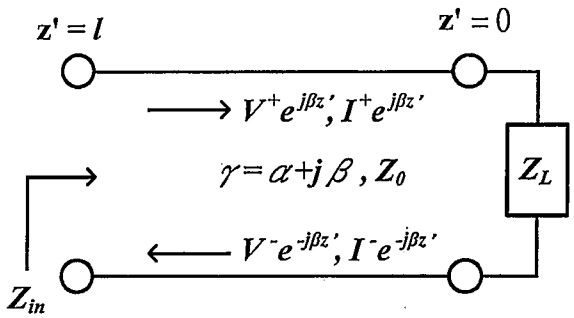


Fig. 6

10. (10%) As shown in Fig. 7, calculate the average power transmitted into the infinite 150 Ω line. The λ/2 line is lossless and the infinitely long line is slightly lossy. (Hint: The input impedance of an infinitely long line is equal to its characteristic impedance so long as the line is not lossless)

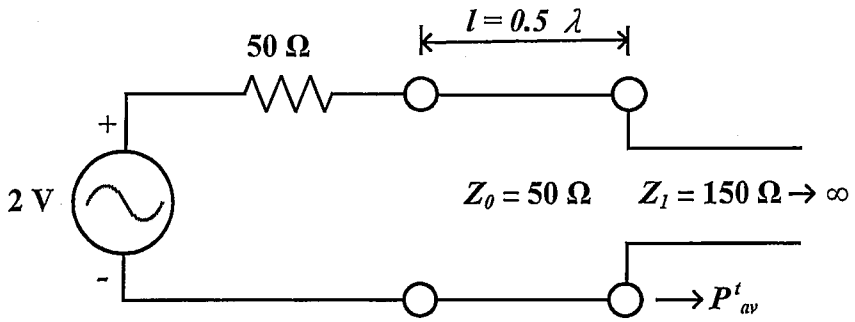


Fig. 7