國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學甲【電機系碩士班甲組、己組、電波領域選考】　　題號：431002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）　　

第一題到第八題為單選題，每題四分。請選出一個最正確選項，答錯不倒扣。

1. When using the *Gaussian elimination* to solve a linear equation $A\mathbf{x} = \mathbf{b}$, elementary row operations (or multiplication by elementary matrices) are applied to the augmented matrix $[A\ \mathbf{b}]$. Actually, there are many places in linear algebra where such a technique plays its role; e.g., the null space of A can be determined by setting $\mathbf{b} = 0$. Which of the following cannot be determined by applying such a technique?
   (A) the range of A
   (B) the inverse of A (if it is nonsingular)
   (C) the QR factorization of A
   (D) the LU factorization of A (if it is square)
   (E) the determinant of A (if it is square).

2. The *linear combination* of a set of vectors is an essential element in linear algebra. We say a set V is invariant under linear combination if the implication " $\forall n \in \mathbb{N}$ and any set of vectors $\{\mathbf{v}_1, \cdots, \mathbf{v}_n\} \subset V \Rightarrow$ the set of all linear combinations $\{c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n | c_i \in \mathbb{R}, \forall i\} \subset V$" holds. And we say a mapping L defined on a set X is invariant under linear combination if the form of linear combination is unchanged under L, or more precisely the statement "$\forall n \in \mathbb{N}$, $\forall c_i \in \mathbb{R}$, and any set of vectors $\{\mathbf{x}_1, \cdots, \mathbf{x}_n\} \subset X$, the identity $L(c_1\mathbf{x}_1 + \cdots + c_n\mathbf{x}_n) = c_1L(\mathbf{x}_1) + \cdots + c_nL(\mathbf{x}_n)$ holds" is true. Which one of the following statements related to linear combination is false
   (A) Let S be a subset of a vector space V. Then S is a subspace of V if S is invariant under linear combination.
   (B) Let $\{V_1, \cdots, V_k\}$ be a set of k subspaces of a vector space W and denote $\text{span}\{V_1, \cdots, V_k\}$ as the set of all linear combinations of the form $c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n$, with each $\mathbf{v}_i$ chosen freely from V_i. Then $\text{span}\{V_1, \cdots, V_k\}$ is also a subspace of W with $\dim(\text{span}\{V_1, \cdots, V_k\}) = \dim(V_1) + \cdots + \dim(V_k)$
   (C) Let A and B be two matrices and denote $C \coloneqq AB$. Then each column of C is a linear combination of all columns of A, and so $rank(C) \leq rank(A)$ is implied.
   (D) A mapping L between two vector spaces is a linear transformation if and only if it is invariant under linear combination.
   (E) Let $(V, \langle\cdot,\cdot\rangle_V)$ be an inner product space. Then $\langle\cdot,\cdot\rangle_V$ is invariant under linear combination at either one of its two arguments.

3. Consider the system represented by the differential equation $\ddot{x} + \dot{x} + kx = 0$. Which of the following is true? (A) the system is critically damped when $k = 1$ (B) the system is underdamped when $k = 1/2$ (C) the system is overdamped when $k = 0.3$ (D) the damping ratio of the system is increased when increasing k (E) none of the above

4. Consider the differential equation $\ddot{x} + b\dot{x} + kx = \cos(\omega t)$ and its sinusoidal solution $x_p(t)$. Which of the following is true? (A) for a fixed b, the amplitude of $x_p(t)$ is maximized when $k = \omega^2$ (B) for a fixed b, the maximal amplitude of $x_p(t)$ is $1/b$ (C) for fixed b and k, the amplitude of $x_p(t)$ always grows as ω increases (D) the period of $x_p(t)$ is 2ω (E) the period of $x_p(t)$ depends on b and k.

5. Consider the differential equation $t\dot{x} + x = \cos(\omega t)$. Which of the following is true? (A) the general solution is $\sin(\omega t) + C$, C is a constant. (B) the particular (forced) solution is periodic with period ω (C) the solution is NOT bounded when t grows to infinity (D) If $x(\pi/\omega) = 1$, then the solution is $(\sin(\omega t) + \pi)/(\omega t)$ (E) none of the above

背面有題

6. Consider a system whose dynamics is governed by a linear constant-coefficient ODE. Suppose the impulse response of this system is $e^{-t} - e^{-3t}$. Which of the following is true? (A) the system is 3rd order (B) the characteristic equation of the system has a pair of complex roots (C) the unit step response has oscillatory behavior (D) the unit step response does NOT converge to a constant value (E) the unit step response is equal to $-e^{-t} + (2 + e^{-3t})/3$

7. Consider the heat equation

$$\frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t), \quad \forall 0 < x < 1,\ t > 0$$
$$u(0,t) = u(1,t) = 0 \quad \forall t > 0$$
$$u(x,0) = f(x), \quad \forall 0 < x < 1$$

(A) the heat equation is nonlinear
(B) without considering the boundary condition, the general solution is $\sum_{n=1}^{\infty} c_n \sin(n\pi x)e^{-4n^2\pi^2 t}$
(C) suppose $f(x) = 7\sin(3\pi x)$. then the solution is $u(x,t) = 7\sin(3\pi x)e^{-4n^2\pi^2 t}$
(D) all of the above are true
(E) none of the above is true

8. Define the del operator $\nabla := \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$.
(A) $\nabla \cdot (\nabla \times \mathbf{F}) = 0$, where $\mathbf{F}$ is a vector field with continuous first and second derivatives.
(B) $\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot (\nabla \times \mathbf{F}) - \mathbf{F} \cdot (\nabla \times \mathbf{G})$, where $\mathbf{F}$ and $\mathbf{G}$ are two smooth vector fields
(C) $\nabla \cdot (\varphi\mathbf{F}) = \nabla\varphi \cdot \mathbf{F} + \varphi(\nabla \cdot \mathbf{F})$, where φ is a smooth scalar field and $\mathbf{F}$ a smooth vector field.
(D) all of the above are true
(E) none of the above is true

第九題到第十六題為單選題，每題四分。請選出一個最正確選項，答錯倒扣一分。第九題到第十六題中，若$z := x + jy$是一個複數，則 x, y 是實數而 j 代表 $\sqrt{-1}$。

9. The Fourier transform of the function $f(t) = \frac{d}{dt}u(t-2)$ is $F(j\omega) = e^{ja\omega}(b + jc)$, where $u(t)$ is an unit-step function. Then which of the following statements is correct?
(A) $a = -2$, $b = 1$, $c = 0$.
(B) $a = 2$, $b = 1$, $c = 0$.
(C) $a > 0$, $b > 1$, $c = 0$.
(D) $a < 2$, $b \neq 1$, $c \neq 0$.
(E) None of the above statements are correct.

10. Given that $f(t)$ has the Fourier transform $F(j\omega)$. Let $f_1(t) = f(-3t-6)$, and let the Fourier transform of $f_1(t)$ be $F_1(j\omega) = aF(jb\omega)e^{jc\omega}$. Then which of the following statements is correct?
(A) $a = 3$, $b = 3$, $c = 2$.
(B) $0 < a < 1$, $-1 < b < 0$, $0 < c \leq 2$.
(C) $a = -3$, $b = 1/3$, $c = 3$.
(D) $a > 0$, $b > 0$, $c < 0$.
(E) None of the above statements are correct.

11. A Fourier transform $F(j\omega)$ of a signal $f(t)$ is shown in Fig. 1. The evaluation of $E=\int_{-\infty}^{\infty}|f(t)|^2dt$ is equal to α. Which of the following statements is correct?

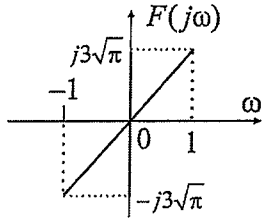

Fig. 1

(A) $\alpha=0$
(B) $\alpha=6\sqrt{\pi}$
(C) $\alpha=3\pi$
(D) $\alpha=3$
(E) None of the above statements are correct.

12. Which one of the following $f(z)$, where $z=x+jy$ is a complex variable, is entire?
(A) $f(z)=e^{y}e^{jx}$
(B) $f(z)=z\,\mathrm{Im}\,z$, Im z stands for the imaginary part of z
(C) $f(z)=x^2+jy^2$
(D) $f(z)=(z^2+j2)e^{-x}e^{-iy}$
(E) None of the above statements are correct.

13. Let $f(z)=\cot z$, and C be a closed path $|z|=4$ in counterclockwise direction. The evaluation of $\int_C f(z)dz$ is $c+jd$. Then which of the following statements is correct?
(A) $c<0,\ d\le 0$
(B) $c>0,\ \mathrm{d}>0$
(C) $c=0,\ 15<d<20$
(D) $c=0,\ 1<d<10$
(E) None of the above statements are correct.

14. Let z be a complex number. Which of the following statements is correct?
(A) $|\sin z|^2$ is an unbounded function.
(B) $|\cos z|^2$ is a bounded function.
(C) $\log(i^2)=2\log i$
(D) $\log(e^z)=z$
(E) None of the above statements are correct.

15. Let $f(z)=z^5+2z^3+3z^2+2$, and let the number of zeros of $f(z)$, counting multiplicities, inside the circle $|z|=2$ be α. Which of the following statements is correct?
(A) $\alpha=3$
(B) $\alpha=5$
(C) α is an even number
(D) $\alpha=1$
(E) None of the above statements are correct.

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16. Let $f(z)=x^2+jy$, and C is the path from $z=0$ to $z=1+j2$ along the parabola $y=x^2$. Compute the value of $\int_C f(z)dz=a+jb$. Then which of the following statements is correct?
(A) $a<0,\ b\le 0$
(B) $a+b=-2/3$
(C) $a+b=2/3$
(D) $a>0,\ b<0$
(E) None of the above statements are correct.

第十七題到第二十二題為多選題，每題六分。請選出所有正確選項；答題完全正確得六分，答錯任何選項則該題以零分計，沒有倒扣。

17. The *linear independence* is without doubt one of the most important concepts in linear algebra. Which of the following statements about this concept are true?
(A) The number of linearly independent columns of any matrix is equal to the number of linearly independent rows of the matrix.
(B) A set of vectors $\{\mathbf{v}_1,\cdots,\mathbf{v}_n\}$ is linearly independent if there exist coefficients $c_1,\cdots,c_n$, all of them are zero, such that $c_1\mathbf{v}_1+\cdots+c_n\mathbf{v}_n=\mathbf{0}$.
(C) The linear independence of a set of vectors is a necessary but not sufficient condition for them to be a basis of a vector space.
(D) A square matrix is diagonalizable if and only if all its eigenvectors are linearly independent.
(E) Let $(\lambda_i,\mathbf{x}_i)$ be the i th eigenvalue-eigenvector-pair of a square matrix. Then all $\lambda_i's$ are distinct if and only if the set $\{\mathbf{x}_1,\cdots,\mathbf{x}_n\}$ of eigenvectors is linearly independent.

18. Let $V=C[-1,1]$ be the vector space of all continuous functions defined on $[-1,1]$, and let V_e and V_o denote, respectively, the set of all even and all odd functions in V. Moreover, define an inner product $\langle f,g\rangle:=\int_{-1}^{1}f(x)g(x)dx$ for any $f(\cdot),\ g(\cdot)\in V$. Which of the following statements are true?
(A) Both V_e and V_o are subspaces of V.
(B) $V=V_e+V_o$ and $V_e\bigcap V_o=\mathbf{0}$.
(C) $V_e=V_o^{\perp}$ and $V_o=V_e^{\perp}$.
(D) $\dim V=\dim V_e+\dim V_o$.
(E) Denote $\text{dist}(f,V_e)$ as the distance induced from the defined inner product between any $f(\cdot)\in V$ and V_e. Then $\text{dist}\left((x+1)^2,V_e\right)=\sqrt{8/3}$.

19. Which of the following statements are true?
(A) Given A and $\mathbf{b}$ of proper dimensions, when $\mathbf{b}\notin R(A)$ the linear equation $A\mathbf{x}=\mathbf{b}$ has no solution. However, the associated LSP (least squares problem) is always solvable and the solution is unique.
(B) Let V be a vector space such that $V=X\oplus Y$. Then only when $X\perp Y$, i.e. the two subspaces are orthogonal, can two projection mappings, say $P:V\to X$ and $Q:V\to Y$, be defined with the complementary property $P+Q=I$, where I indicates the identity mapping on V.
(C) (continue from (B)) The projections P and Q become orthogonal projections when $X\perp Y$. Moreover, once the bases for X and Y are chosen, their union forms a basis for V, and the

two projection matrices associated with projections P and Q with respect to this set of bases are all symmetric.

(D) Any projection mapping is a linear transformation that is definitely onto, but may not be one-to-one.

(E) Consider the vector space $C[-1, 1]$ with an inner product $\langle f, g\rangle := \int_{-1}^{1} f(x)g(x)dx$. Then the set $\{\mathbf{u}_1, \mathbf{u}_2\}$ with $\mathbf{u}_1 = 1/\sqrt{2}$ and $\mathbf{u}_2 = (\sqrt{6}/2)x$ forms an orthonormal set in $C[-1, 1]$. Moreover, the best least squares approximation to $h(x) = x^2$ by a linear function is $\hat{h}(x) = (\sqrt{2}/3) + (\sqrt{6}/4)x$.

20. Let $\mathcal{L}(\cdot)$ denote the Laplace transform. Which of the followings are true?

(A) $\mathcal{L}(\alpha_1 \cdot f_1 + \alpha_2 \cdot f_2) = \alpha_1 \cdot \mathcal{L}(f_1) + \alpha_2 \cdot \mathcal{L}(f_2)$, for all $\alpha_1,\ \alpha_2 \in \mathbb{R}$ and for all functions $f_1,\ f_2$

(B) if $\mathcal{L}(f) = F(s)$, then $\mathcal{L}(\frac{d}{dt}f) = sF(s)$

(C) if $\mathcal{L}(f) = F(s)$, then $\mathcal{L}(e^{\alpha t} \cdot f) = F(s)/(s + \alpha)$

(D) if $\mathcal{L}(f) = F(s)$, then $\mathcal{L}(t^2 \cdot f) = \frac{d^2}{ds^2}F(s)$

(E) if $\mathcal{L}(f) = F(s)$ and $\mathcal{L}(g) = G(s)$, then $\mathcal{L}(f \cdot g) = (F * G)(s)$, where $*$ denotes convolution operation

21. Consider the homogeneous system $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = A\begin{bmatrix}x_1\\ x_2\end{bmatrix}$, where $A = \begin{bmatrix}a & -2\\ 2 & 1\end{bmatrix}$

(A) the system is linear

(B) the system is time invariant

(C) the system has more than one equilibrium

(D) when $a < -4$, any initial condition results in a solution which diverges to infinity

(E) when $-4 < a < -1$, any initial condition results in a solution which converges to the origin

22. Consider the autonomous system $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}x_1^2 - x_2^2\\ x_1^2 + x_2^2 - 8\end{bmatrix}$

(A) the system is linear

(B) the system has two equilibria

(C) $(x_1, x_2) = (2, -2),\ (-2, -2)$ are equiliria of the system

(D) $(x_1, x_2) = (-2, -2)$ is the only stable equilibrium of the system

(E) $(x_1, x_2) = (2, -2)$ is a saddle equilibrium of the system.

End of Examination

國立中山大學 106 學年度碩士暨碩士專班招生考試試題

科目名稱：電子學【電機系碩士班甲組】　　題號：431009

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　

1. (20%) Figure 1 shows the CS amplifier. The threshold voltage of transistor is –0.5 V. (a) Select a value for Rs to bias the transistor at I_D = 0.4 mA and $|V_{OV}|$ = 0.4 V. Assume v_{sig} to have a zero DC component. (b) Select value for R_D that results in v_o/v_{sig} = –8 V/V. (c) Find the largest sinusoid v_{sig} peak that the amplifier can handle while remaining in the saturation region. (d) If to obtain reasonably linear operation, v_{sig} peak is limited to 40 mV, what value can R_D be increased to while maintaining saturation-region operation? (5%*4)

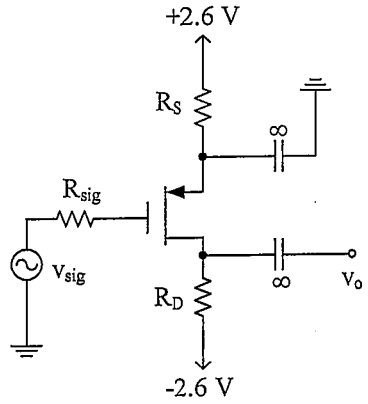

Figure 1

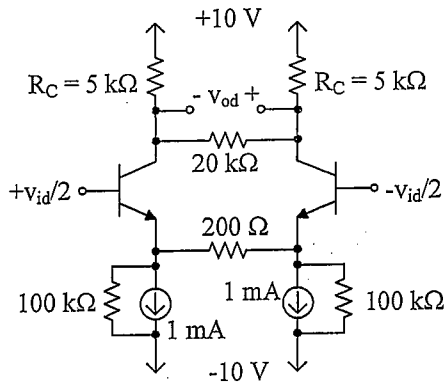

Figure 2

2. (20%) Figure 2 shows the differential amplifier. Please find (a) the differential gain, (b) the differential input resistance, (c) the common-mode gain assuming the resistance R_C have 2% tolerance, and (d) the common-mode input resistance. For these transistors, β = 100, thermal voltage is 25.9 mV and Early voltage V_A = 100 V.. (5%*4)

3. (25%) Figure 3 shows a three-stage amplifier. (a) Find the DC bias collector current in each of the three transistors and DC bias output voltage V_o. Assume $|V_{BE}|$ = 0.7 V, β = 100, thermal voltage is 25.9 mV and neglect the Early effect for all BJTs. (b) Find the input resistance R_{in}, output resistance R_{out}, and voltage gain v_o/v_i. (10%, 5%*3)

4. (20%) For the CC-CB amplifier of Figure 4, let I = 1 mA, β = 100, C_π = 8 pF, C_μ = 3 pF, R_{sig} = 15 kΩ, R_L = 20 kΩ, and thermal voltage is 25.9 mV. Find (a) the low-frequency overall voltage gain A_M, (b) the frequencies of the poles for the high frequency response, and (c) the 3-dB frequency f_H. All BJTs have the same β, C_π and C_μ. (5%, 10%, 5%)

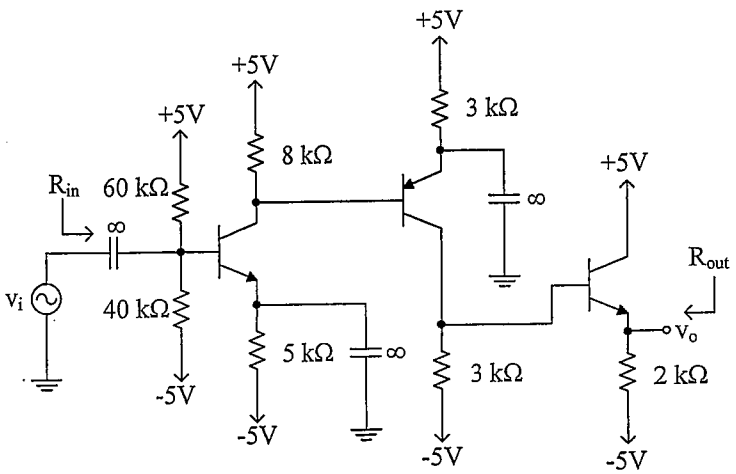

Figure 3

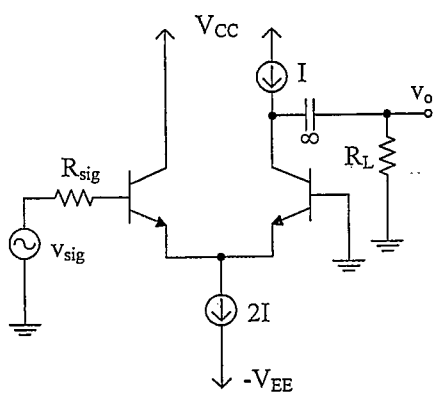

Figure 4

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5. (15%) Please only use the diode and capacitor components to draw the circuits of peak rectifier, DC restorer and voltage doubler to come out the stable output voltage: (a) $-V_a$, (b) $-V_a + V_a \sin(\omega t)$, (c) $-2V_a$, with a stable sinusoid input voltage $V_a \sin(\omega t)$ as shown in Figure 5. (5%*3)

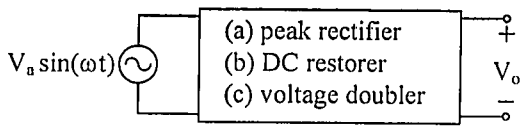


Figure 5

國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：半導體概論【電機系碩士班甲組】　　題號：431012

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. The resistivity of a silicon bulk is reduced by 1.5 Ω-cm after doping 5×10^{16} cm^{-3} donor atoms. The electron mobilities of Si are 1300 and 1100 cm^2/V-s before and after the doping process. Calculate the initial electron density in this material before the donors are added. $T = 300$ K. (20%)

2. Tungsten (W) has a high melting point and can be used in semiconductor processing. It has body-centered cubic structure and the density is 19.25 g/cm^3. Each W atom has a mass of 183.84 amu. (1 amu = 1.66×10^{-27} kg)
   (a) Determine the lattice constant of the cubic unit cell. Express your answer in angstrom.
   (b) If the atom in the center just touches the atoms at the corners of the cube, find the volume of each atom. (10%, 10%)

3. Answer the following questions about an *n-p-n* transistor.
   (a) Write down all the different modes of operation for this bipolar transistor. Describe the biasing conditions of each junction.
   (b) For each of the modes that you write, draw the minority carrier concentration profiles in emitter, base, and collector regions of the transistor. (10%, 10%)

4. A *p*-type GaAs is doped with $N_A = 5\times10^{16}$ cm^{-3}. Assume for the recombination lifetime of both electrons and holes are $\tau_n = \tau_p = 3\times10^{-7}$ s. The sample is under illumination resulting in a constant and spatially uniform generation rate of electron-hole pairs $G = 2\times10^{21}$ cm^{-3} s^{-1}. $T = 300$ K.
   (a) Calculate the steady-state electron density.
   (b) Calculate the change in conductivity due to the illumination. The electron and hole mobilities are $\mu_n = 8500$ cm^2/V-s and $\mu_p = 400$ cm^2/V-s, respectively. (10%, 10%)

5. For an abrupt p^+-*n* silicon diode, the doping concentration in the *n*-region is 5×10^{16} cm^{-3}. The width of the *n*-region is 3 μm. Assuming this width is much smaller than the hole diffusion length. Calculate the reverse saturation current at 300 K. The area of the diode is 100μm× 100μm and the hole mobility is 350 cm^2/V-s. (20%)

國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學乙【電機系碩士班乙組】　　題號：431001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）　　

第一題到第六題為單選題，每題四分。請選出一個最正確選項，答錯不倒扣。

1. When using the *Gaussian elimination* to solve a linear equation $A\mathbf{x} = \mathbf{b}$, elementary row operations (or multiplication by elementary matrices) are applied to the augmented matrix $[A\ \mathbf{b}]$. Actually, there are many places in linear algebra where such a technique plays its role; e.g., the null space of A can be determined by setting $\mathbf{b} = 0$. Which of the following cannot be determined by applying such a technique?
   (A) the range of A
   (B) the inverse of A (if it is nonsingular)
   (C) the QR factorization of A
   (D) the LU factorization of A (if it is square)
   (E) the determinant of A (if it is square).

2. Consider the system represented by the differential equation $\ddot{x} + \dot{x} + kx = 0$. Which of the following is true? (A) the system is critically damped when $k = 1$ (B) the system is underdamped when $k = 1/2$ (C) the system is overdamped when $k = 0.3$ (D) the damping ratio of the system is increased when increasing k (E) none of the above

3. Consider the differential equation $\ddot{x} + b\dot{x} + kx = \cos(\omega t)$ and its sinusoidal solution $x_p(t)$. Which of the following is true? (A) for a fixed b, the amplitude of $x_p(t)$ is maximized when $k = \omega^2$ (B) for a fixed b, the maximal amplitude of $x_p(t)$ is $1/b$ (C) for fixed b and k, the amplitude of $x_p(t)$ always grows as ω increases (D) the period of $x_p(t)$ is 2ω (E) the period of $x_p(t)$ depends on b and k.

4. Consider the differential equation $t\dot{x} + x = \cos(\omega t)$. Which of the following is true? (A) the general solution is $\sin(\omega t) + C$, C is a constant. (B) the particular (forced) solution is periodic with period ω (C) the solution is NOT bounded when t grows to infinity (D) If $x(\pi/\omega) = 1$, then the solution is $(\sin(\omega t) + \pi)/(\omega t)$ (E) none of the above

5. Consider a system whose dynamics is governed by a linear constant-coefficient ODE. Suppose the impulse response of this system is $\mathrm{e}^{-t} - \mathrm{e}^{-3t}$. Which of the following is true? (A) the system is 2nd order (B) the system is 3rd order (C) the characteristic equation of the system has a pair of complex roots (D) one root of the characteristic equation is less than -5 (E) response to a step function does NOT converge to a constant value

6. Consider the system described in Question 5 again. The unit step response of this system
   (A) has oscillatory behavior
   (B) converges to 1
   (C) converges to 1/3
   (D) is $1/4 - \mathrm{e}^{-3t}/3 + \mathrm{e}^{-t}$
   (E) is $-\mathrm{e}^{-t} + (2 + \mathrm{e}^{-3t})/3$

第七題到第十題為單選題，每題五分。請選出一個最正確選項，答錯不倒扣。

7. What is the solution to the ODE $t\dot{x} = 4t - 3x$? (A) $x(t) \equiv$ constant (B) $x(t) = -ct^3$ (C) $x(t) = ct^{-3} + t$ (D) $x(t) = c\mathrm{e}^{-3t} + 4t$ (E) none of the above

背面有題

8. Consider the heat equation

$$\frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t), \quad \forall 0 < x < 1,\ t > 0$$
$$u(0,t) = u(1,t) = 0 \quad \forall t > 0$$
$$u(x,0) = f(x), \quad \forall 0 < x < 1$$

(A) the heat equation is nonlinear

(B) without considering the boundary condition, the general solution is $\sum_{n=1}^{\infty} c_n \sin(n\pi x)\mathrm{e}^{-4n^2\pi^2 t}$

(C) suppose $f(x) = 7\sin(3\pi x)$. then the solution is $u(x,t) = 7\sin(3\pi x)\mathrm{e}^{-4n^2\pi^2 t}$

(D) all of the above are true

(E) none of the above is true

9. The *linear combination* of a set of vectors is an essential element in linear algebra. We say a set V is invariant under linear combination if the implication " $\forall n \in \mathbb{N}$ and any set of vectors $\{\mathbf{v}_1, \cdots, \mathbf{v}_n\} \subset V \Rightarrow$ the set of all linear combinations $\{c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n | c_i \in \mathbb{R}, \forall i\} \subset V$" holds. And we say a mapping L defined on a set X is invariant under linear combination if the form of linear combination is unchanged under L, or more precisely the statement "$\forall n \in \mathbb{N}$, $\forall c_i \in \mathbb{R}$, and any set of vectors $\{\mathbf{x}_1, \cdots, \mathbf{x}_n\} \subset X$, the identity $L(c_1\mathbf{x}_1 + \cdots + c_n\mathbf{x}_n) = c_1L(\mathbf{x}_1) + \cdots + c_nL(\mathbf{x}_n)$ holds" is true. Which one of the following statements related to linear combination is false?

(A) Let S be a subset of a vector space V. Then S is a subspace of V if S is invariant under linear combination.

(B) Let $\{V_1, \cdots, V_k\}$ be a set of k subspaces of a vector space W and denote $\mathrm{span}\{V_1, \cdots, V_k\}$ as the set of all linear combinations of the form $c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n$, with each $\mathbf{v}_i$ chosen freely from V_i. Then $\mathrm{span}\{V_1, \cdots, V_k\}$ is also a subspace of W with $\dim(\mathrm{span}\{V_1, \cdots, V_k\}) = \dim(V_1) + \cdots + \dim(V_k)$

(C) Let A and B be two matrices and denote $C := AB$. Then each column of C is a linear combination of all columns of A, and so $rank(C) \le rank(A)$ is implied.

(D) A mapping L between two vector spaces is a linear transformation if and only if it is invariant under linear combination.

(E) Let $(V, \langle\cdot,\cdot\rangle_V)$ be an inner product space. Then $\langle\cdot,\cdot\rangle_V$ is invariant under linear combination at either one of its two arguments.

10. Let U, V, W be vector spaces and $T: U \to V$, $S: V \to W$ be linear transformations. Also denote the set of all linear transformations from V to W by $\mathcal{L}(V,W)$ (e.g. $S \in \mathcal{L}(V,W)$). Which one of the following statements is false?

(A) The set $(\mathcal{L}(V,W), +, \bullet)$, where $(\alpha \bullet S_1 + \beta \bullet S_2)(\mathbf{v}) = \alpha \cdot S_1(\mathbf{v}) + \beta \cdot S_2(\mathbf{v})$ for any $\mathbf{v} \in V$, $S_i \in \mathcal{L}(V,W)$, and $\alpha, \beta \in \mathbb{R}$, is also a vector space with $\dim\mathcal{L}(V,W) = \dim V + \dim W$.

(B) The mapping $ST(\cdot)$ defined as $ST(\mathbf{u}) := S(T(\mathbf{u}))$ for any $\mathbf{u} \in U$ is also a linear transformation from U to W.

(C) If both S and T are one-to-one, then their combination $ST(\cdot)$ is also one-to-one.

(D) If $S: V \to W$ is invertible, then the mapping S^{-1} from W to V is also a linear transformation.

(E) Suppose that the combination $ST(\cdot)$ is onto. Then at least one of S and T is onto.

第十一題到第十六題為多選題，每題六分。請選出所有正確選項；答題完全正確得滿分，答錯任何選項則該題以零分計，沒有倒扣。

11. The *linear independence* is without doubt one of the most important concepts in linear algebra. Which of the following statements about this concept are true?
   (A) The number of linearly independent columns of any matrix is equal to the number of linearly independent rows of the matrix.
   (B) A set of vectors $\{\mathbf{v}_1,\cdots,\mathbf{v}_n\}$ is linearly independent if there exist coefficients $c_1,\cdots,c_n$, all of them are zero, such that $c_1\mathbf{v}_1+\cdots+c_n\mathbf{v}_n=\mathbf{0}$.
   (C) The linear independence of a set of vectors is a necessary but not sufficient condition for them to be a basis of a vector space.
   (D) A square matrix is diagonalizable if and only if all its eigenvectors are linearly independent.
   (E) Let $(\lambda_i,\mathbf{x}_i)$ be the ith eigenvalue-eigenvector-pair of a square matrix. Then all $\lambda_i's$ are distinct if and only if the set $\{\mathbf{x}_1,\cdots,\mathbf{x}_n\}$ of eigenvectors is linearly independent.

12. Let $V=C[-1,1]$ be the vector space of all continuous functions defined on $[-1,1]$, and let V_e and V_o denote, respectively, the set of all even and all odd functions in V. Moreover, define an inner product $\langle f,g\rangle:=\int_{-1}^{1}f(x)g(x)dx$ for any $f(\cdot),\ g(\cdot)\in V$. Which of the following statements are true?
   (A) Both V_e and V_o are subspaces of V.
   (B) $V=V_e+V_o$ and $V_e\bigcap V_o=\mathbf{0}$.
   (C) $V_e=V_o^{\perp}$ and $V_o=V_e^{\perp}$.
   (D) $\dim V=\dim V_e+\dim V_o$.
   (E) Denote $\text{dist}(f,V_e)$ as the distance induced from the defined inner product between any $f(\cdot)\in V$ and V_e. Then $\text{dist}\left((x+1)^2,V_e\right)=\sqrt{8/3}$.

13. Which of the following statements are true?
   (A) Given A and $\mathbf{b}$ of proper dimensions, when $\mathbf{b}\notin R(A)$ the linear equation $A\mathbf{x}=\mathbf{b}$ has no solution. However, the associated LSP (least squares problem) is always solvable and the solution is unique.
   (B) Let V be a vector space such that $V=X\oplus Y$. Then only when $X\perp Y$, i.e. the two subspaces are orthogonal, can two projection mappings, say $P:V\to X$ and $Q:V\to Y$, be defined with the complementary property $P+Q=I$, where I indicates the identity mapping on V.
   (C) (continue from (B)) The projections P and Q become orthogonal projections when $X\perp Y$. Moreover, once the bases for X and Y are chosen, their union forms a basis for V, and the two projection matrices associated with projections P and Q with respect to this set of bases are all symmetric.
   (D) Any projection mapping is a linear transformation that is definitely onto, but may not be one-to-one.
   (E) Consider the vector space $C[-1,1]$ with an inner product $\langle f,g\rangle:=\int_{-1}^{1}f(x)g(x)dx$. Then the set $\{\mathbf{u}_1,\mathbf{u}_2\}$ with $\mathbf{u}_1=1/\sqrt{2}$ and $\mathbf{u}_2=(\sqrt{6}/2)x$ forms an orthonormal set in $C[-1,1]$. Moreover, the best least squares approximation to $h(x)=x^2$ by a linear function is $\hat{h}(x)=(\sqrt{2}/3)+(\sqrt{6}/4)x$.

14. Let $E := \{\mathbf{u}_1, \cdots, \mathbf{u}_n\}$ be a basis of $\mathbb{R}^n$ and $F := \{\mathbf{v}_1, \cdots, \mathbf{v}_n\} \subset \mathbb{R}^n$ with the property $\forall i, j = 1, \cdots, n$ $\mathbf{u}_i^T \mathbf{v}_j = 1$ when $i = j$, and $\mathbf{u}_i^T \mathbf{v}_j = 0$ when $i \neq j$. Which of the following statements are true?
(A) F is also a basis of $\mathbb{R}^n$, and the coordinate vector of any $\mathbf{x} \in \mathbb{R}^n$ with respect to base F is $\begin{bmatrix} \mathbf{x}^T\mathbf{v}_1 \\ \vdots \\ \mathbf{x}^T\mathbf{v}_n \end{bmatrix}$.
(B) Any $\mathbf{x} \in \mathbb{R}^n$ can be represented as $\mathbf{x} = c_1\mathbf{u}_1 + \cdots + c_n\mathbf{u}_n$ with $c_i = \mathbf{x}^T\mathbf{u}_i$ for each i.
(C) Denote the transition matrix from base E to base F by S. Then $S(i, j) = \mathbf{u}_i^T\mathbf{u}_j$ for $i, j = 1, \cdots, n$.
(D) The transition matrix from base F to base E can be described by $([\mathbf{v}_1]_E \cdots [\mathbf{v}_n]_E)$, where $[\mathbf{v}_i]_E$ denotes the coordinate vector of $\mathbf{v}_i$ with respect to base E.
(E) Denote $U := [\mathbf{u}_1 \cdots \mathbf{u}_n]$ and $V := [\mathbf{v}_1 \cdots \mathbf{v}_n]$. Then $U\mathbf{x} = \lambda\mathbf{x}$ for some $\mathbf{x} \neq \mathbf{0}$ iff $(\bar{\mathbf{x}})^T V = (\bar{\lambda})^{-1}(\bar{\mathbf{x}})^T$, where the upper bar means to take the complex conjugate.

15. For any given $0 \neq \mathrm{w} \in \mathbb{R}^n$, let's consider the matrix $H_\alpha = I - \alpha(\mathbf{w}^T\mathbf{w})^{-1}\mathbf{w}\mathbf{w}^T$ parameterized by a real scalar α. Which of the following statements are true?
(A) $H_\alpha = H_\alpha^2$, i.e. H_α is an idempotent matrix, if and only if $\alpha \in \{0, 1, 2\}$.
(B) $H_\alpha^2\mathrm{x} = \mathrm{x}$ for any $\mathrm{x} \in \mathbb{R}^n$ if and only if $\alpha \in \{0, 1, 2\}$.
(C) H_α is singular if and only if $\alpha = 1$.
(D) For any $\mathrm{x} \in \mathbb{R}^n$, $H_\alpha\mathrm{x} \in \mathrm{w}^\perp$, the orthogonal complement of $\mathbf{w}$, if and only if $\alpha = 1$.
(E) For any $\mathrm{x} \in \mathbb{R}^n$, $\min_{\alpha \in \mathbb{R}} \|H_\alpha\mathrm{x}\|_2 = \|H_1\mathrm{x}\|_2$, i.e. $H_\alpha\mathrm{x}$ has the smallest $2-$norm if and only if $\alpha = 1$.

16. Let $A = A^H := (\bar{A})^T$ be a nonzero matrix of size $n \times n$ with its real part and imaginary part being denoted by B and C, respec 7. tively. Moreover, let be a symmetric matrix with and as its four blocks. Which of the following statements are true?
(A) Matrix C has at least one real eigenvalues.
(B) Let λ and μ be any two eigenvalues of B and C, respectively. Then $\lambda + \mu \neq 0$.
(C) Any eigenvalue of A is also an eigenvalue of Ω.
(D) Any eigenvalue of Ω is also an eigenvalue of A.
(E) Let $\left(\lambda, \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}\right)$ be an eigenvalue-eigenvector-pair of Ω. Then $(\lambda, \mathbf{x} + i\mathbf{y})$ is an eigenvalue-eigenvector-pair of A.

國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學乙【電機系碩士班乙組】　　題號：431001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）　　共5頁第5頁

第十七題到第二十題為多選題，每題五分。請選出所有正確選項；答對一個選項得一分，答錯一個選項倒扣零點五分。

17. Let $\mathcal{L}(\cdot)$ denote the Laplace transform. Which of the followings are true?
(A) $\mathcal{L}(\alpha_1 \cdot f_1 + \alpha_2 \cdot f_2) = \alpha_1 \cdot \mathcal{L}(f_1) + \alpha_2 \cdot \mathcal{L}(f_2)$, for all $\alpha_1,\ \alpha_2 \in \mathbb{R}$ and for all functions $f_1,\ f_2$
(B) if $\mathcal{L}(f) = F(s)$, then $\mathcal{L}(\frac{d}{dt}f) = sF(s)$
(C) if $\mathcal{L}(f) = F(s)$, then $\mathcal{L}(e^{\alpha t} \cdot f) = F(s)/(s+\alpha)$
(D) if $\mathcal{L}(f) = F(s)$, then $\mathcal{L}(t^2 \cdot f) = \frac{d^2}{ds^2}F(s)$
(E) if $\mathcal{L}(f) = F(s)$ and $\mathcal{L}(g) = G(s)$, then $\mathcal{L}(f \cdot g) = (F * G)(s)$, where $*$ denotes convolution operation

18. Consider the differential equation $m\ddot{x}(t) + b\dot{x}(t) + kx(t) = u(t)$. Suppose $\frac{1}{2}t\sin(2t)$ is a solution when $u(t) = 4\cos(2t)$. Which of the followings are true?
(A) $m = 1$
(B) $b = 0$
(C) $k = 8$
(D) $2t\sin(t)$ is a solution when $u(t) = 8\cos(2t)$
(E) $\frac{1}{2}(t - \frac{1}{2})\sin(2t-1)$ is a solution when $u(t) = 4\cos(2t-1)$

19. Consider the homogeneous system $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = A\begin{bmatrix}x_1\\ x_2\end{bmatrix}$, where $A = \begin{bmatrix}a & -2\\ 2 & 1\end{bmatrix}$
(A) the system is linear
(B) the system is time invariant
(C) the system has more than one equilibrium
(D) when $a < -4$, any initial condition results in a solution which diverges to infinity
(E) when $-4 < a < -1$, any initial condition results in a solution which converges to the origin

20. Consider the autonomous system $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}x_1^2 - x_2^2\\ x_1^2 + x_2^2 - 8\end{bmatrix}$
(A) the system is linear
(B) the system has two equilibria
(C) $(x_1, x_2) = (2, -2),\ (-2, -2)$ are equiliria of the system
(D) $(x_1, x_2) = (-2, -2)$ is the only stable equilibrium of the system
(E) $(x_1, x_2) = (2, -2)$ is a saddle equilibrium of the system.

End of Examination

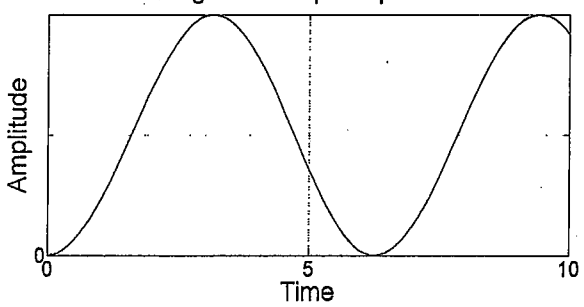
國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：控制系統【電機系碩士班乙組】　　題號：431008

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）　　

以下第一題到第八題，每題請選出一個正確答案。每題答對得三分，答錯倒扣一分

Figure 1: Step Response

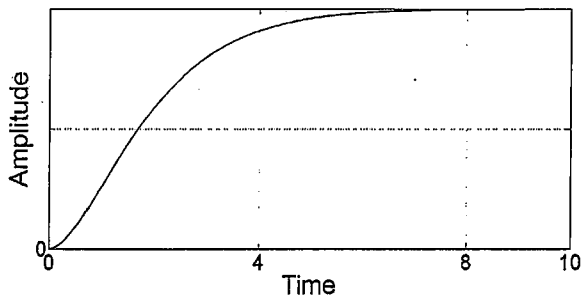


(1) Figure 1 可為以下哪個轉移函數之步階響應

(A) $\frac{1}{s+1}$ (B) $\frac{1}{s^2+1}$ (C) $\frac{1}{s^2+0.1s+1}$

(D) $\frac{1}{s^2+2s+1}$ (E) 以上皆非

Figure 2: Step Response

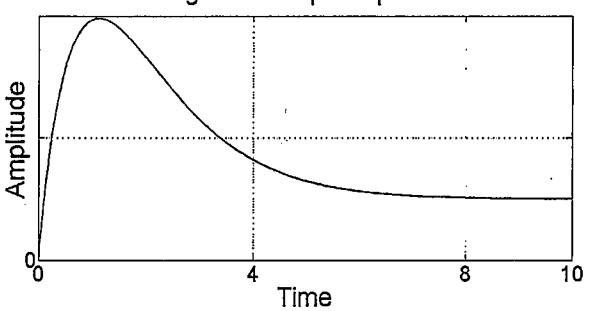


(2) Figure 2 可為以下哪個轉移函數之步階響應

(A) $\frac{s}{s+1}$ (B) $\frac{1}{s^2+0.5s+1}$ (C) $\frac{s+0.1}{s^2+s+1}$

(D) $\frac{s+1}{s}$ (E) 以上皆非

Figure 3: Step Response

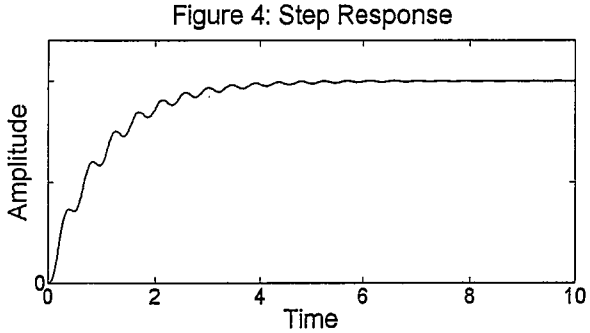


(3) Figure 3 可為以下哪個轉移函數之步階響應

(A) $\frac{1-s}{s+1}$ (B) $\frac{1}{s^2+1}$ (C) $\frac{1-s}{s^2+s+1}$

(D) $\frac{s+0.1}{s^2+2s+1}$ (E) 以上皆非

Figure 4: Step Response



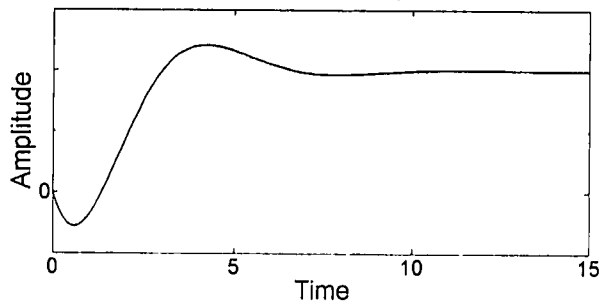
(4) Figure 4 可為以下哪個轉移函數之步階響應

(A) $\frac{200}{s^3+2s^2+200s+200}$ (B) $\frac{2}{s^2+5s+2}$

(C) $\frac{1-s}{s^2+2s+1}$ (D) $\frac{s+1}{s+2}$ (E) 以上皆非

背面有題

Figure 5: Step Response

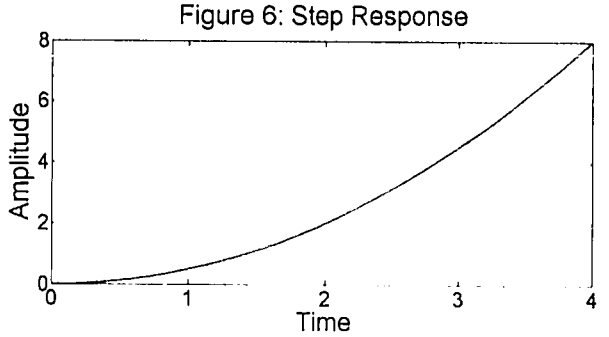


(5) Figure 5 可為以下哪個轉移函數之步階響應

(A) $\frac{s+2}{s+1}$ (B) $\frac{2s+1}{s^2+s+1}$ (C) $\frac{1}{s^2+2s+1}$

(D) $\frac{1-s}{s^2+s+1}$ (E) 以上皆非

Figure 6: Step Response

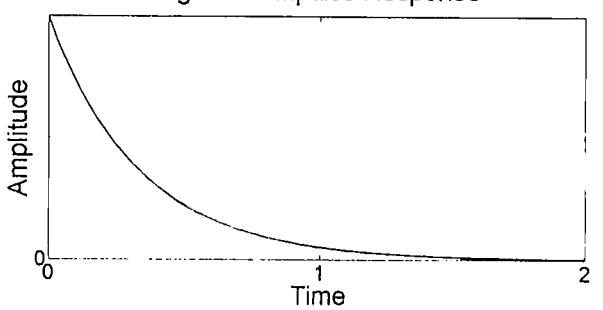


(6) Figure 6 可為以下哪個轉移函數之步階響應

(A) $\frac{1}{s-1}$ (B) $\frac{s+1}{s}$ (C) $\frac{1}{s^2-0.1s+1}$

(D) $\frac{1-s}{s^3+s+1}$ (E) 以上皆非

Figure 7: Impulse Response

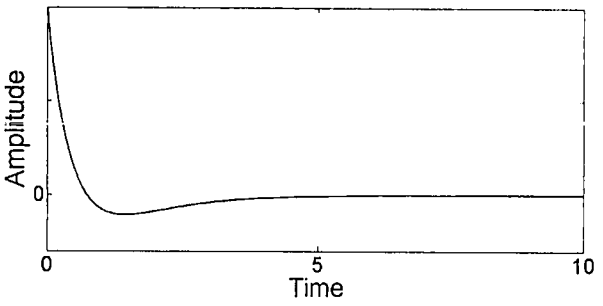


(7) Figure 7 可為以下哪個轉移函數之脈衝響應

(A) $\frac{2}{s+3}$ (B) $\frac{1}{s^2}$ (C) $\frac{2}{s^3+2s^2+2s+1}$

(D) $\frac{s-1}{s^2+s+5}$ (E) 以上皆非

Figure 8: Impulse Response



(8)左圖可為以下哪個轉移函數之脈衝響應

(A) $\frac{s+1}{s-1}$ (B) $\frac{10s+1}{s^2+3s+2}$ (C) $\frac{1}{s^2+4}$

(D) $\frac{1}{s^2+0.1s+1}$ (E) 以上皆非

以下第九到第十五題互有關聯，每題請選出一個最正確答案。每題答對得三分，答錯倒扣一分。

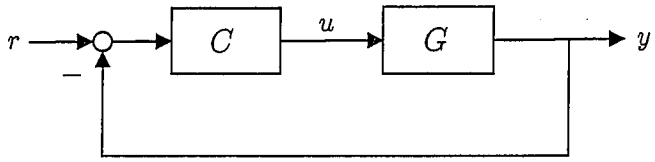
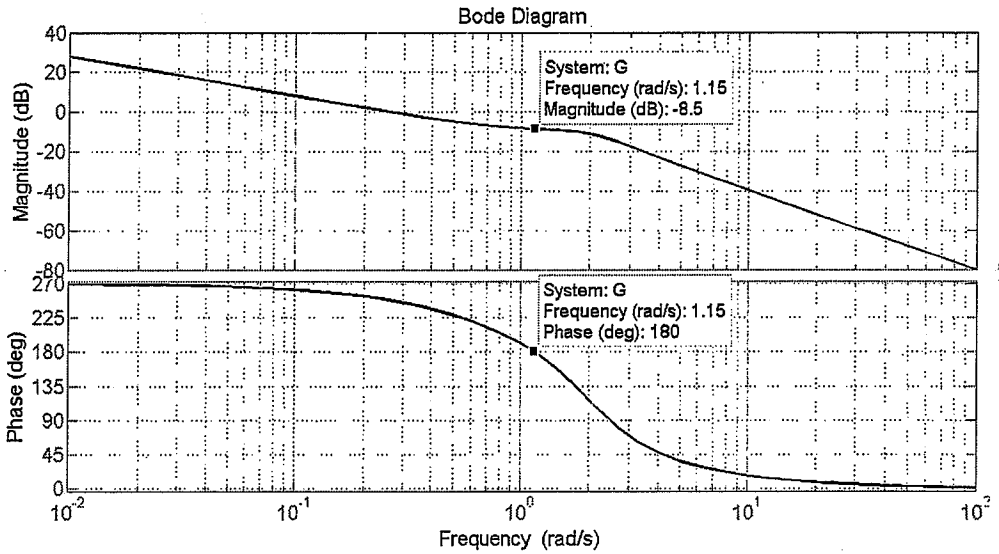


Figure 9: a standard feedback system



考慮圖 Figure 9 所示之回授控制系統，其中受控系統 G 為一個三階系統，且該系統沒有極點在右半平面。系統 G 的 Bode plot 如上圖所示。

(9) G 的相對階數（relative degree）為何？ (A) 0 (B) 1 (C) 2 (D) 3 (E) 以上皆非。

(10) G 有幾個極點在原點？ (A) 0 個 (B) 1 個 (C) 2 個 (D) 無法判斷 (E) 以上皆非。

(11) G 有幾個零點在右半平面？ (A) 0 個 (B) 1 個 (C) 2 個 (D) 無法判斷 (E) 以上皆非。

(12) 已知當 $C=1$ 時閉迴路系統為穩定。請問此時 r 到 y 之轉移函數的頻寬大約為何？
(A) 0.5 rad/s (B) 0.01 rad/s (C) 5 rad/s (D) 10 rad/s (E) 以上皆非。

(13) 已知當 $C=1$ 時閉迴路系統為穩定。請問若此時 r 為振幅為 β 的步階輸入，請問 y 之終值為何？ (A) 0.5β (B) 0.1β (C) 0.8β (D) 1.2β (E) 以上皆非。

(14) 已知當 $C=1$ 時閉迴路系統為穩定。此時該系統無法有效追蹤以下哪類訊號（追蹤誤差將發散）？ (A) 弦波訊號 (B) 斜坡訊號 (C) 拋物線訊號 (D) 以上皆是 (E) 以上皆非。

(15) 令 $C \equiv K$，$K > 0$。請問當K增加到多大時，閉迴路會變為不穩定？
(A) 無論K多大閉迴路系統都會穩定 (B) 介於 1 與 2 之間 (C) 當 $K = 2.5$ (D) 當 $K > 3$ (E) 以上皆非。

背面有題

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※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）　　共5頁第4頁

以下第十六到第二十題互有關聯，每題請選出一個最正確答案。每題答對得三分，答錯倒扣一分

再次考慮第三頁圖 Figure 9 所示之回授控制系統。這次讓我們假設受控系統 G 的動態由以下常微分方程式支配：

$$\ddot{y}(t) + b_1\dot{y}(t) + b_0 y(t) = u(t-\tau)$$

(16) 假設 $\tau > 0$，$b_1 > 0$，且 $b_1^2 < 4b_0$。此時，若訊號 u 為單位步階訊號且 y 之初值為零，則下列有關訊號 y 的敘述，何者不正確？ (A) $y(t) = 0, t \in [0, \tau)$ (B) y 收斂到終值（steady state）前，有震盪現象 (C) y 收斂到終值前，沒有超越終值的現象 (D) $y(t) \geq 0, \forall\, t \in [0, \infty)$ (E) y 的終值為 $1/b_0$

(17) 假設 $\tau > 0$，$C \equiv K$，$K > 0$。此時閉迴路系統有幾個極點？ (A) 零個 (B) 一個 (C) 兩個 (D) 三個 (E) 無窮多個

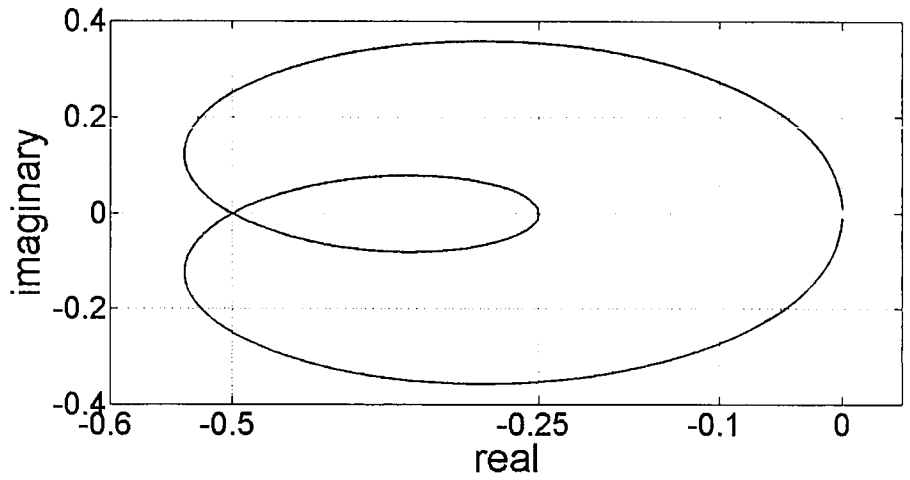
(18) 假設 $\tau > 0$, $b_1 > 0$, $b_0 > 0$，$C \equiv K$，$K > 0$。關於這個閉迴路系統，以下敘述何者正確？ (A) 無論 τ, b_1, b_0 之正值為何，系統 G 之增益邊界都是有限的 (B) 無論 τ, b_1, b_0 之正值為何，系統 G 之相位邊界都是無限的 (C) 若閉迴路系統為穩定，則步階響應的穩態誤差必然為零 (D) 無論 τ, b_1, b_0, K 之正值為何，閉迴路系統絕不可能為穩定 (E) 以上皆非

(19) 假設 $\tau = 0$, $b_1 = 0$, $b_0 > 0$。以下敘述何者不正確？ (A) 系統 G 之增益邊界是無限的 (B) 選擇控制器 C為 PD 控制器時，閉迴路系統可能會穩定 (C) 選擇控制器 C 為 PID 控制器時，閉迴路系統可能會穩定 (D) 選擇控制器 C 為 PI 控制器時，閉迴路系統可能會穩定 (E) 選擇控制器 C 為 lead compensator時，閉迴路系統可能會穩定

(20) 假設 $\tau = 0$, $b_1 = 0$, $b_0 = 4$，$C \equiv K(s+1)$，$K > 0$。以下敘述何者正確？ (A) 當K 太大時，閉迴路系統為不穩定 (B) 當 $r(t) = \sin(2t)$ 時，閉迴路系統之追蹤誤差的穩態值必然為零 (C) 當 $r(t) = \cos(2t)$ 時，閉迴路系統之追蹤誤差沒有穩態值 (D) 當 $r(t)$ 為斜坡訊號時，閉迴路系統之追蹤誤差為一非零之定值 (E) 以上皆非

以下第二十一到第二十三題互有關聯，每題請選出兩個正確答案。每題答對得五分，答錯不倒扣

再次考慮第三頁圖 Figure 9 所示之回授控制系統。這次我們假設受控系統 G 的 Nyquist Plot 如下



國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：控制系統【電機系碩士班乙組】　　題號：431008

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）　　共5頁第5頁

已知受控系統 G 知轉移函數形式為 $\frac{s+b}{s^2+as+a^2}$，a, b 為實數。以下問題每題有兩個正確答案

(21) 受控系統 G 有 (A) 穩定零點 (B) 不穩定零點 (C) 兩個穩定極點 (D) 兩個不穩定極點 (E) 一個穩定極點、一個不穩定極點

(22) 假設控制器 $C \equiv K$，$K > 0$。以下何者正確？ (A) 當 $1 < K < 2$，閉迴路系統為穩定 (B) 當 $K = 2$，閉迴路系統有一個極點在虛軸上 (C) 當 $2 < K < 4$，閉迴路系統為穩定 (D) 當 $K = 4$，閉迴路系統有一對共厄極點 (E) 當 $K > 4$，閉迴路系統有一個不穩定極點

(23) 假設控制器 $C \equiv K$，$K > 0$，且K之值讓閉迴路系統為穩定。以下何者正確？ (A) 閉迴路系統的步階響應有 undershoot (B) 閉迴路系統的步階響應有負的終值 (C) 閉迴路系統的步階響應之穩態誤差可藉由增大K之值來無限降低 (D) 該系統有無窮大的相位邊界 (E) 閉迴路系統為極小相位系統

以下第二十四到第二十八題為多選題，各題相互獨立。每題有五個選項，請選出所有正確選項。每個選項答對得一分，每題滿分五分；答錯一個選項倒扣零點五分。

(24) 以下關於回授控制機制的敘述，何者為真？ (A) 可讓不穩定系統變穩定 (B) 可讓穩定系統變不穩定 (C) 可減少系統抵禦外來干擾的能力 (D) 可改變系統零點的位置 (E) 可增加系統的相位邊界

(25) 以下關於 PD 控制器的敘述，何者為真？ (A) 通常會增加穩定度邊界 (B) 適合抑制高頻雜訊 (C) 通常會降低步階響應之上升時間 (D) 通常可增加欠阻尼二階系統之阻尼係數 (E) 是一種相位超前補償器

(26) 以下關於 PI 控制器的敘述，何者為真？ (A) 通常會增加穩定度邊界 (B) 不可以消除或改善步階響應之穩態誤差 (C) 是一種相位落後補償器 (D) 不適合抑制低頻雜訊 (E) 可用來降低步階響應之最大超越量

(27) 以下敘述何者為真？ (A) 相對階數為1的穩定系統都有無窮大的增益邊界 (B) 相對階數為2的極小相位穩定系統有無窮大的增益邊界 (C) 任何穩定的極小相位系統都有無窮大的增益邊界 (D) 任何穩定系統都有可能有無窮大的相位邊界 (E) 回授系統中只要有訊號傳輸延遲，系統就不可能有無窮大的的相位邊界

(28) 以下關於線性非時變狀態變數系統（$\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + Du(t)$）的敘述，何者為真？ (A) u 到 y 之轉移函數必為 proper (B) 若該系統為可控制（controllable），則 u 到 y 之轉移函數的階數必然與狀態變數之個數相同 (C) 若該系統為可觀測（observable），則 u 到 y 之轉移函數的階數必然沒有不穩定的零點 (D) 系統的可控制性（controllability）無法由回授機制來改變 (E) 系統的可控制性（controllability）與可觀測性（observability）之間沒有任何必然的關聯

End of Examination

國立中山大學 106 學年度碩士暨碩士專班招生考試試題

科目名稱：資料結構【電機系碩士班丙組】　　題號：431004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. [15] Suppose we have a byte-addressable machine, i.e., each byte is accessed via an address. Let the locations of an array be allocated in a row-major manner. In a program segment, we declare an array *PP* by

   int PP[20,30];

   Let each integer take 4 bytes and the location of this array start with address 1000. Please answer the following questions:

   1.1 [3] How many integers can be stored in the array *PP*?
   1.2 [3] How many bytes does the array *PP* itself take?
   1.3 [3] What is the starting address of the element *PP[10,5]*?
   1.4 [3] What is the starting address of the element *PP[5,10]*?
   1.5 [3] What is the starting address of the last element in *PP*?

2. [15] Suppose we use a singly linked list to store integers in its nodes. Assume that, initially, 100 is stored in the first node starting at address 2000, 200 is stored in the second node starting at address 500, 300 is stored in the third node starting at address 1000, and 400 is stored in the fourth node starting at address 100. Let the starting address of the first node of the list is stored in the variable named head. Let an integer take 4 bytes and a pointer (or address) take 8 bytes. Please answer the following questions:

   2.1 [3] What is the size, in the number of bytes, of each node of the list?
   2.2 [3] What is stored in the variable head?
   2.3 [3] What is the content of the first node of the list?
   2.4 [3] What is the content of the last node of the list?
   2.5 [3] Suppose we delete the second node from the list. After deletion, what is the content of the first node of the resulting list?

3. [15] Usually, there are three ways to display the nodes of a binary tree: preorder, inorder, and postorder. Consider Figure 1 and answer the following questions:

   3.1 [5] What is the fifth node of the preorder sequence of this tree?
   3.2 [5] What is the fifth node of the inorder sequence of this tree?
   3.3 [5] What is the fifth node of the postorder sequence of this tree?

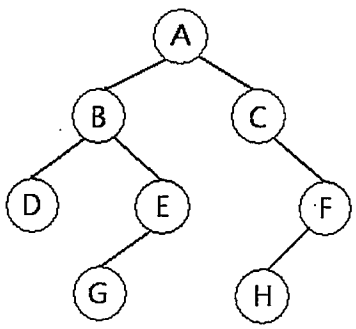


Figure 1

背面有題

國立中山大學 106 學年度碩士暨碩士專班招生考試試題

科目名稱：資料結構【電機系碩士班丙組】　　題號：431004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

4. [15] Initially, the array *AA* shown in Figure 2 stores a max heap with 12 integers. Note that A[0] is not used. Suppose we Insert 105 into the heap and then make it into a max heap.

	120	90	100	65	20	80	60	55	25	10	5	40			
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Figure 2

Please answer the following questions:

4.1 [3] What is the content of A[3] of the resulting array?

4.2 [3] What is the content of A[6] of the resulting array?

4.3 [3] What is the content of A[10] of the resulting array?

4.4 [3] What is the content of A[13] of the resulting array?

4.5 [3] We delete the root node from the heap and make it into a max heap again. What is the content of the A[6] of the resulting array?

5. [15] Let one-node binary tree have a depth of 1. Suppose we have a complete binary tree *T* of depth 100.

5.1 [3] What is the minimum number of nodes in *T*?

5.2 [3] What is the maximum number of nodes in *T*?

5.3 [3] There are *n* possible complete binary trees of depth 100. What is *n*?

5.4 [3] Suppose we number the nodes *T* by starting with the node on level 1, and continuing with the nodes on level 2, and so on. Nodes on any level are numbered from left to right. Let the first node be numbered 1, the second node be numbered 2, and so on. For a node numbered 200, what is its left node numbered?

5.5 [3] Following above, for a node numbered 3001, what is its parent node numbered?

6. [10] Suppose we have the following 11 integers:

44, 30, 95, 33, 50, 82, 18, 55, 70, 64, 26

Please create a hash table AA with 13 entries declared by

int AA[13];

Let's insert the integers one by one and from the left to the right into the hash table. Assume the hash function is h(k) = k%13 and linear probing is used for collision resolution. Note that % yields the remainder when one integer is divided by another, for example, 20%13 = 7 and 5%13 = 5. Please answer the following questions:

6.1 [2] What is the content of A[0]?

6.2 [2] What is the content of A[3]?

6.3 [2] What is the content of A[6]?

6.4 [2] What is the content of A[9]?

6.5 [2] What is the content of A[12]?

7. [15] Suppose we have 12 integers: 60, 40, 25, 10, 50, 80, 15, 65, 30, 90, 20, 55. Construct a binary search tree by inserting the integers, starting from 60, one after another. Please answer the following questions for the binary search tree:

7.1 [3] What is the left child of the root node?

7.2 [3] What is the right child of the node containing 80?

7.3 [3] What is the left child of the node containing 25?

7.4 [3] What is the right child of the node containing 55?

7.5 [3] What is the height of the binary tree? Let one-node tree have a depth of 1.

國立中山大學 106 學年度碩士暨碩士專班招生考試試題

科目名稱：離散數學【電機系碩士班丙組】　　題號：431011

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

（答題需將推導過程與原因寫出，回答到要點原因與推導的嚴謹性為主要評分考量）

1. (10%) Given Boolean variables p, q, and r where $\neg p$ is the complement of p, $\wedge$ is logical AND, $\vee$ is logical OR, $\rightarrow$ is logical imply, and $\Leftrightarrow$ is equivalence, prove that the following formula is true.

   $$p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$$

2. (15%) Given a directed graph G as in Figure 1, derive a relation matrix R of the graph, and derive the matrix of transitive closure R^* of relation R.

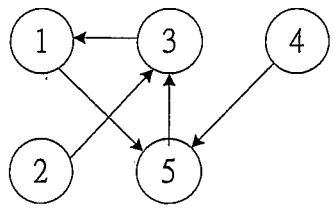


Figure 1

3. (15%) In a 2D n x n checkerboard as shown in Figure 2, given two integers p and q ($1 \le p \le q \le n$), write the formula of the number of all possible squares in the checkerboard with side length s satisfying $p \le s \le q$. Write the result in a formula of n, p, and q.

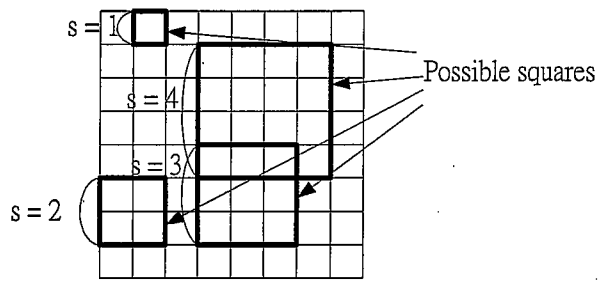


Figure 2

4. (15%) Given a bag having 4 white balls and 3 red balls, let us proceed a drawing run that a drawing takes a ball from the bag, record its color, put it back, and do such drawing for 5 times. Assume that the probability of each distinct red ball being drawn is two times of that of a distinct white ball being drawn. Calculate the probability to draw exactly 3 white balls and 2 red balls in the run.

5. (10%) A complete graph K_n is a graph $G(V,E)$ with n vertices ($n \ge 1$) that have an edge between each pair of vertices in the vertex set. Write the formula of the number of **all** complete subgraphs K_p existing in K_n. ($1 \le p \le n$) The formula is a function of n.

6. (10%) Write an algorithm *queue_partitioning(q, k)* to reorder all data elements in a queue q. Assume that the queue q has n integer elements initially. Given an integer $k > 1$, the reordering will form k ordered partitions of these integers. The resulting queue should be in the order of partition 0, partition 1, ... , and partition k-1. The i^{th} partition of these integer elements contains all integer elements e in the original queue such that $e \bmod k = i$. (In this algorithm, you can utilize an empty queue $q2$ as a local variable.) You can use a function *length*(q) to get the length of the queue q.

   For illustration, Figure 3 shows an example with an initial queue state, $k = 3$, ordered partitions, and a final queue state.

背面有題

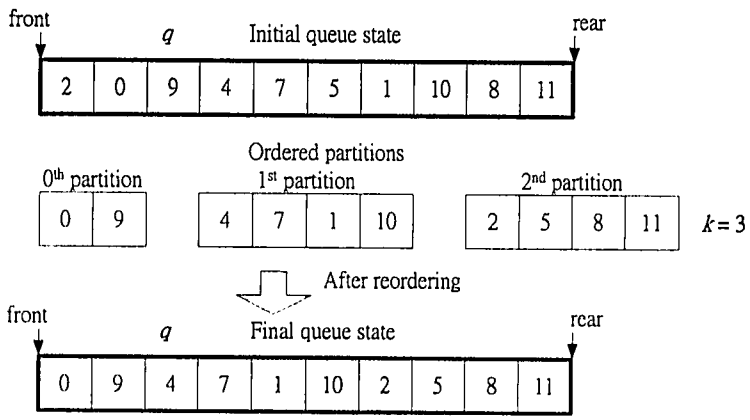


Figure 3

7. (15%) A binary expression tree data structure uses the node data element given as follows. A node either represents a value or represents an arithmetic operator in { +, –, *, / }. An evaluation of an example expression tree is shown in Figure 4. Write a recursive algorithm *evaluate(root)* to perform such expression evaluation task where *root* points to the root element of the expression tree.

```
struct node {
    int    specifier;     // 0: the node represents a number in the value field
                          // 1 - 4: the node represents an operator:
                          //        1: + (add), 2: - (subtract), 3: * (multiply), 4: / (divide)
    int    value;
    struct node *  left;  // pointer to the root node of the left subtree
    struct node *  right; // pointer to the root node of the right subtree
}
```

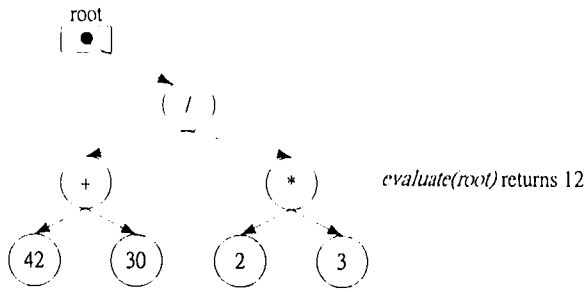


Figure 4

8. (10%) In an undirected graph $G(V, E)$ with n vertices, these is an integer data d_i in each vertex v_i. Given a starting vertex S in the vertex set V, write an algorithm that computes the sum of associated data d_i's of all reachable vertices in V from S. (i.e. A reachable vertex T from the vertex S is a vertex that has a path from S to T.)

國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：電路學【電機系碩士班丁組】　　題號：431006

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（單選題共20題）　

每題正確答案得5分，錯誤答案倒扣1分，未作答者，不給分亦不扣分

1. A Lithium Ion battery module is rated at 12V/6Ah. Two modules, connected in series, are discharged by 0.5C constant current for 1 hour. Determine the total energy released from the battery modules.
   Ⓐ、144Wh
   Ⓑ、72Wh
   Ⓒ、36Wh
   Ⓓ、18Wh
2. In a DC circuit, a load is connected to a Thevenin's equivalent circuit (Thevenin voltage=200V, Thevenin resistance=10Ω). Determine power supplied by Thevenin voltage when the maximum power is consumed by the load.
   Ⓐ、2000W
   Ⓑ、1000W
   Ⓒ、500W
   Ⓓ、250W
3. Determine power consumption of two resistors in parallel connection. Assume the resistor is 10Ω for each and voltage across on the resistor is 10 V.
   Ⓐ、40W
   Ⓑ、20W
   Ⓒ、10W
   Ⓓ、5W
4. Determine the time constant of a RC first-order circuit with C=2F and R=2Ω.
   Ⓐ、4s
   Ⓑ、2s
   Ⓒ、0.5s
   Ⓓ、0.25s
5. A series RLC second-order circuit is given with parameters L=4H, C=0.25F, R=40Ω. Choose a false statement.
   Ⓐ、resonant frequency is 1 rad/s
   Ⓑ、damping factor is 10 Np/s
   Ⓒ、natural frequencies are -0.101 and -9.899 Np/s
   Ⓓ、the natural response is overdamped
6. In an AC circuit, choose a false statement.
   Ⓐ、the impedance of a resistor increases with increasing frequency
   Ⓑ、the impedance of an inductor increases with increasing frequency
   Ⓒ、the impedance is defined as voltage phasor divided by current phasor
   Ⓓ、the impedance is not a phasor
7. In TAIWAN, single-phase three-wire 110V is common in domestic power system. Choose a true statement.
   Ⓐ、Voltage RMS value is 156 V.
   Ⓑ、Voltage RMS value is 110 V
   Ⓒ、Voltage RMS value is 78V
   Ⓓ、System frequency is 50Hz

背面有題

國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：電路學【電機系碩士班丁組】　　題號：431006

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（單選題共20題）　

每題正確答案得5分，錯誤答案倒扣1分，未作答者，不給分亦不扣分

8. How to calculate the average power consumed by an AC circuit?
   Ⓐ、voltage phase times current phasor
   Ⓑ、current phasor square times impedance
   Ⓒ、voltage phasor times current phasor complex conjugate
   Ⓓ、current phasor square times resistance
9. What is the definition of power factor?
   Ⓐ、average power divided by apparent power
   Ⓑ、average power divided by reactive power
   Ⓒ、cosine function of angle different between voltage and current
   Ⓓ、cosine function of impedance angle
10. Determine the power factor of a load with impedance 10-j10Ω.
   Ⓐ、0.707 leading
   Ⓑ、0.707 lagging
   Ⓒ、0.866 leading
   Ⓓ、0.866 lagging
11. A PV inverter is operated at grid-connected mode with output power 1kW. Assume grid voltage is $100V_{rms}$. Choose a false statement.
   Ⓐ、power factor is 1
   Ⓑ、reactive power is 1kVAR
   Ⓒ、output current is 10A
   Ⓓ、apparent power is 1kVA
12. A delta-connected resistive load 100Ω per phase is connected to a balanced three-phase three-wire 200V circuit. Choose a true statement.
   Ⓐ、power factor is 0.5
   Ⓑ、reactive power is 1200VA
   Ⓒ、average power is 1200W
   Ⓓ、line current is in phase with the line voltage
13. For an ideal transformer, choose a false statement.
   Ⓐ、input average power is equal to output average power
   Ⓑ、input reactive power is equal to output reactive power
   Ⓒ、input apparent power is equal to output apparent power
   Ⓓ、input complex power is equal to output complex power
14. For a two-winding coupling device in an AC circuit, choose a false statement.
   Ⓐ、stored energy in the coupling device is zero
   Ⓑ、coupling coefficient is less than 1
   Ⓒ、induced voltage is determined by the mutual inductance
   Ⓓ、transformer is a magnetic coupling device
15. For a RLC parallel resonant circuit, choose a false statement.
   Ⓐ、resonant condition is that the admittance of the RLC circuit is pure conductance
   Ⓑ、resonance frequency is determined both L and C
   Ⓒ、quality factor can be increased by increasing R
   Ⓓ、current flowing between L and C is equal to zero at resonance

16. Voltage across on a load with 10Ω reactance is 100cos(377t) V. Determine average power of the load.
   Ⓐ、1000W
   Ⓑ、500W
   Ⓒ、250W
   Ⓓ、0W
17. An ideal buck converter is operated in CCM mode. Input voltage is 20V, output voltage is 10V and load resistance is 10Ω. Determine the average of the input current.
   Ⓐ、0.5 A
   Ⓑ、1 A
   Ⓒ、2 A
   Ⓓ、4 A
18. An ideal boost converter is operated in CCM mode. Input voltage is 10V and output voltage is 50V. Determine the average voltage across on the inductor.
   Ⓐ、40V
   Ⓑ、20V
   Ⓒ、10V
   Ⓓ、0V
19. A 50hp, 250V, 1200 rpm dc shunt motor with compensating windings has an armature resistance of 0.06Ω. Its field circuit has a total resistance 50Ω, which produces a no-load speed of 1200rpm. If input current is 100A, choose a false statement.
   Ⓐ、armature current=95 A
   Ⓑ、armature voltage=244.3 V
   Ⓒ、motor speed=1115 rpm
   Ⓓ、induced torque=190 N·m
20. A two-pole, 60Hz induction motor supplies 15kW to a load at a speed of 2950 rpm. Choose a false statement.
   Ⓐ、synchronous speed=3000 rpm
   Ⓑ、slip=1.67%
   Ⓒ、induced torque=96 N·m
   Ⓓ、motor speed is 2900 rpm if the applied torque is double

國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：電子學【電波領域聯合】　　題號：482003

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）　　

1. (30%) Figure 1 shows a Colpitts oscillator. Assume $\beta = 100$, $V_{BE} = 0.7\,\text{V}$, and $V_T = 25\,\text{mV}$ for BJT Q. **(a)(10%)** Estimate the quality factor of the equivalent small-signal LCR circuit seen at the collector of Q. **(b) (10%)** If the sustained oscillation occurs, what will be the oscillation frequency (in Hz)? **(c) (10%)** Calculate the small-signal transconductance g_m of the transistor Q, and determine if the feedback gain is large enough to cause oscillation. Justify your answer.

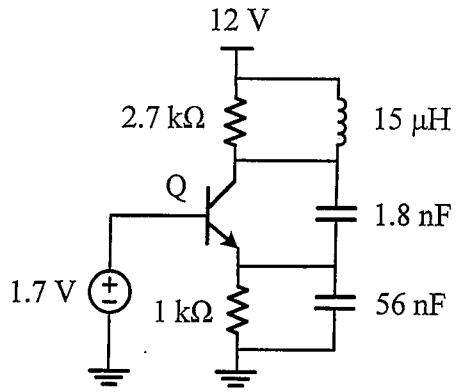


Fig. 1

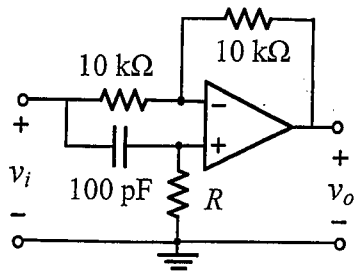


Fig. 2

2. (20%) Suppose we want to convert $v_i(t) = \cos(10^6 t)$ to $v_o(t) = A \times \sin(10^6 t)$ by a phase shift circuit in Fig. 2. Determine the value of R, and what will be the output amplitude A in this case?

3. (20%) A series RC circuit is utilized to equalize the impedance of an inductive load in Fig. 3. Determine the value of C in Fig. 3 so that the overall impedance Z becomes a pure resistance of 50 ohm in the entire frequency band.

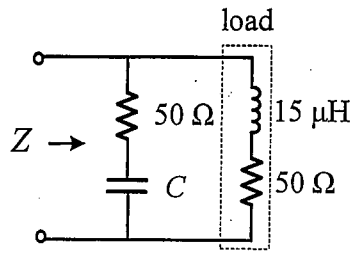


Fig. 3

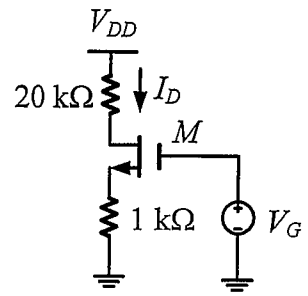


Fig. 4

4. (30%) Figure 4 shows a MOSFET transistor M that makes a constant current source for a 20-kΩ load. The body terminal and source terminal of MOSFET M are tied together. Assume that V_{DD}= 15 V, and the transistor M has the device parameters: W/L = 80, $\mu_n C_{ox}$ = 50 μA/V^2, V_{TH} =0.7 V, λ = 0.1. Suppose we want to have I_D = 0.5 mA. **(a) (10%)** Determine V_G. **(b) (10%)** Determine the minimum allowed V_{DD} that still assures that M works properly as a constant current source. **(c) (10%)** Suppose the 15V supply voltage has 1% variations on it (i.e., $\Delta V_{DD} = \pm 0.15$V). Estimate the percentage change in current I_D. Note: $I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$ for M in saturation.

國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：電磁學【電機系碩士班戊組、電波領域聯合】　　題號：482004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. (20%) The radius of the solid inner conductor and the inner radius of the very thin outer conductor of an air coaxial transmission line are r_i and r_o, respectively. A uniform current I flows in the inner conductor and returns via the thin outer conductor in the other direction. The permeability of free space μ_0 is $4\pi\times10^{-7}$ H/m. Determine the stored magnetic energy per unit length of the line.

2. (20%) Determine the capacitance of a spherical capacitor with an inner conducting sphere of radius a and an outer conductor of radius b. The permittivity of a dielectric medium between these two concentric spherical conductors is ε.

3. (20%)
   (a) (10%) Write down the **integral form** of time-harmonic Maxwell's equations.
   (b) (10%) Starting from these equations, **derive** all the boundary conditions at the interface between two different materials.

4. (20%) A uniform plane wave in air is incident on a lossless dielectric material at a 45° angle, as shown in Fig. P4. The transmitted wave propagates in a 30° direction with respect to the normal. The frequency is 300 MHz.
   (a) (6%) Find ε_2 in terms of ε_0.
   (b) (7%) Find the reflection coefficient and transmission coefficient.
   (c) (7%)Obtain the mathematical expressions for the incident, reflected and transmitted fields.

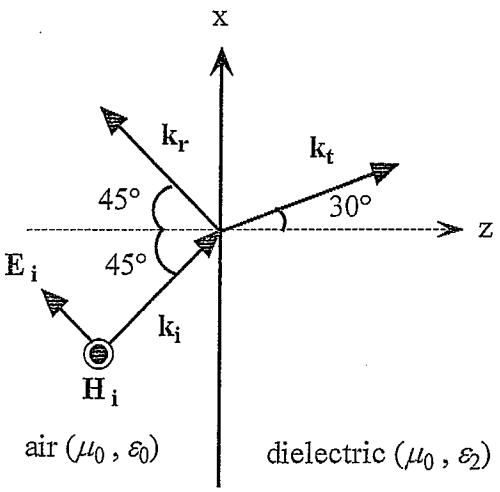


Fig. P4

5. (20%) Briefly answer the following questions.
   (a) (3%) What is the meaning of the polarization of a plane wave?
   (b) (2%) What is a uniform plane wave?
   (c) (3%) What is the skin effect? Discuss your knowledge about skin effect.

背面有題

國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：電磁學【電機系碩士班戊組、電波領域聯合】　　題號：482004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

(d)(2%) What is called a TEM wave?

(e)(3%) What is the dispersion effect for electromagnetic wave propagation?

(f) (2%) As far as you know, what are the factors that can cause the dispersion effect?

(g)(3%) What is a Smith chart and why is it still useful even though we can just use computers to do the transmission line calculations?

(h)(2%) What is a distortionless transmission line?

國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：計算機結構【電機系碩士班己組】　　題號：431007

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

[Problem 1] (20%) Terminology Explanation

(a) Pipeline Processing (b) Superscalar Processor (c) RISC (d) GPGPU

[Problem 2] (20%)

Suppose we are considering a change to an instruction set. The base machine is a load-store machine. Measurements of the load-store machine showing the instruction mix and clock cycle counts per instructions are given in the following table:

Instruction Type	Frequency	Clock Cycle Count
ALU Operations	40%	1
Loads	25%	2
Stores	15%	2
Branches	20%	2

Let's assume that 30% of the ALU operations directly use a loaded operand that is not used again. We propose adding ALU instructions that have one source operand in memory. These new register-memory instructions have a clock cycle count of 2. Suppose that the extended instruction set increases the clock cycle count for branches by 1, but it does not affect the clock cycle time. Would this change improve CPU performance? Explain your answer.

[Problem 3] (20%) A set associative cache has a block size of four 32-bit words and a set size of 4. The cache can accommodate a total of 256K words. The main memory size that is cacheable is 1024M * 32 bits. Design the cache structure and show how the processor's addresses are interpreted.

[Problem 4] (20%) Given the 8-bits adder (named Add8), the 2-to-1 8-bits multiplexers (named MUX8_2to1) and the basic gates such as NOT, AND, OR, NAND, and NOR, you are asked to design an ALU in function block diagrams, which must match the following requirements:

(1) Support add, sub, and sgt (set on great than) functions. Their operation selection bits (op_sel) are as follows: add(00), sub(10), sgt(11),

(2) Report the result status in sign, zero, overflow, and carry bits.

[Problem 5] (20%) Use the following code fragment:

```
Loop:   LW      R1, 0(R2)
        ADDI    R1, R1, #2
        SW      0(R2), R1
        ADDI    R2, R2, #5
        SUB     R4, R3, R2
        BNEZ    R4, Loop
```

Assume the initial value of R3 is R2+100. Use the five-stage instruction pipeline (IF, DEC, EXE, MEM, WB) and assume all memory accesses are one cycle operation. Furthermore, branches are resolved in MEM stage.

(a) (10%) Show the timing of this instruction sequence for the five-stage instruction pipeline with normal forwarding and bypassing hardware. Assume that branch is handled by predicting it has not taken. How many cycles does this loop take to execute?

(b) (10%) Assuming the five-stage instruction pipeline with a single-cycle delayed branch and normal forwarding and bypassing hardware, schedule the instructions in the loop including the branch-delay slot. You may reorder instructions and modify the individual instruction operands, but do not undertake other loop transformations that change the number of op-code of instructions in the loop. Show a pipeline timing diagram and compute the number of cycles needed to execute the entire loop.