1. Multiple Choice (12%)

Instructions:

- There are 4 questions, each of which is associated with 5 possible responses.
- For each of questions, select **ONE most appropriate** response.
- For each response you provide, **you will be awarded 3 marks if the response is correct and -3 marks if the response is incorrect (答錯一題倒扣三分).**
- You get 0 mark if no response is provided.

(1.1) Let $L[\cdot]$ denotes the Laplace transform.

(A) The Laplace transform is a linear operation.

(B) If $L[f(t)] = F(s)$, then $L[t^2 f(t)] = \frac{d^2}{ds^2}F(s)$.

(C) If $L[f(t)] = F(s)$, then $L\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0)$.

(D) If $L[f(t)] = F(s)$ and $L[g(t)] = G(s)$, then $L[f * g(t)] = F(s)G(s)$, where $*$ denotes the convolution integral.

(E) All of the above statements are TRUE.

(1.2) For what ω does the sinusoidal solution of $\ddot{x} + \dot{x} + x = \cos(\omega t)$ have the biggest amplitude?

(A) 2 (B) 1 (C) $\sqrt{2}$ (D) $1/\sqrt{2}$ (E) $1/2$.

(1.3) Consider the ODE $\ddot{x} + x\dot{x}^2 + (x^2 - 1) = 0$, where x is a real function.

(A) This is a time-invariant ODE.

(B) This ODE is nonlinear.

(C) This ODE has two equilibria.

(D) The equilibria of this ODE are -1 and 1.

(E) All of the above are TRUE.

(1.4) For what value of (a, b) will the solutions to $\ddot{y} + a\dot{y} + by = 0$ exhibit oscillatory behavior ?

(A) (1,2) (B) (1,0) (C) $(1, -1)$ (D) (2,1) (E) All of the above

2. (13%) Consider the following system of differential equations:

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -2x_1(t) - 3x_2(t) + u(t)$$
$$y(t) = 2x_1(t) - x_2(t)$$

with initial conditions $x_1(0) = x_2(0) = 0$. The external forcing function $u(t)$ is as follows

$$u(t) = \begin{cases} 1 & \text{if } t \geq 1 \\ 0 & \text{if } t < 1 \end{cases}$$

See Figure 1 for an illustration.

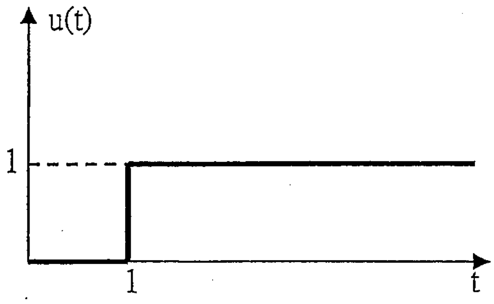


Figure 1: illustration of u(t).

(2.1) (10%) Find the corresponding response y(t).

(2.2) (3%) Calculate the peak value and the steady state value of y(t).

3. (20%) Let $\{\mathbf{x},\mathbf{y},\mathbf{z}\}$ be a set of linearly independent vectors in $\mathbb{R}^n$, and let $S := Span(\mathbf{x},\mathbf{y})$ and $T := Span(\mathbf{y},\mathbf{z})$. Define matrix $A := \mathbf{x}\mathbf{y}^T + \mathbf{y}\mathbf{z}^T$. Obviously, the sets S, T, their orthogonal complements $S^\perp$, $T^\perp$, and the four sets associated with matrix A, i.e. the two ranges $R(A)$ and $R(A^T)$ and the two null spaces $N(A)$ and $N(A^T)$, are all subspaces of $\mathbb{R}^n$.

This problem has three questions. The first one is a MULTIPLE-choice question, for which you don't need to give any derivation, but **you need to give detailed derivations for the other two questions**. In the multiple-choice question, the total score is evenly divided into each correct statement, and your each correct choice will get the partial score. However, the penalty for each wrong choice is equal to the score of each correct choice. (所以同時選了一個對的答案和一個錯的答案時，淨得分為0；但是扣分僅扣到該小題 0 分為止。另外為方便改題、請將選擇題的答案寫在此題做答處即可，不要寫到別處，以免漏改。)

(3.1) What are the possible relationships associated with S and $S^\perp$? (6%)
(A) $S^\perp \subset N(A^T)$
(B) $N(A^T) \subset S^\perp$
(C) $S^\perp \subset N(A)$
(D) $S \subset R(A)$
(E) $R(A^T) \subset S$

(3.2) Similar to the sub-question (3.1), please find out all possible relationships of T associated with the subspaces in the set $\{R(A), R(A^T), N(A), N(A^T)\}$. Give detailed arguments for your answers. (6%)

(3.3) Now let $(\lambda, \mathbf{v})$ be an eigenpair of matrix A with $\lambda \neq 0$. Then from the definition of A, it can be shown that $\mathbf{v}$ lies in certain subspace of $\mathbb{R}^n$ and λ is an eigenvalue of another matrix, denoted by $B \in \mathbb{R}^{m\times m}$ with $m = rank(A)$. Please (i) (2%)

indicate the subspace of $\mathbb{R}^n$ where the eigenvector $\mathbf{v}$ lies, and (ii)(6%) use vectors $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ to describe the matrix B.

4. (25%) Let P_2 denote the vector space of all polynomials of degree less than 2. Consider the transformation $L: P_2 \to \mathbb{R}^2$ defined by $L(p(x)) := \begin{bmatrix} \int_0^\alpha p(x) \\ p(\beta) \end{bmatrix}$ with **undecided parameters** $\alpha > 0$ and $\beta \in \mathbb{R}$. Let A be the matrix representation of transformation L with respect to the ordered bases $E = [1, x]$ and $E' = \left[\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right]$ for P_2 and $\mathbb{R}^2$, respectively.

以下小題僅需依序寫下答案即可，不需做任何推導。

(4.1) Find the set of $\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \in \mathbb{R}^2$ such that matrix A becomes singular. (5%)

(4.2) Let's define an inner product for P_2 by $\langle p(x), q(x)\rangle := \sum_{i=1}^{2} p(x_i)q(x_i)$, for arbitrary $p(x), q(x) \in P_2$, with $\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}\gamma\\1\end{bmatrix}$ and $\gamma \neq 1$ an **undecided parameter.** Find the orthonormal basis, denoted by $F := [\mathbf{f}_1, \mathbf{f}_2]$, of P_2 generated from basis E given above to satisfy the subspace equality constraints $Span(\mathbf{f}_1) = Span(1)$ and $Span(\mathbf{f}_1, \mathbf{f}_2) = Span(1, x)$. (8%)

(4.3) Let B denote the matrix representation of transformation L with respect to the ordered bases F computed in (4.2) and $F' = \left[\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}\right]$ for P_2 and $\mathbb{R}^2$, respectively. Find the matrix B. (6%)

(4.4) Now suppose $\alpha = \sqrt{2}$ and $\beta = 0$. Find all possible values of γ such that the set of eigenvalues of B is $\{1, \sqrt{2}\}$. (6%)

5.(a)(7%)Let $f(z)$ be a complex function defined by

$$f(z) = \begin{cases} \bar{z}^2 / z, & \text{if } z \neq 0 \\ 0, & \text{if } z = 0 \end{cases},$$

where $\bar{z}$ denotes the complex conjugate of the complex variable z. Does the function $f(z)$ satisfy the Cauchy-Riemann equations? Give your reason (no credit will be given if there is no explanation).

(b)(8%) Does the derivative of $f(z)$ at $z = 0$, i.e., $f'(0)$, exist? Give your reason (no credit will be given if there is no explanation).

6. (15%) Using the theory of Residues, compute the inverse $f(t)$, $-\infty < t < \infty$, of the Fourier transform

$$F(\omega) = \frac{2a}{a^2 + \omega^2}, \quad a > 0.$$

國立中山大學101學年度碩士暨碩士專班招生考試試題

科目：離散數學【電機系碩士班丙組選考】

題號：4061
共1頁 第1頁

考生請注意：必須寫出作答過程或得到答案之理由，只寫答案不予計分。

1. Let a and n be integers. Show: If $a \mid n$ and $a \mid (n+2)$, then $a \mid 2$. (10%)

2. Show: For all $n \in Z$. $(n^3 - n) \bmod 3 = 0$. (10%)

3. If a vending machine dispenses only 3-cent and 8-cent stamps, then any monetary value of 14 cents or greater can be obtained from this machine.
   (a) How could 20 cents be obtained? (5%)
   (b) Prove the general result. Hint: by Principle of Mathematical Induction. (15%)

4. Draw a Hasse diagram for the "divides" relation | on the set of positive divisors of 30. (10%)

5. Let V={1, 2, 3, 4}, E={{1, 4}, {2, 4}, {3, 4}}, and graph G=(V, E).
   (a) Draw G=(V, E). (5%)
   (b) Determine the edge set for the subgraph of G induced by W={1, 2, 3}. (5%)
   (c) What is the distance from 2 to 3? (5%)
   (d) Is G bipartite? (5%)

6. (a) Draw the graph $K_{4,4}$. (5%)
   (b) Find a Hamiltonian cycle in $K_{4,4}$.(Draw it out.) (5%)

7. (a) How many functions are there from an n-element set onto {0, 1}? (5%)
   (b) How many strings can be formed by ordering the letters *PARAGRAPH* ? (5%)

8. Show two partitions of 15 and draw their corresponding Ferrers graphs. (10%)

1. Multiple Choice (21%)

Instructions:

- There are 7 questions, each of which is associated with 5 possible responses.
- For each of questions, select **ONE most appropriate** response.
- For each response you provide, **you will be awarded 3 marks if the response is correct and -3 marks if the response is incorrect (答錯一題倒扣三分).**
- You get 0 mark if no response is provided.

(1.1) Consider the ODE $\ddot{x} + \dot{x}^2 + (x^3 - 1) = 0$, where x is a real function.

(A) This is a time-varying ODE.
(B) This ODE is nonlinear.
(C) This ODE has two equilibria.
(D) The equilibria of this ODE are 0 and 1.
(E) None of the above is TRUE.

(1.2) What is the amplitude of the sinusoidal solution of $\ddot{x} + \dot{x} + 2x = 2\sin(t)$?

(A) 2 (B) 1 (C) $\sqrt{2}$ (D) $1/\sqrt{2}$ (E) None of the above.

(1.3) Let $\boldsymbol{L}[\cdot]$ denotes the Laplace transform.

(A) The Laplace transform is a linear operation.
(B) If $\boldsymbol{L}[f(t)] = F(s)$, then $\boldsymbol{L}[t^2 f(t)] = \frac{d^2}{ds^2}F(s)$.
(C) If $\boldsymbol{L}[f(t)] = F(s)$, then $\boldsymbol{L}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0)$.
(D) If $\boldsymbol{L}[f(t)] = F(s)$ and $\boldsymbol{L}[g(t)] = G(s)$, then $\boldsymbol{L}[f * g(t)] = F(s)G(s)$, where * denotes the convolution integral.
(E) All of the above statements are TRUE.

(1.4) Define the *del operator* $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$.

(A) $\nabla \cdot (\nabla \times \mathbf{F}) = 0$, where **F** is a vector field with continuous first and second derivatives.
(B) $\nabla \times \nabla F = 0$, where F is a scalar function with continuous first and second derivatives.
(C) $\nabla \cdot (\nabla F \times \nabla G) = 0$, where F and G are scalar functions with continuous first and second derivatives.
(D) All of the above statements are TRUE.
(E) None of the above statements is TRUE.

(1.5) The point on the plane $2x + y - z = 6$ which is closest to the origin is

(A) (2,1,-1) (B) (2,2,0) (C) (3,0,0) (D) (1,1,-2) (E) None of the above

(1.6) For what value of b will the solutions to $\ddot{y} + b\dot{y} + y = 0$ exhibit oscillatory behavior ?

(A) 1 (B) 2 (C) 3 (D) -2 (E) All of the above

(1.7) What is the work done by the vector field $\mathbf{F} = -y^3\mathbf{i} + x^3\mathbf{j}$ around the circle of radius 1, centered at the origin, oriented counter-clockwise ?

(A) π (B) $-\pi$ (C) $\frac{3}{2}\pi$ (D) 2π (E) None of the above

2. (9%) Evaluate the following integral

$$\int_0^1 \int_{x^2}^1 xe^{-y^2}\, dydx$$

3. (15%) Find the solution to the following heat equation:

$$\frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) \qquad \forall 0 < x < 1, t > 0$$
$$u(0,t) = 0, \qquad u(1,t) = 1 \qquad \forall t > 0$$
$$u(x,0) = x + \sin(\pi x) \qquad \forall 0 < x < 1$$

4. (12%) Let $\{\mathbf{x},\mathbf{y},\mathbf{z}\}$ be a set of linearly independent vectors in $\mathbb{R}^n$ and let $S := Span(\mathbf{x},\mathbf{y})$. Define matrix $A := \mathbf{x}\mathbf{y}^T + \mathbf{y}\mathbf{z}^T$. Obviously, the sets S, its orthogonal complement $S^\perp$, and the four sets associated with matrix A, i.e. the two ranges $R(A)$ and $R(A^T)$ and the two null spaces $N(A)$ and $N(A^T)$, are all subspaces of $\mathbb{R}^n$.

This problem has two questions. The first one is a MULTIPLE-choice question, which you don't need to give any derivation. But you **need to give detailed derivations for the second question.** For the multiple-choice question, the total score is evenly divided into each correct statement, and your each correct choice will get the partial score. However, the penalty for each wrong choice is equal to the score of each correct choice. (所以同時選了一個對的答案和一個錯的答案時，淨得分為0；但是扣分僅扣到該小題 0 分為止。另外、為方便改題，請將選擇題的答案寫在此題做答處即可，不要寫到別處，以免漏改。)

(4.1) What are the possible relationships associated with S and $S^\perp$? (6%)

(A) $S^\perp \subset N(A^T)$

(B) $N(A^T) \subset S^\perp$

(C) $S^\perp \subset N(A)$

(D) $S \subset R(A)$

(E) $R(A^T) \subset S$

(4.2) Now let $(\lambda, \mathbf{v})$ be an eigenpair of matrix A^T with $\lambda \neq 0$. Then from the definition of A, it can be shown that $\mathbf{v}$ lies in certain subspace of $\mathbb{R}^n$ and λ is an eigenvalue of another matrix, denoted by $B \in \mathbb{R}^{m\times m}$ with $m = rank(A)$. Please (i) (2%) indicate the subspace of $\mathbb{R}^n$ where the eigenvector $\mathbf{v}$ lies, and (ii) (4%) use vectors $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ to describe the matrix B.

5. (13%) Let P_2 denote the vector space of all polynomials of degree less than 2. Consider the transformation $L: P_2 \to \mathbb{R}^2$ defined by $L(p(x)) := \begin{bmatrix} \int_0^\alpha p(x) \\ p(\beta) \end{bmatrix}$ with **undecided parameters** $\alpha > 0$ and $\beta \in \mathbb{R}$. Let A be the matrix representation of transformation L with respect to the ordered bases $E = [1,\ x]$ and $E' = \left[\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right]$ for P_2 and $\mathbb{R}^2$, respectively.

以下小題僅需依序寫下答案即可，不需做任何推導。

(5.1) Find the set of $\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \in \mathbb{R}^2$ such that matrix A becomes singular. (4%)

(5.2) Let's define an inner product for P_2 by $\langle p(x), q(x)\rangle := \sum_{i=1}^{2} p(x_i)q(x_i)$, for arbitrary $p(x), q(x) \in P_2$, with $\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}\gamma\\ 1\end{bmatrix}$ and $\gamma \neq 1$ an **undecided parameter**. Find the orthonormal basis, denoted by $F := [\mathbf{f}_1, \mathbf{f}_2]$, of P_2 generated from basis E given above to satisfy the subspace equality constraints $Span(\mathbf{f}_1) = Span(1)$ and $Span(\mathbf{f}_1, \mathbf{f}_2) = Span(1, x)$. (5%)

(5.3) Let B denote the matrix representation of transformation L with respect to the ordered bases F computed in (4.2) and $F' = \left[\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}\right]$ for P_2 and $\mathbb{R}^2$, respectively. Find the matrix B. (4%)

6. (a)(7%) Let $f(z)$ be a complex function defined by

$$f(z) = \begin{cases} \bar{z}^2 / z, & \text{if } z \neq 0 \\ 0, & \text{if } z = 0 \end{cases},$$

where $\bar{z}$ denotes the complex conjugate of the complex variable z. Does the function $f(z)$ satisfy the Cauchy-Riemann equations? Give your reason (no credit will be given if there is no explanation).

(b)(8%) Does the derivative of $f(z)$ at $z = 0$, i.e., $f'(0)$, exist? Give your reason (no credit will be given if there is no explanation).

7. (15%) Using the theory of Residues, compute the inverse $f(t)$, $-\infty < t < \infty$, of the Fourier transform

$$F(\omega) = \frac{2a}{a^2 + \omega^2}, \quad a > 0.$$

國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：線性代數【電機系碩士班己組】

1. (5%)(單選題) Given $n \times n$ matrices **A, B** and **S**. Among the following statements, which are **true**?

   [i] $tr(\mathbf{AB}) \neq tr(\mathbf{BA})$
   [ii] $tr(\mathbf{SAS}^{-1}) = tr(\mathbf{A})$
   [iii] $\mathbf{AB} - \mathbf{BA} \neq \mathbf{I}$
   [iv] $det(\propto \mathbf{A}) = \propto^n det(\mathbf{A})$, $\propto$ is constant
   [v] $det\,(\mathbf{A}^T) \neq det\,(\mathbf{A})$

   (a) i、ii、iii (b) i、ii、v (c) i、iii、iv (d) ii、iii、iv (e) iii、iv、v

2. (5%)(單選題) Let $\mathbf{A}_T$ be the matrix representation of the linear transformation T.
   $T : \mathbb{R}^3 \to \mathbb{R}^3$

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x - 2y + 3z \\ -2x + 4y - 6z \\ 3x - 6y + 9z \end{bmatrix}$$

   Let $\mathbf{A}_T$ be the matrix representation of the linear transformation T.
   Which statement is **not** correct?

   (a) The matrix $\mathbf{A}_T$ is linearly dependent.
   (b) The kernel of T is span$\left\{\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}\right\}$.
   (c) The range of T is span$\left\{\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}\right\}$.
   (d) The nullity of T is 2.
   (e) The rank of T is 1.

3. (10%) Find a singular value decomposition of $\mathbf{A} = \begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{bmatrix}$.

4. (15%) Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

   (i) Find the characteristic function of **A**. (5%)
   (ii) Compute $\mathbf{A}^5 - 7\mathbf{A}^4 + 13\mathbf{A}^3 - 13\mathbf{A}^2 + 7\mathbf{A} - \mathbf{I}$. (10%)

5. (15%) Let $\{f_1(x), f_2(x), f_3(x)\}$ be a basis for a vector space, where

$$f_1(x) = \begin{cases} 1 & if\ 0 \le x \le 1 \\ 0 & otherwise \end{cases}, f_2(x) = \begin{cases} 1 & if\ 1 \le x \le 2 \\ 0 & otherwise \end{cases},$$
$$f_3(x) = \begin{cases} 1 & if\ 2 \le x \le 3 \\ 0 & otherwise \end{cases}.$$

Define a linear transformation $L[.]$ having the following properties:

$$L[f_1(x)] = \begin{cases} 1 & if\ 0 \le x \le 2 \\ 0 & otherwise \end{cases}, L[f_2(x)] = \begin{cases} 1 & if\ 1 \le x \le 3 \\ 0 & otherwise \end{cases}$$
$$L[f_3(x)] = \begin{cases} 1 & if\ 0 \le x \le 3 \\ 0 & otherwise \end{cases}$$

(i) Find the matrix representation of L with respect to $\{f_1(\mathrm{x}), f_2(\mathrm{x}), f_3(\mathrm{x})\}$. (5%)

(ii) If we use another basis $\{g_1(\mathrm{x}), g_2(\mathrm{x}), g_3(\mathrm{x})\}$, where

$$g_1(x) = \begin{cases} 1 & if\ 0 \le x \le 3 \\ 0 & otherwise \end{cases}, g_2(x) = \begin{cases} 1 & if\ 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$
$$g_3(x) = \begin{cases} 1 & if\ 2 \le x \le 3 \\ 0 & otherwise \end{cases}$$

Find the matrix representation of L with respect to $\{g_1(x), g_2(x), g_3(x)\}$. (10%)

6. (30%) Consider the following 4×4 matrix **A**

$$\mathbf{A} = \sum_{i=1}^{4} \lambda_i \mathbf{u}_i \mathbf{u}_i^H$$

where λ_i is real and nonzero, and

$$\boldsymbol{u}_1 = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \boldsymbol{u}_2 = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \quad \boldsymbol{u}_3 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad \boldsymbol{u}_4 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}.$$

(i) Prove that **A** is Hermitian (5%)
(ii) Prove that $\boldsymbol{u}_i$ (i = 1,2,3,4) are eigenvector of the matrix **A** (5%)
(iii) Find the inverse matrix of the matrix **A** (5%)
(iv) Find the condition that matrix **A** is positive definite (5%)
(v) Find the condition that matrix **A** is unitary (5%)
(vi) Find a matrix **L** such that $\mathbf{L}\mathbf{L}^H = \mathbf{A}$ (5%)

7. (20%) Consider three vectors :

$$\boldsymbol{u}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \qquad \boldsymbol{u}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \qquad \boldsymbol{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$

(i) Apply the Gram-Schmidt process to $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, $\boldsymbol{u}_3$ to form a set of orthonormal bases. (5%)

(ii) Find the orthogonal projection of a vector $\boldsymbol{b} = [2 \quad -1 \quad 3 \quad 1 \quad 1]^T$ on the space spanned by $\boldsymbol{u}_1$,$\boldsymbol{u}_2$,$\boldsymbol{u}_3$. (5%)

(iii) Find the QR decomposition of (5%)

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(iv) Find a solution of $\boldsymbol{x} = [x_1 \quad x_2 \quad x_3]^T$, such that $\|\mathbf{U}\boldsymbol{x} - \boldsymbol{b}\|^2$ is minimized. (5%)

國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：資料結構【電機系碩士班丙組選考】

題號：4062
共 4 頁 第 1 頁

注意事項

一、不可於試題紙上作答。

二、When you are asked to derive the time complexity of a program (or algorithm) in the form of $O(f(n))$, the function $f(n)$ should be expressed in the ***simplest*** and ***tightest*** form. For example, if running time $T(n) = 2n^2 + 3$, you should write $T(n) = O(n^2)$. In other words, you will get 0 points if you write $T(n) = O(n^3)$ or $T(n) = O(2n^2 + 3)$.

1. (a) [5 points] Given the size n of the input data, where n is a positive integer, we assume that the running time of a program is $O(f(n))$. State the *formal* definition of $O(f(n))$.

   (b) [5 points] Given the size n of the input data, where n is a positive integer, we assume that the program requires the running time $T(n) = O(f(n))$, where

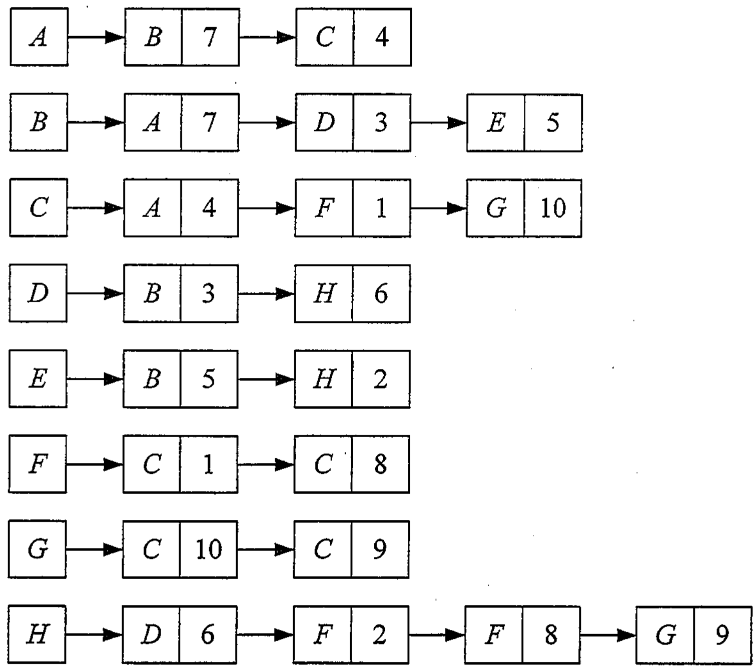
$$T(n) = \log 1 + \log 2 + \log 3 + \cdots + \log n = \sum_{x=1}^{n} \log x.$$

   Derive the function $f(n)$.

2. [10 points] What would be the contents of queue `Q` after the following pseudo code is executed? The contents of the input file are `8, 16, 15, 4, 0, 4, 6, 8, 17, 24, 0, 33, 47, 28, 9, 0`. Note that your answer should point out where is the front of the queue.

```
Q = create_Queue();
S = create_Stack();
while (not end of file) {
    x = read_an_input_number();
    if ( x != 0 )
        push_Stack(S, x);
    else {
        x = pop_Stack(S);
        x = pop_Stack(S);
        while ( S is not empty ) {
            x = pop_Stack(S);
            add_Queue(Q, x);
            // Add the integer x into the queue Q.
        }
        delete_Queue(Q);
        // Delete one element from the queue Q.
    }
}
```

3. On the basis of the adjacency list shown below, answer the following questions. Note that the value field in the element of the adjacency list contains the edge weight of the graph.



Vertex list Adjacency list

(a) [5 points] Draw the graph *G* whose adjacency list is defined above.
(b) [5 points] Temporarily ignore the edge weight and use the DFS (depth first search) to traverse the graph *G* from the vertex *A*.
(c) [5 points] Temporarily ignore the edge weight and use the BFS (breadth first search) to traverse the graph *G* from the vertex *A*.
(d) [10 points] Star from the vertex *A* and use Prim's algorithm to find the minimum cost spanning tree of the graph *G*. Show the actions step by step.

4. Insert a sequence of keys `{7, 9, 16, 30, 49, 82, 5, 33, 31, 6, 2, 1}`, in that order, into a data structure which has no keys initially.
(a) [5 points] Construct a binary search tree for that sequence.
(b) [5 points] Construct a max heap for that sequence.
(c) [5 points] Tell us the resultant max heap after deleting `9` and `49` from the max heap obtained in question 4(b).

5. (a) [5 points] Given an unsorted integer array of size n, does the binary search algorithm outperform the sequential search algorithm? Use the big-O notation to justify your answer.

(b) [10 points] Given an integer array **A** of size n, the following pseudo code shows the insertion sort algorithm. Now, suppose that we want to sort an integer array **B** of size 100. In other words, the array **B** contains 100 integers. Derive the worst case running time of `insertion_sort(B)` in terms of big-O notation. (Note that you will get 0 points if you just give the answer directly.)

```
void insertion_sort(array A) {
  int i, j, key;

  for (j = 2; j <= array_size(A); j++) {
     key = A[j];
     // Insert A[j] into the sorted sequence A[1..j-1].
     i = j-1;
     while ( (i > 0) && (A[i] > key) ) {
        A[i+1] = A[i];
        i = i-1;
     }
     A[i+1] = key;
  }
}
```

6. The sequence L_n of Lucas numbers is defined as follows.

$$L_n = \begin{cases} 2 & \text{if } n = 0, \\ 1 & \text{if } n = 1, \\ L_{n-1} + L_{n-2} & \text{if } n \geq 2. \end{cases}$$

The LUCAS NUMBER PROBLEM is defined as "Given an integer $n \geq 0$, output the n-th Lucas number L_n." The following recursive function `Lucas(int n)` can solve the LUCAS NUMBER PROBLEM.

```
int Lucas(int n) {
  if (n==0)
     return 2;
  else if (n==1)
     return 1;
  else
     return Lucas(n-1) + Lucas(n-2);
}
```

(a) [10 points] Prove that, when $n \geq 2$, the running time $T(n)$ of `Lucas(n)` is larger than $\left(\frac{1+\sqrt{5}}{2}\right)^{n-2}$. Note that the running time is measured in units of instruction step.

(b) [5 points] Now, tell us whether the LUCAS NUMBER PROBLEM is NP-complete? Explain your reasons. (Note that you will get 0 points if you do not present any reasons.)

7. [10 points] Given the preorder sequence, *A B D H J E C F G*, and the inorder sequence, *H D J B E A F C G*, please (1) draw the corresponding binary tree, and (2) show its postorder sequence.

1. (15%) A Si *p-n* junction diode with a cross-sectional area of 3×10^{-4} cm^2. The parameters of the diode are: acceptor concentration $N_A = 6 \times 10^{16}$ cm^{-3}, donor concentration $N_D = 2 \times 10^{16}$ cm^{-3}, intrinsic carrier concentration $n_i = 9.65 \times 10^9$ cm^{-3}, diffusion constant of electrons $D_n = 21$ cm^2/s, diffusion constant of holes $D_p = 10$ cm^2/s, carriers lifetime: $\tau_p = \tau_n = 5 \times 10^{-7}$ s. (i) Calculate the built-in voltage at 300 K. (5%) (ii) Calculate the ideal reverse saturation current. (10%)

2. (10%) Consider an n-channel enhancement-mode MOSFET with the following parameters: threshold voltage $V_{TN} = 0.75$ V, channel width $W = 100$ μm, channel length $L = 10$ μm, electron mobility $\mu_n = 650$ cm^2/V-s, oxide thickness $t_{ox} = 500$ Å, and oxide permittivity $\varepsilon_{ox} = (3.9)(8.85\times10^{-14})$ F/cm. (i) Determine the drain current when $V_{GS} = V_{DS} = 2.5$ V, for the transistor biased in the saturation region. (5%) (ii) Determine the drain current when $V_{GS} = 2.5$ V and $V_{DS} = 1$ V, for the transistor biased in the triode (non-saturation) region. (5%)

3. (25%) In the circuit shown in Figure 1, the transistor has a β of 250. (i) What is the dc voltage at the collector? (5%) (ii) Find the input resistances R_{ib} and R_{in} and the overall voltage gain (v_o/v_{sig}). (15%) (iii) For an output signal of ±0.5 V, what values of v_{sig} and v_b are required? (5%)

4. (18%) Analyze the circuit of Figure 2 to determine the small-signal voltage gain V_o/V_s, the input resistance R_{in}, and the output resistance $R_{out} = R_{of}$. The transistor has $\beta = 100$.

5. (18%) Find the voltage gain v_O/v_{Id} for the difference amplifier of Figure 3 for the case $R_1 = R_3 = 10$ kΩ and $R_2 = R_4 = 100$ kΩ. What is the differential input resistance R_{id}? If the two key resistance ratios (R_2/R_1) and (R_4/R_3) are different from each other by 1%, what do you expect the common-mode gain A_{cm} to be? Also, find the CMRR in this case.

6. (14%) A BJT is specified to have a maximum power dissipation P_{D0} of 2 W at an ambient temperature T_{A0} of 25°C, and a maximum junction temperature T_{Jmax} of 150°C. Find the following: (a)The thermal resistance θ_{JA}. (5%) (b) The maximum power that can be safely dissipated at an ambient temperature of 50°C. (5%) (c) The junction temperature if the device is operating at $T_A = 25$°C and is dissipating 1 W. (4%)

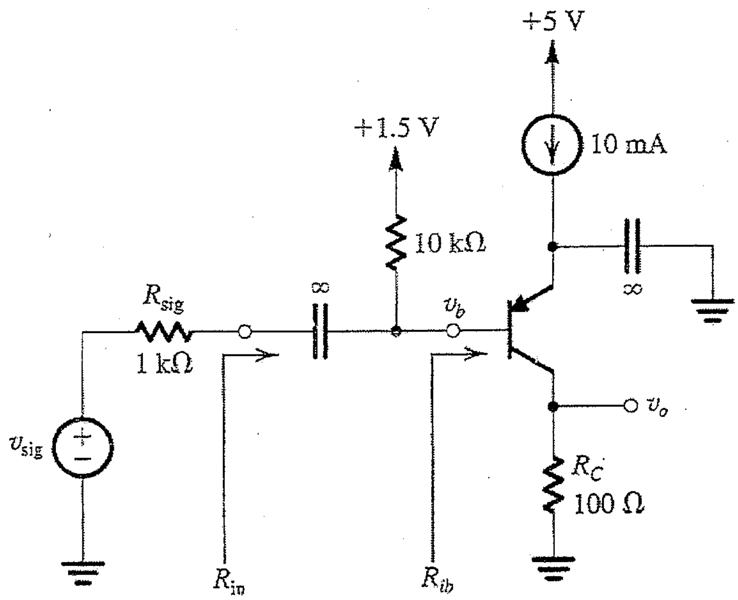


Figure 1

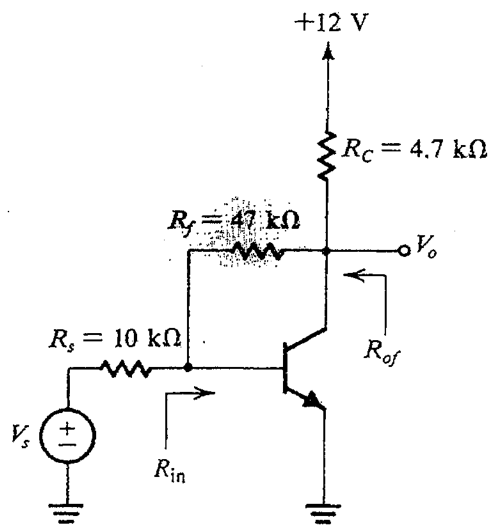


Figure 2

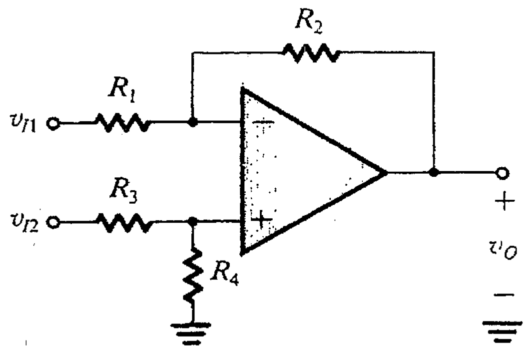
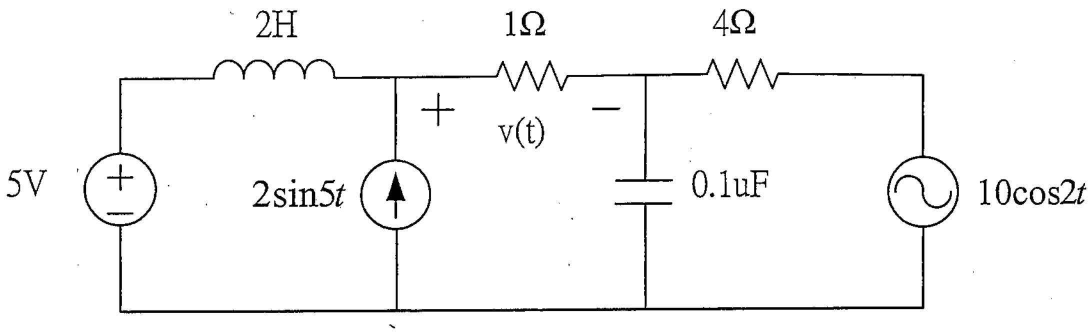


Figure 3

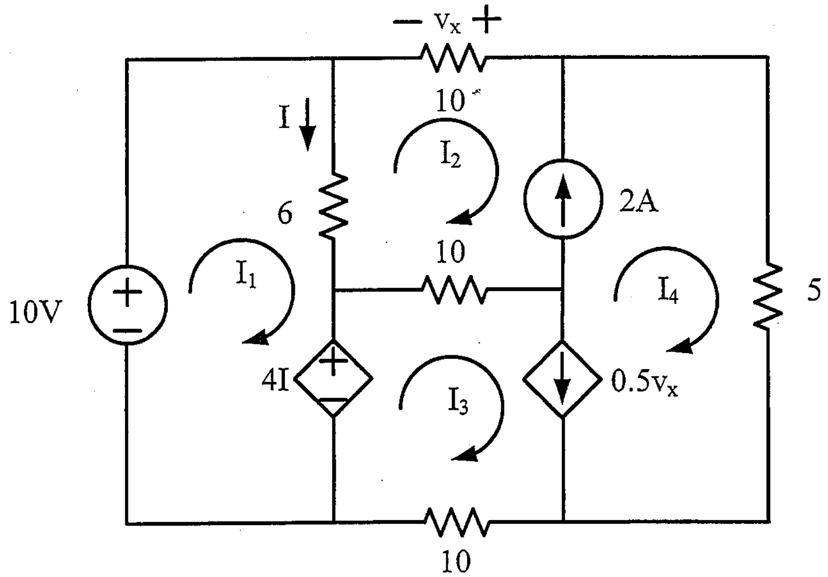
1. (10pt) A balanced delta-connected load is connected to the generator by a balanced transmission line with impedance of 0.1+j0.2Ω per phase. The load is rated at 500kW, 0.866 power factor lagging, 440V_{rms} line voltage. Find line current and line voltage of the generator, respectively.

2. (10pt)Find v(t) in the sinusoidal steady state.



3. (20pt) Phase voltage and phase current of a load are given as $v(t)=100\cos\omega t$ V, $i(t)=10\sin(\omega t+60^{o})+2\sin(3\omega t+60^{o})$ A. Find power factor, instantaneous power, average power, and apparent power of the load.

4. (20pt)Use mesh analysis to find I_1, I_2, I_3, I_4,. Voltage, current, and resistance units are V, A, and Ω, respectively.

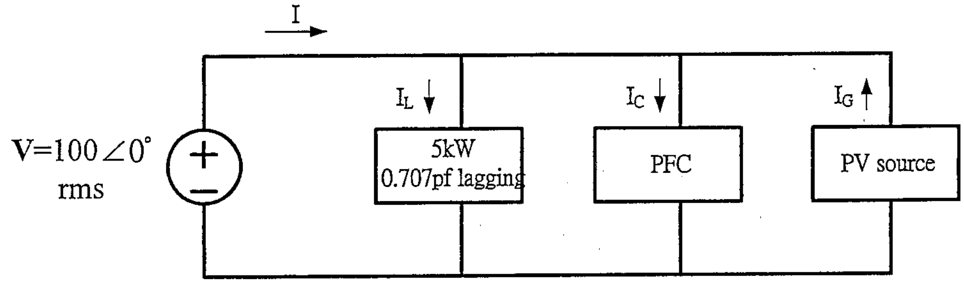


5. (20pt) A power-factor-correction (PFC) capacitor is designed to improve power factor of the load (5kW) equal to 0.9. Assume the PV is a current source inverter to deliver 2kW average power to the system.

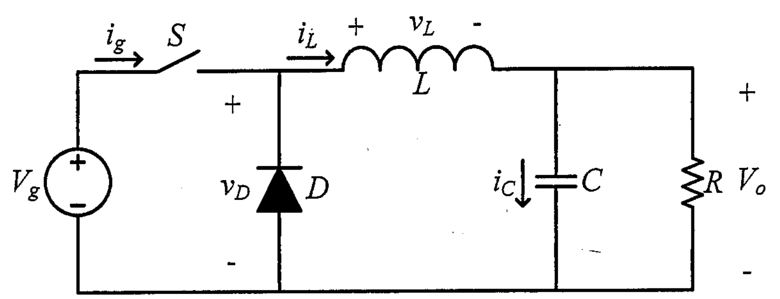
(A)Calculate I, I_L, I_c, and I_G.

(B)Construct phasor diagram of V, I, I_L, I_c, and I_G.

(C)Construct power triangle diagram.



6. (20pt) A buck converter is given with the following parameters: Vg=40V, Vo=30V, the switching frequency of S is 40kHz, output power is 100W. Draw waveforms of v_D, v_L, i_C, i_L,i_g for L=40uH and L=10uH, respectively.



[Problem 1] Short answer questions:

(a) What are the differences between a combinational circuit and a sequential circuit? (4%)

(b) What are the differences between a latch and a flip-flop? (2%)

(c) What are the setup time and the hold time of a D flip-flop? (4%)

(d) What is a clocked synchronous sequential circuit? (3%)

(e) What are the differences between Mealy and Moore finite state machines? (2%)

[Problem 2] Draw the logic diagram of the following Boolean function using a multiplexer:

(a) $F(A, B, C, D) = A'B'C'+A'B'D'+A'C'D+ABCD'+AB'C'D'+AB'CD$. (5%)

(b) $F(A, B, C, D) = (A+B+D')(A+C'+D')(A'+B+C')$. (5%)

Please note that the input code words (*A*, *B*, *C*, *D*) and their complements can be used directly as fan-in in the logic circuit, and the details of the multiplexer are no need to show.

[Problem 3] 8-4-(-2)-(-1) is one 4-bit binary code that can be used to represent decimal digit, as listed in Table 1. Design a combinational circuit using minimum number of logic gates and literals that can check if the decimal input encoded by 8-4-(-2)-(-1) is a prime number, i.e., a positive integer that is greater than 1 and can be exactly divided by only 1 and itself. That is, the output *f* of the circuit equals 1 if and only if the decimal input is a prime number. You need to show the truth table of this circuit, the logic simplification process and the final logic diagram. Please note that the input code words (*w*, *x*, *y*, *z*) and their complements can be used directly as fan-in in the final logic diagram, and the unused input code words can be used as don't care conditions for logic simplification. (15%)

Table 1

Decimal digit	8-4-(-2)-(-1) code
0	0000
1	0111
2	0110
3	0101
4	0100
5	1011
6	1010
7	1001
8	1000
9	1111

[Problem 4] Implement the following Boolean function

$$F(x, y, z) = \sum(0, 1, 3, 5, 6)$$

(a) using only OR and inverter gates. (5%)

(b) using only NAND and inverter gates. (5%)

Please note that you only need to show the final Boolean function and how you derive the function.

國立中山大學101學年度碩士暨碩士專班招生考試試題

科目：數位電路【電機系碩士班丙組選考、庚組】

[Problem 5] A sequential circuit has three flip-flops X, Y and Z, a one-bit data input I, a one-bit clock input CLK; and a one-bit data output O. The state transition diagram of this circuit is shown in Figure 1. Please draw the logic diagram of this circuit using D flip-flops and minimum numbers of logic gates and literals. Please note that the details of the D flip-flop are no need to show, and the unused states can be used as don't care conditions for logic simplification. (15%)

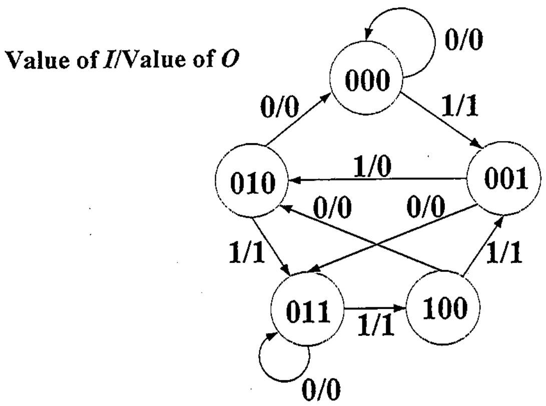


Figure 1

[Problem 6] Design an asynchronously resettable positive edge-triggered finite state machine that has a one-bit data input x, a one-bit clock input clk, a one-bit reset input rst and a one-bit data output y. The output is 1 only when an input sequence 110 or 101 appears. For example, the input sequence x=0110101010110 results in the output y=0001101010101.

(a) Draw the state transition diagram and define each state clearly. (10%)

(b) Write RTL Verilog/VHDL codes to implement the finite state machine you designed in (a). (10%)

[Problem 7] Please use four 1-bit full adder and a few logic gates (AND, OR, NOT, XOR) to design a 4-bit signed adder-subtractor. The details of the 1-bit adder are no need to show. This circuit can perform addition or subtraction operation, controlled by a mode input M. When M=0, the circuit performs addition ($A + B$ where A and B are both 4-bit signed inputs, i.e., {A_3, A_2, A_1 and A_0} and {B_3, B_2, B_1 and B_0}), and when M=1, the circuit performs subtraction ($A - B$, i.e., A + the 2's complement of B). The outputs of this circuit include

(1) One 4-bit sum output S {S_3, S_2, S_1 and S_0}

(2) One 1-bit output E to indicate that the value of sum is odd or even: E=1(0) for even (odd) value

(3) One 1-bit output P to indicate that the parity of sum is odd or even: P=1(0) for even (odd) parity

(4) One 1-bit output O to indicate whether overflow occurs: O=1(0) means overflow happens (does not happen)

(5) One 1-bit carry-out output C_out. (15%)

國立中山大學101學年度碩士暨碩士專班招生考試試題

科目：機率【電機系碩士班己組】

題號：4067
共1頁第1頁

1. (15%) Given any two real-valued random variables X_1 and X_2 with finite second moment. Here, $\mathrm{E}\{\cdot\}$ takes the expectation with respect to X_1 and X_2. If the statement is true, please write a circle ("o") on your ANSWER SHEET. If the statement is wrong, then mark it as ("x") on your ANSWER SHEET. You do NOT need to provide any justification.
   (a) ( ). $(\mathrm{E}\{X_1X_2\})^2 \leq \mathrm{E}\{X_1^2\}\mathrm{E}\{X_2^2\}$;
   (b) ( ). $\mathrm{E}\{c_1X_1 + c_2X_2\} \neq c_1\mathrm{E}\{X_1\} + c_2\mathrm{E}\{X_2\}$, where c_1 and c_2 are constant values;
   (c) ( ). $\mathrm{E}\{(X_1 + X_2)^2\} \leq \mathrm{E}\{X_1^2\} + \mathrm{E}\{X_2^2\}$.

2. (15%). Let Y be a binomial distribution with parameters n and p ; i.e., the probability distribution function of Y is given by $P(Y = y) = \binom{n}{y}p^y(1-p)^{n-y}$, $y = 0, 1, 2, \cdots, n$. Please find
   (a) the mean of Y,
   (b) the variance of Y,
   (c) the probability generating function of Y.

3. (10%) The joint probability density function of the random variable (X_1, X_2) is given by

$$f(x_1, x_2) = \begin{cases} c(x_1 + x_2) & 0 < x_1 < x_2 < 1, \\ 0 & \text{otherwise.} \end{cases}$$

   Are X_1 and X_2 stochastically independent? Why?

4. (10%) Let X and Y be independent normal random variables with zero mean and unit variance. Find the value of $\mathrm{E}\{X^2Y + XY^2 + X^2Y^2\}$, in which $\mathrm{E}\{\cdot\}$ takes the expectation with respect to X and Y.

5. (20%) Explain the following terms in detail: central limit theorem, negative correlated, Bayes' theorem, Tchebycheff inequality.

6. (10%) Let X and Y be independent normal random variables with zero mean and standard deviation σ. If $X\cos(\omega t) + Y\sin(\omega t) = R\cos(\omega t - \varphi)$. Find the probability density functions of random variables R and φ respectively.

7. (10%) Let X be an exponential random variable with parameter λ. Find the mean and variance of $2X$.

8. (10%) Let Y be a uniform random variable in the range [a, b]. Find the mean and variance of $6Y$.

國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：半導體概論【電機系碩士班甲組】

題號：4068

1. Formulate the ideal diode current-voltage characteristics, $I_D = I_S \cdot \exp(V_D/\eta V_T - 1)$, with respect to the Diode cross-sectional area A, the carrier diffusion lengths: L_n and L_p, the carrier diffusion coefficients: D_n and D_p, the equilibrium minority-carrier densities: n_{po} and p_{no}, and the diode voltage bias: V_D. Where η is ideal factor of the diode. (20%)

2. Show the fabrication process and draw its corresponding I_D - V_D and $I_D - V_G$ current characteristics of an Enhancement mode N-channel MOSFET (*ENH nMOS*) and a Depletion mode N-channel MOSFET (*DEP nMOS*) respectively. (Note: You need point out the main differences between them) (20%)

3. A MOSFET has a threshold voltage of V_T= 0.5 V, a subthreshold swing (*SS*) of 100 mV/decade, and a drain current of 0.1 μA at V_T. What is the subthreshold leakage current at V_G = 0 V? (20%)

4. Nowadays for CMOS IC industry we normally need a buffered layer placed between a high-*k* Ta_2O_3 and the silicon substrate. Please calculate the effective oxide thickness (*EOT*) when the stacked gate dielectric is Ta_2O_3 (k = 25) with a thickness of 7.5 nm on a buffered nitride layer (k = 7) and a thickness of 1 nm). Also calculate *EOT* for a buffered oxide layer (k = 3.9) and a thickness of 0.5 nm). (20%)

5. Consider an n-channel MOSFET with source and drain doping concentrations of $N_D = 10^{19}$ cm^{-3} and a channel region doping of $N_A = 3 \times 10^{16}$ cm^{-3}. Assume a channel length of L = 0.6 μm, $\varepsilon_s = 11.9 \times 8.85 \times 10^{-14}$ F/cm, $n_i = 1.5 \times 10^{10}$ cm^{-3} and assume the source and the body are at ground potential. Assuming the abrupt junction approximation please calculate the theoretical punch-through voltage. (20%)

國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：計算機結構【電機系碩士班丙組、庚組】

題號：4069

[Problem 1] Terminology Explanation (20%)
(a) Multi-core Processor (b) Muti-threading (c) Parallel Processing
(d) Branch Prediction (e) Superscalar Processor

[Problem 2] (a) Describe the definition of Amdahl's law. (5%)
(b) Suppose we enhance a machine making all floating-point instructions run 10 times faster. If the execution time of some benchmark before the floating-point enhancement is 80 seconds, what will the speedup be if three-forth of the 80 seconds are spent executing floating-point instructions? (15 %)

[Problem 3] A set associative cache has a block size of four 32-bit words and a set size of 4. The cache can accommodate a total of 16K words. The main memory size that is cacheable is 1M * 32 bits. Design the cache structure and show how the processor's addresses are interpreted. (20%)

[Problem 4] Suppose we are considering a change to an instruction set. The base machine is a load-store machine. Measurements of the load-store machine showing the instruction mix and clock cycle counts per instructions are given in the following table:

Instruction Type	Frequency	Clock Cycle Count
ALU Operations	40%	1
Loads	20%	4
Stores	15%	4
Branches	25%	2

Let's assume that 25% of the ALU operations directly use a loaded operand that is not used again.

We propose adding ALU instructions that have one source operand in memory. These new register-memory instructions have a clock cycle count of 4. Suppose that the extended instruction set increases the clock cycle count for branches by 1, but it does not affect the clock cycle time. Would this change improve CPU performance? Explain your answer. (20%)

[Problem 5] (a) Briefly describe the Delayed branch scheme. (5%)
(b) Give three delayed-branch scheduling strategies and the requirement of each strategy. Also, describe situations in which the three strategies improve performance. (15%)

國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：訊號與系統【電機系碩士班己組】

題號：4070
共 1 頁 第 1 頁

1. [10] Define the linear system. Give an <u>example</u> and a <u>counter-example</u> (反例)and then explain them.

2. [10] Define the time-invariant system. Give an <u>example</u> and a <u>counter-example</u>(反例) and then explain them.

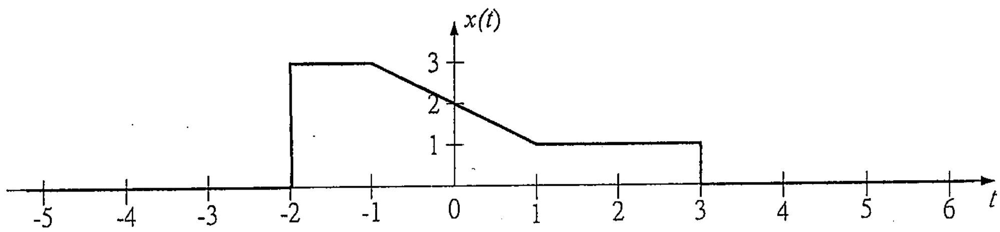
3. [10] Define the linear-time-invariant system. What is the advantage of this system in applications?

4. [10] Fourier Transform
   A. [5] Compute the Fourier transform of the signal Sin(10t + 1).
   B. [5] Draw the spectrum for the above signal too.

5. [10] Prove and explain the sampling theorem on signals. Please give an example.

6. [20] Discrete-Time Fourier Transform
   A. [5] Find the Fourier transform of the signal $x_1[n]=\frac{1}{3^n}u[n]$.
   B. [5] Using the result of A, find the Fourier transform of $x_2[n]=\frac{1}{3^n}u[n+3]$.
   C. [10] Using the result of A, find the Fourier transform of $x_3[n]=\frac{1}{3^{|n|}}$.

7. [30] Consider a continuous-time system $y(t)=x(t-2)+x(1-2t)$ with the input signal:



Please answer the following questions.
   A. [10] Determine whether or not the system is <u>memoryless</u>, <u>causal</u>, or <u>stable</u>. Please explain your reasons.
   B. [10] Decompose the input signal $x(t)$ as an even signal and an odd signal. That is, $x(t)= x_e(t)+x_o(t)$, where $x_e(t)$ is even and $x_o(t)$ is odd. Sketch $x_e(t)$ and $x_o(t)$ respectively.
   C. [10] Sketch the output signal $y(t)$.

Problem 1 (30%) Figure 1 shows a circuit to be controlled, where current u is the control input and voltage y is the output.

(a)(10%) Find the transfer function of the circuit.

(b)(10%) Estimate resistance R and capacitance C from the measured output waveform y in Fig. 2 when the input u is a unit step function.

(c)(10%) Suppose that we want to regulate the output y to 1 V and thus devise a control circuit as shown in Fig. 3 with the reference voltage r being set to 1 V. Please draw the corresponding block diagram of the circuit in Fig. 3 and choose from the following list of types of control it belongs to (multiple choices): feedforward control, feedback control, P control, I control, D control, PI control, PID control.

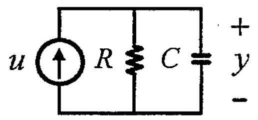


Fig. 1

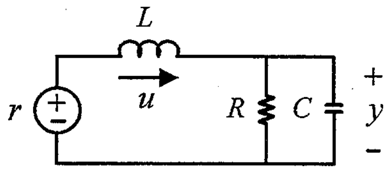


Fig. 3

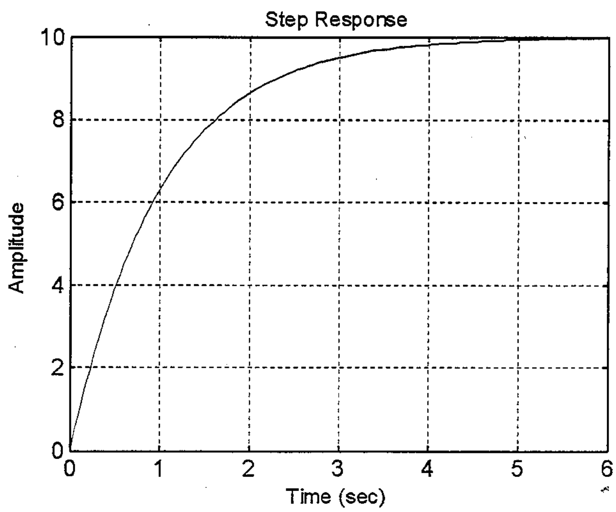


Fig. 2

Problem 2 (20%) A simple model of a car in motion is shown in Fig. 4, where the driving force f is 100 N, the air drag and friction is modeled as a linear damper with a damping coefficient b=10 N·s/m, and the car weighs m=100 kg.

(a)(10%) Derive a state-space model of the system with the car speed v as an output and the driving force f as an input.

(b)(10%) Determine the car speed using the state transition matrix under the assumption of zero initial state.

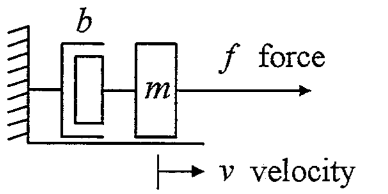


Fig. 4

Problem 3 (20%) Figure 6 shows the Bode diagram of the op-amp of Fig. 5.

(a) (10%) Given a sinusoidal input $u(t)=\sin(30t)$, estimate the steady state output $y(t)$.

(b) (10%) Use this op-amp to build a non-inverting amplifier as shown in Fig. 7. Is it stable? Estimate the phase margin of the resulting non-inverting amplifier.

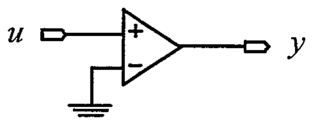


Fig. 5

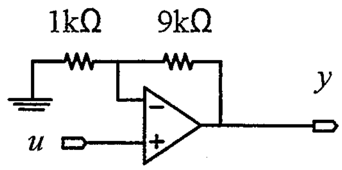


Fig. 7

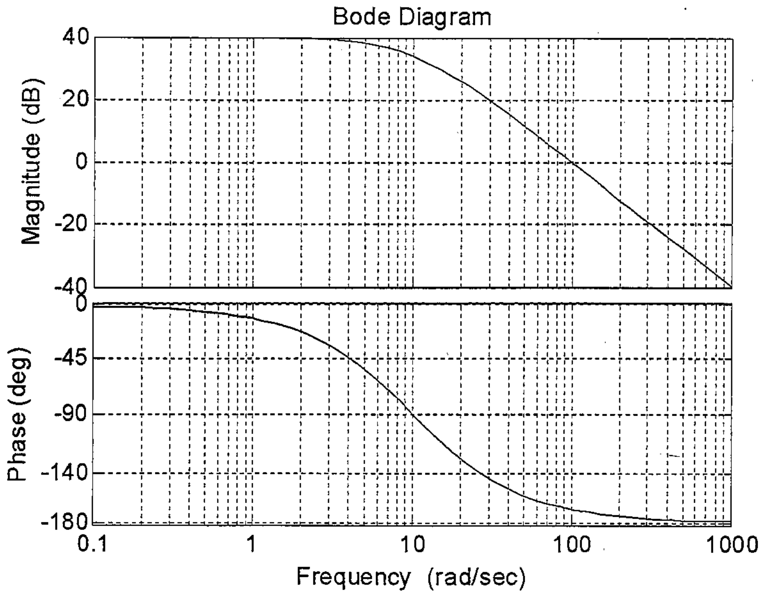


Fig. 6

Problem 4 (15%) Given a feedback system shown in Fig. 8.

(a) (5%) Determine the value of K to satisfy a damping ratio of 1.

(b) (10%) With K=1 and r=1, find the steady state regulation error e.

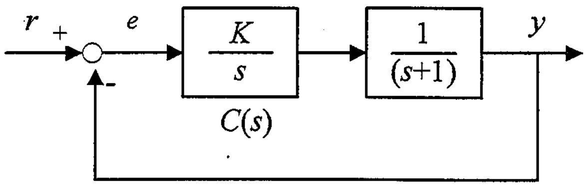


Fig. 8

Problem 5 (15%) A system is described by the following input-output relationship.

$$y(t)=\int_0^t e^{-2\tau}\sin(\tau)u(t-\tau)d\tau\,.$$

(a) (5%) Is it a linear system? Justify your answer.

(b) (10%) Determine the poles of the system.

1. Discuss four basic losses such as core loss that occur in AC machine and draw the power flow diagrams from P_{in} to P_{out} for a three-phase AC generator and a three-phase AC motor, respectively. (20%)

2. Describe the solution procedure of a power flow problem solved by a Gauss-Seidel technique. (including the procedure for swing bus, PV bus and PQ bus) (15%)

3. A 200 hp, 440 V, 0.8-PF-leading, Δ-connected synchronous motor has an armature resistance of $0.22\,\Omega$ and a synchronous reactance of $3.0\,\Omega$. Its efficiency at full load is 89%.
   (a) What is the input power, line current and phase current to the motor at rated conditions? (10%)
   (b) What is the internal generated voltage of the motor at rated conditions? (10%)

4. The fuel cost function in $/h for three thermal plants are given by
   $$C_1(P_1) = 625 + 7.3P_1 + 0.0025P_1^2$$
   $$C_2(P_2) = 345 + 7.2P_2 + 0.004P_2^2$$
   $$C_3(P_3) = 527 + 6.74P_3 + 0.003P_3^2$$
   where P_1, P_2 and P_3 are in MW. Assume that all three units operate economically to meet the total plant load of 450MW, find the incremental cost and the required output of each plant. (20%)

5. Fig. 1 shows a three-bus power system, calculate
   (a) Admittance matrix ($\boldsymbol{Y_{bus}}$). (10%)
   (b) Impedance matrix ($\boldsymbol{Z_{bus}}$). (10%)
   (c) If a balanced short-circuit fault occurred on bus 2, find the short-circuit current (The voltage at bus 2 is 1.0 p.u.). (5%)

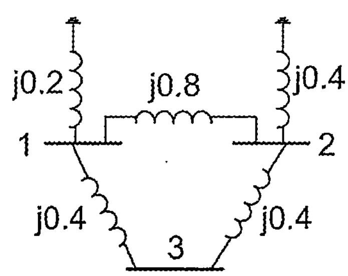


Fig. 1: Three-Bus Power System

國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：電磁學【電機系碩士班戊組、電波領域聯合】

題號：4073

共 1 頁 第 1 頁

1. (20%) A long straight conducting wire of radius a, length l and conductivity σ carries a direct current I. Find
   (a) the current density and the electric field intensity in the wire. (10%)
   (b) the magnetic field intensity at a distance $a/2$ from the axis of the wire. (5%)
   (c) the power loss. (5%)

2. (20%) The boundary condition for the tangential **H** is $\mathbf{a}_{n2} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$, where $\mathbf{a}_{n2}$ is the outward unit normal from medium 2 at the interface. Given in air a surface current density $\mathbf{J}_s = \mathbf{a}_x\, 3$ A/m flowing on the xy-plane, calculate
   (a) the 3 components B_x, B_y, B_z of the magnetic flux density **B** at (1,1,1). (15%)
   (b) the curl of **B** at (1,1,1). (5%)

3. (8%) Determine the polarization of the following electric fields:
   (a) $\mathbf{E} = \mathbf{a}_z E_0 \cos(\omega t - \beta y) + \mathbf{a}_x E_0 \sin(\omega t - \beta y)$ (2%)
   (b) $\mathbf{E} = \mathbf{a}_y E_0 \cos(\omega t + \beta x) + \mathbf{a}_z E_0 \sin(\omega t + \beta x)$ (2%)
   (c) $\mathbf{E} = \mathbf{a}_x E_0 \cos(\omega t - \beta y) - \mathbf{a}_z E_0 \sin(\omega t + \beta y)$ (2%)
   (d) $\mathbf{E} = \mathbf{a}_x E_0 \cos(\omega t - \beta y) + \mathbf{a}_z E_0 \cos(\omega t - \beta y)$ (2%)

4. (20%) Consider a rigid square conducting loop located in the xy-plane with its vertices at $(x, 1, 0)$, $(x, 3, 0)$, $(x + 2, 1, 0)$, and $(x + 2, 3, 0)$. A magnetic field given by
$$\mathbf{B} = \mathbf{a}_z B_0 \cos \pi(x - v_0 t) \ \ \text{Wb/m}^2$$
   exists in the space.
   (a) Find the expression for the emf induced around the loop. (10%)
   (b) What would be the induced emf if the loop is moving with the velocity $\mathbf{v} = \mathbf{a}_x v_0$ m/s instead of being stationary? (10%)

5. (12%) An electromagnetic wave from an underwater source with perpendicular polarization is incident on a water-air interface at $_i = 30$. Using $_r = 81$ and $_r = 1$ for water, find
   (a) critical angle $_c$, (3%)
   (b) reflection coefficient $_\perp$, (3%)
   (c) transmission coefficient $_\perp$, (3%)
   (d) attenuation for each wavelength into the air. (3%)

6. (20%) This problem goes through the procedure to find the TE modes inside a metallic rectangular waveguide of dimensions $a \times b$, with $a > b$.
   (a) First, write down the time-harmonic Maxwell's equations. (4%)
   (b) Derive the equations expressing the transverse components of electric and magnetic fields in terms of the longitudinal components. (8%)
   (c) Solve the longitudinal component from the Helmholtz equation and related boundary conditions, and then the rest of the field components from the longitudinal component. (8%)