國立中山大學100學年度碩士班招生考試試題

科目：工程數學甲【電機系碩士班甲組、丙組選考、丁組、戊組、庚組】

Problem 1 Multiple Choice (30%)

Instructions:

- There are 10 questions, each of which is associated with 4 possible responses.
- For each of questions, select **ONE most appropriate** response.
- For each response you provide, **you will be awarded 3 marks if the response is correct and -3 marks if the response is incorrect (答錯一題倒扣三分).**
- You get 0 mark if no response is provided.

(1.1) What is the solution of the ODE $\frac{dy}{dx} = 2xy^2$ for which y(0)=1?

(A) $y(x) = (1+x)^{-1}$. (B) $y(x) = (1-x^2)^{-1}$. (C) $y(x) = e^{-x}$. (D) $y(x) = 1 + \sqrt[3]{x^4}$.

(1.2) What is the amplitude of the sinusoidal solution of $\frac{dx}{dt} + 2x = 5\sin(3t)$?

(A) $\frac{2}{5}$ (B) 1 (C) $\sqrt{7}$ (D) $\frac{5}{\sqrt{13}}$

(1.3) Let $\boldsymbol{L}[\cdot]$ denotes the Laplace transform.

(A) The Laplace transform is a linear operation.

(B) If $\boldsymbol{L}[f(t)] = F(s)$, then $\boldsymbol{L}[f(t-1)] = F(s-1)$.

(C) $\boldsymbol{L}[(t-2)] = e^{-s}/s$.

(D) None of the above statements is FALSE.

(1.4) What is the general solution to the ODE $t\frac{dx}{dt} = 4t - 3x$?

(A) $x(t) =$ constant. (B) $x(t) = -ct^3$. (C) $x(t) = ct^{-3} + t$. (D) $x(t) = -ce^{3t} + 4t$.

(1.5) Let $\boldsymbol{L}[\cdot]$ denotes the Laplace transform.

(A) $\boldsymbol{L}[t^{-0.5}] = \sqrt{\frac{\pi}{s}}$.

(B) If $\boldsymbol{L}[f(t)] = F(s)$, then $\boldsymbol{L}\left[f\left(\frac{t}{a}\right)\right] = F(as)$.

(C) If $\boldsymbol{L}[f(t)] = F(s)$, then $\boldsymbol{L}\left[\frac{df}{dt}(t)\right] = sF(s)$.

(D) All of the above statements are TRUE.

(1.6) Define the *del operator* $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$, and consider $\varphi(x,y,z) = (\cos z^3)e^{\sqrt{x+y}}$.

(A) $\nabla \cdot \nabla\varphi = \varphi$. (B) $\nabla \times \nabla\varphi = 0$. (C) $\mathbf{R} \cdot \nabla\varphi = 0$, where $\mathbf{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

(D) None of the above statements is TRUE.

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科目：工程數學甲【電機系碩士班甲組、丙組選考、丁組、戊組、庚組】

(1.7) Consider the del operator defined in (1.6).

(A) $\nabla\cdot(\nabla\times\mathbf{F})=0$, where $\mathbf{F}$ is a vector field with continuous first and second derivatives.

(B) $\nabla\cdot(\mathbf{F}\times\mathbf{G})=\mathbf{G}\cdot(\nabla\times\mathbf{F})-\mathbf{F}\cdot(\nabla\times\mathbf{G})$, where $\mathbf{F}$ and $\mathbf{G}$ are two smooth vector fields.

(C) $\nabla\cdot(\varphi\mathbf{F})=\nabla\varphi\cdot\mathbf{F}+\varphi(\nabla\cdot\mathbf{F})$, where φ and $\mathbf{F}$ are smooth scalar and vector fields, respectively.

(D) All of the above statements are TRUE.

(1.8) Consider the ODE $\ddot{x}+x^2\dot{x}+x(x^2-1)=0$.

(A) This is a linear ODE.

(B) This is a time-varying ODE.

(C) This ODE has three equilibria.

(D) The equilibria of this ODE are ±1 and ±2.

(1.9) Consider the heat equation

$$\frac{\partial u}{\partial t}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t)\qquad \forall 0<x<1,t>0 \qquad (1.9.1)$$

$$u(0,t)=u(1,t)=0\qquad \forall t>0 \qquad (1.9.2)$$

$$u(x,0)=f(x)\qquad \forall 0<x<1 \qquad (1.9.3)$$

(A) Without considering the boundary condition (1.9.3), a general solution to the heat equation is $u(x,t)=\sum_{n=1}^{\infty}C_n\sin(n\pi x)\,e^{-4n^2\pi^2 t}$.

(B) Suppose $f(x)=7\sin(3\pi x)$. Then $u(x,t)=7\sin(3\pi x)\,e^{-4n^2\pi^2 t}$.

(C) Both of the above statements are TRUE.

(D) None of the above statement is TRUE.

(1.10) Consider the wave equation

$$\frac{\partial^2 w}{\partial t^2}(x,t)=3\frac{\partial^2 w}{\partial x^2}(x,t)\qquad \forall 0<x<1,t>0 \qquad (1.10.1)$$

$$w(0,t)=w(1,t)=0\qquad \forall t>0 \qquad (1.10.2)$$

$$w(x,0)=f(x)\qquad \forall 0<x<1 \qquad (1.10.3)$$

$$\frac{\partial w}{\partial t}(x,0)=0\qquad \forall 0<x<1 \qquad (1.10.4)$$

(A) Without considering the boundary condition (1.10.3), a general solution to the wave equation is $w(x,t)=\sum_{n=1}^{\infty}C_n\sin(n\pi x)\cos(n\pi t)$.

(B) Suppose $f(x)=17\sin(9\pi x)$. Then $w(x,t)=17\sin(9\pi x)\,(\sin(27\pi t)+1)$.

(C) Both of the above statements are TRUE.

(D) None of the above statement is TRUE.

國立中山大學100學年度碩士班招生考試試題

科目：工程數學甲【電機系碩士班甲組、丙組選考、丁組、戊組、庚組】

Problem 2 (15%)

Consider the vector function $\mathbf{F}(x, y) = (y^3 - 12y)\mathbf{i} + (15x - x^3)\mathbf{j}$.

(2.1) (10%) Find the simple closed curve C for which the integral

$$\oint_C \mathbf{F} \cdot d\mathbf{r}$$

(with positive orientation) will have the largest positive value. (Hint: Use Green's Theorem)

(2.2) (5%) Compute this largest positive value.

Problem 3 (13%)

This problem has two sub-problems. **Please give your answers in details.**

(a) Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \in \mathbb{R}^{2\times 2}$ and define a scalar-valued function $f_A(\mathbf{x}, \mathbf{y}) := \mathbf{x}^T A \mathbf{y}$ for any $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$. If $f_A(\mathbf{x}, \mathbf{y})$ is an inner product on $\mathbb{R}^2$, then what conclusions can be made on all the entries a_{ij}? (6%)

(b) Consider the inner product space $\left(\mathbb{R}^{2\times 2}, \langle \cdot, \cdot \rangle\right)$ with $\langle A, B \rangle := trace(AB^T)$ defined for matrices A and B in $\mathbb{R}^{2\times 2}$. Let S be the subspace of $\left(\mathbb{R}^{2\times 2}, \langle \cdot, \cdot \rangle\right)$ defined as $S := \left\{ A \in \mathbb{R}^{2\times 2} \,\middle|\, A = A^T \right\}$. Let E denote the standard basis for S and F denote an orthonormal basis for S that is derived from basis E. What are bases E and F? (7%)

Problem 4 (12%)

Given real numbers a_i, b_i, c_i for $i = 1, 2$, let L be a transformation from V to W, with $V = \mathbb{R}^2 = W$, defined by

$$L(\mathbf{r}) := \begin{bmatrix} a_1 r_1 + b_1 r_2 + c_1 \\ a_2 r_1 + b_2 r_2 + c_2 \end{bmatrix}, \quad \forall\, \mathbf{r} := \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \in \mathbb{R}^2.$$

Suppose that c_1 and c_2 are of the values so that $L: V \to W$ is linear.

以下小題僅需依序寫下答案即可，不需做任何推導。

(a) Let E be the standard basis for $\mathbb{R}^2$. Find the matrix A representing L with respect to basis E for both V and W. (5%)

(b) Let $F := \{\mathbf{f}_1, \mathbf{f}_2\}$ be another basis for $\mathbb{R}^2$ and let Q denote the matrix representation of L with respect to basis F for V and basis E for W, respectively. Denote $P := \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$. Please write the algebraic relationship between Q, P, $\mathbf{f}_1$, and $\mathbf{f}_2$. (7%)

Problem 5 (15%)

Compute the least upper bound of the integral

$$\left| \int_C (e^z - \bar{z})\, dz \right|$$

where z is a complex variable, $\bar{z}$ is its complex conjugate, and C denotes the boundary of a triangle with vertices at the points $i3$, -4 and 0, oriented in counterclockwise direction.

Problem 6 (15%)

Let $F(\omega)$ be the Fourier transform of $f(t)$. Compute $\mathcal{F}(i \cdot t \cdot f(t))$, where $\mathcal{F}$ stands for the Fourier transform and $i = \sqrt{-1}$. Write down you answer in terms of $F(\omega)$, and each calculation step is also required.

國立中山大學100學年度碩士班招生考試試題

科目：電子學【電機系碩士班甲組、乙組、戊組】

1. (25%) Consider a source follower such as that in Fig. 1 Specifically,
   (a) Please derive R_{in}, A_{vo}, A_v, and R_o with r_o taken into account. (4*5%)
   (b) Please derive the overall small-signal voltage gain G_v with r_o taken into account. (1*5%)

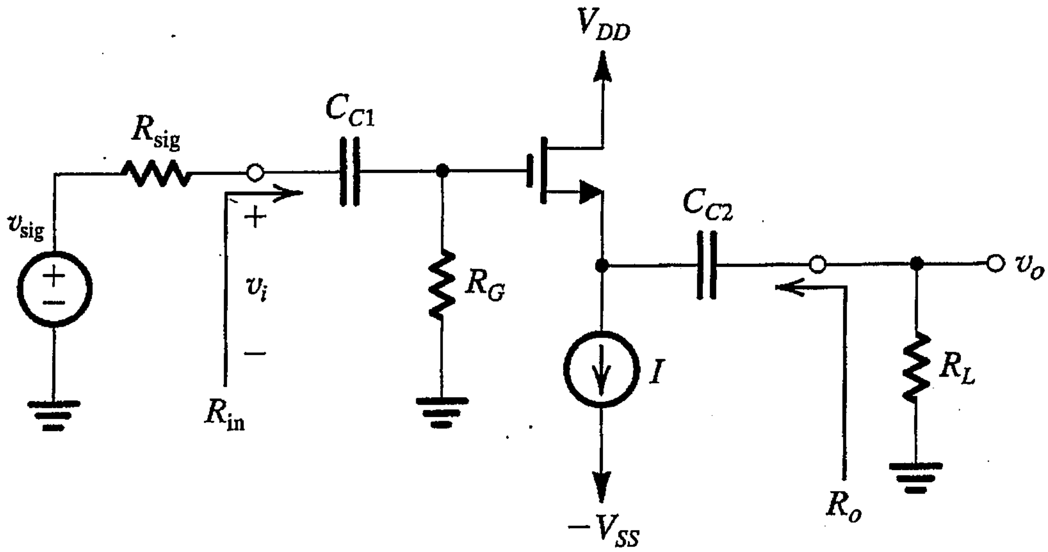

Fig. 1

2. (20%) Please find the three break frequencies and the low *3dB* frequency of the common emitter amplifier with an emitter resistance as shown in Fig. 2. (4*5%)

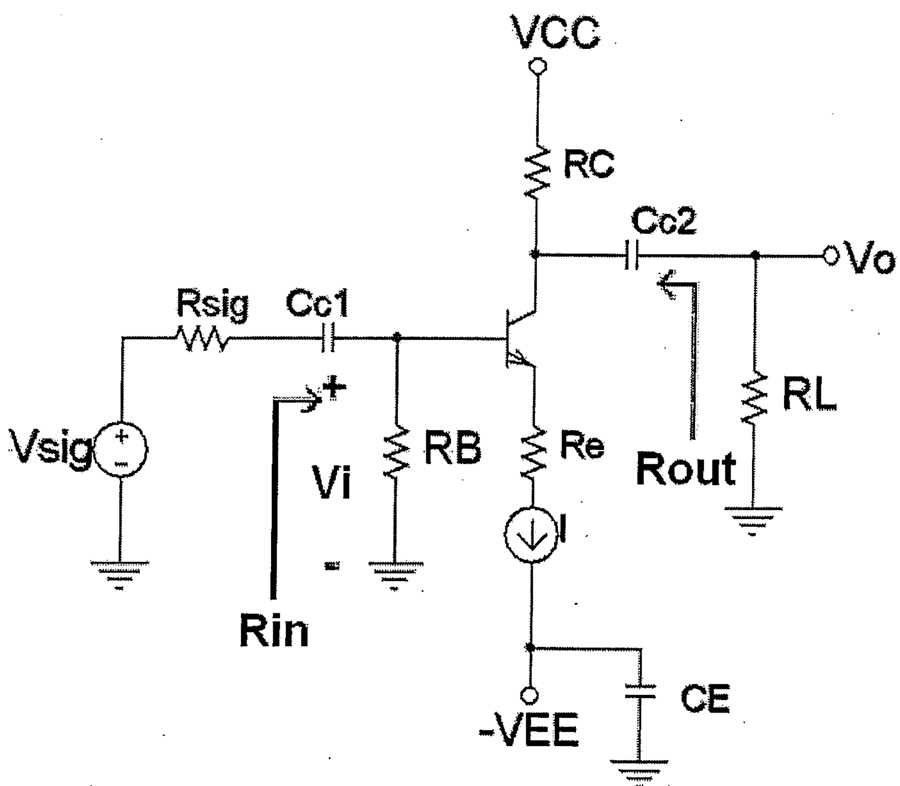

Fig. 2

國立中山大學100學年度碩士班招生考試試題

科目：電子學【電機系碩士班甲組、乙組、戊組】

3. (20%) Design and draw a three input CMOS NAND and CMOS NOR gates respectively. Also, please find out the $(W/L)_P/(W/L)_N$ ratios of the designed CMOS NAND and CMOS NOR gates respectively so that their PMOS part and NMOS part have same MOS transconductance parameters, where W/L is aspect ratio. (2*5%, 1*10%)

4. (15%) In Fig. 4, give that the input resistance is 10 KΩ and the differential voltage gain is 100, please find out (a) $R_1 = R_3 = ?$, and (b) $R_2 = R_4 = ?$ (2*7.5%)

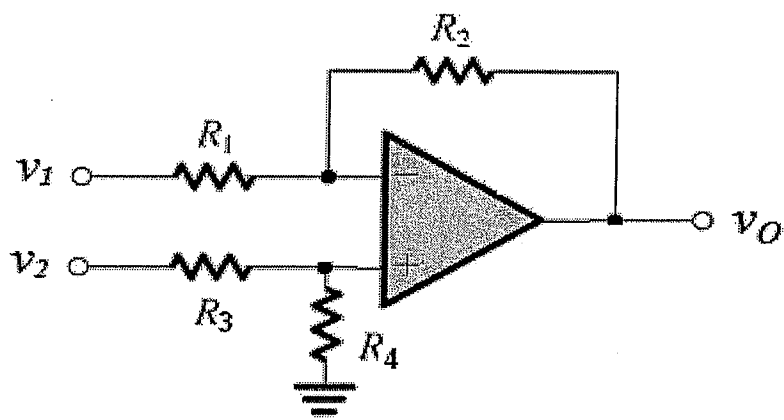

Fig. 4

5. (20%) Fig. 5 shows an operation amplifier and its small equivalent circuit. Please derive its output resistance R_o and its dc open-loop gain A_v with respect to the circuit transistor's g_m and r_o. (2*10%)

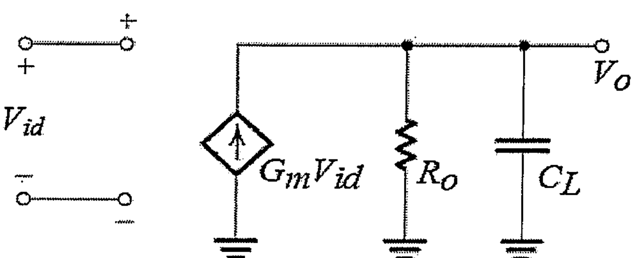

Fig 5.(b)

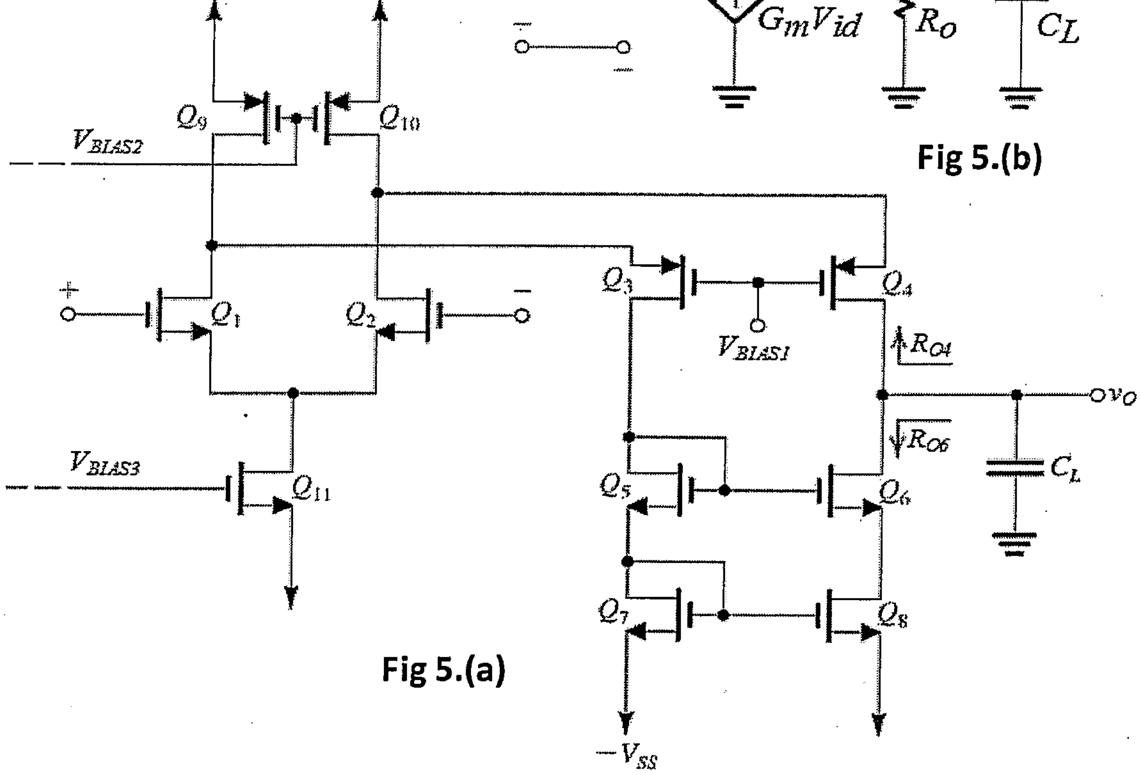

Fig 5.(a)

國立中山大學100學年度碩士班招生考試試題

科目：半導體概論【電機系碩士班甲組】

1. The lattice constant of Ge is 5.65× 10^{-8} cm for a diamond crystal structure. Calculate

   (a) the distance from the center of one Ge atom to the center of its nearest neighbor. (10%)

   (b) the number density of Ge atoms on <110> plane (# per cm^2) (10%)

2. An Au-n-GaAs Schottky Contact is at T=300K^o with N_d =2× 10^{16} cm^{-3} (ϕ_m =5.1 Volt, χ =4.07 Volt, N_c =4.7× 10^{17} cm^{-3} , $\epsilon = 13.1\epsilon_o$,$\epsilon_o = 8.85 \times 10^{-14}$ F/cm). Calculate

   (a) the depletion region width for a revrese bias voltage of 0.5V. (10%)

   (b) the maximum electric filed in the above condition .(10%)

3. Consider the p-n-p bipolar junction transistor with base width W_b . The base doping concentration is N_d and the base hole diffusion coefficient is D_p . The emitter doping concentration is N_a, the emitter electron diffusion coefficient is D_n, and the emitter width is W_e which is much smaller than the electron diffusion length in the emitter. Derive the expression of the emitter injection efficiency γ. (20%)

4. A MOS transistor is fabricated on a p-type silicon substrate with N_a =3× 10^{15} cm^{-3} . The oxide thickness is t_{ox} =600× 10^{-8} cm and the equivalent fixed oxide charge is Q'_{SS} =1.5× 10^{11} cm^{-2} . Calculate the threshold voltage when the source/bulk bias voltage V_{SB} is equal to 0.6 V for an n^+ -polysilicon gate. (Si : $n_i = 1.5 \times 10^{10} cm^{-3}$, $\epsilon_{Si} = 11.8\ \epsilon_o, \epsilon_{SiO_2} = 3.9\ \epsilon_o$, $\epsilon_o = 8.85 \times 10^{-14}\ F/cm$,$E_g$= 1.12eV . Note: kT/q = 0.0259 V,$q = 1.6 \times 10^{-19} C$) (20%)

5. A direct semiconductor has the recombination rate R=α (pn-n_i^2) where α =1× 10^{-8} cm^3/s and n_i = 10^{10} cm^{-3} . The semiconductor is doped with N_d =2× 10^{15} cm^{-3}. The sample is uniformly exposed to a steady optical generation rate of g_{op} = 1× 10^{22} EHP/cm^3 -s. For this excitation, calculate the electron concentration n. (20%)

國立中山大學100學年度碩士班招生考試試題

科目：工程數學乙【電機系碩士班乙組】

Problem 1 Multiple Choice (15%)

Instructions

- There are 5 questions, each of which is associated with 4 possible responses.
- For each question, select **ONE most appropriate response.**
- For each response you select, you will be awarded 3 marks if the response is most appropriate and **-3 marks if the response is not** (答錯一題倒扣三分).
- You get 0 mark if no response is provided.

(1.1) Let $\mathfrak{L}[\cdot]$ denotes the Laplace transform. (3%)

(A) $\mathfrak{L}[\cdot]$ obeys the superposition principle.

(B) If $\mathfrak{L}[f(t)] = F(s)$, then for any positive constant a, $\mathfrak{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$.

(C) If $\mathfrak{L}[f(t)] = F(s)$, then $\mathfrak{L}[\dot{f}(t)] = sF(s) - f(0)$.

(D) All of the above statements are TRUE.

(1.2) Let $\mathfrak{L}^{-1}[\cdot]$ denotes the inverse Laplace transform. (3%)

(A) $\mathfrak{L}^{-1}[\cdot]$ does not obey the superposition principle.

(B) If $\mathfrak{L}^{-1}[F(s)] = f(t)$, then $\mathfrak{L}^{-1}[F(s+a)] = \mathrm{e}^{-at}f(t)$.

(C) If $\mathfrak{L}^{-1}[F(s)] = f(t)$ and $\mathfrak{L}^{-1}[G(s)] = g(t)$, then $\mathfrak{L}^{-1}[F(s)G(s)] = f(t)g(t)$.

(D) All of the above statements are FALSE.

(1.3) What is the general solution to the ODE $t\dot{x}(t) = 4t - 3x(t)$? (3%)

(A) $x(t) = t + ct^{-3}$. (B) $x(t) =$ constant. (C) $x(t) = -c\mathrm{e}^{3t} + 4t$.

(D) None of the above.

(1.4) What is the amplitude of the sinusoidal solution of $\ddot{x}(t) + 2\dot{x}(t) + x(t) = \sin(3t)$? (3%)

(A) 1 (B) $\frac{1}{3}$ (C) $\frac{1}{10}$ (D) $\frac{1}{100}$

(1.5) Consider the ODE $\ddot{x}(t) + \dot{x}(t) + x(t)(1 - x(t)^2) = 0$. (3%)

(A) This is a nonlinear ODE.

(B) This ODE has three equilibria 0, ± 1.

(C) If the initial conditions are small enough, the solution of this ODE converges to 0.

(D) All of the above statements are TRUE.

國立中山大學100學年度碩士班招生考試試題

科目：工程數學乙【電機系碩士班乙組】

Problem 2 (10%)

(2.1) (8%) Consider the following differential equation defined on the nonnegative real axis:

$$\dot{y}(t) + 2y(t) = u(t), \quad y(0) = 0, \tag{1}$$

where

$$u(t) = \begin{cases} 0 & \text{if } 0 \le t < 1 \\ -1 & \text{if } 1 \le t \le 2 \\ t-3 & \text{if } 2 \le t \le 5 \\ 2 & \text{if } t \ge 5 \end{cases}$$

See Figure 1 for an illustration. Find $y(t)$ that satisfies equation (1).

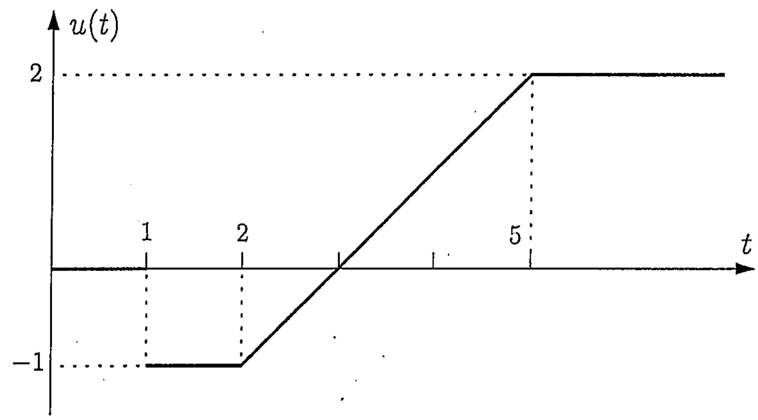

Figure 1: Time history of function u.

(2.2) (2%) Calculate the peak value and the steady state value of y that satisfies equation (1).

Problem 3 (25%) This problem has three sub-problems. **Please give your answers in details.**

(3.1) (6%) Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \in \mathbb{R}^{2\times 2}$ and define a scalar-valued function $f_A(\mathrm{x}, \mathrm{y}) := \mathrm{x}^T A \mathrm{y}$ for any x^T, $\mathrm{y} \in \mathbb{R}^2$. If $f_A(\mathrm{x}, \mathrm{y})$ is an inner product on $\mathbb{R}^2$, then what conclusions can be made on all the entries a_{ij}?

(3.2) (7%) Consider the inner product space $(\mathbb{R}^{2\times 2}, \langle\cdot,\cdot\rangle)$ with $\langle A, B\rangle := trace(AB^T)$ defined for matrices A and B in $\mathbb{R}^{2\times 2}$. Let S be the subspace of $(\mathbb{R}^{2\times 2}, \langle\cdot,\cdot\rangle)$ defined as $S := \left\{A \in \mathbb{R}^{2\times 2} | A = A^T\right\}$. Let E denote the standard basis for S and F denote an orthonormal basis for S that is derived from basis E. What are bases E and F?

(3.3) (12%) Let V be an inner product space of dimension n with elements defined over the field $\mathbb{R}$, and let $B = \{\mathrm{v}_1, \cdots, \mathrm{v}_n\}$ be an ordered basis for V. Moreover, let p and q be any two vectors in V. Define $\Phi \in \mathbb{R}^{n\times n}$ by $\phi_{ij} := \langle \mathrm{v}_j, \mathrm{v}_i\rangle$ for each i and j. Derive an equality to describe the relationship between the coordinate vectors $[\mathrm{p}]_B$ and $[\mathrm{q}]_B$, the matrix Φ, and the inner product $\langle \mathrm{p}, \mathrm{q}\rangle$.

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科目：工程數學乙【電機系碩士班乙組】

Problem 4 (20%) Given real numbers a_i, b_i, c_i for $i = 1, 2$, let L be a transformation from V to W, with $V = \mathbb{R}^2 = W$, defined by

$$L(\mathrm{r}) := \begin{bmatrix} a_1 r_1 + b_1 r_2 + c_1 \\ a_2 r_1 + b_2 r_2 + c_2 \end{bmatrix}, \quad \forall\, \mathbf{r} := \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \in \mathbb{R}^2.$$

Suppose that c_1 and c_2 are of the values so that $L : V \mapsto W$ is linear.

以下小題僅需依序寫下答案即可, 不需做任何推導.

(4.1) (5%) Let E be the standard basis for $\mathbb{R}^2$. Find the matrix A representing L with respect to basis E for both V and W.

(4.2) (7%) Let $F := \{\mathrm{f}_1, \mathrm{f}_2\}$ be another basis for $\mathbb{R}^2$ and let Q denote the matrix representation of L with respect to basis F for V and basis E for W, respectively. Denote $P = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$. Please write the algebraic relationship between Q, P, f_1, and f_2.

(4.3) (8%) Now let $a_1 = 1 = b_1 = b_2$ and $a_2 = 0$, i.e. let $P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Find the set of all ordered bases $G := \{\mathrm{g}_1, \mathrm{g}_2\}$ for $\mathbb{R}^2$ such that the matrix representation of L with respect to basis G for both V and W is exactly the matrix P.

Problem 5 (15%) Compute the least upper bound of the integral

$$\left| \int_C (\mathrm{e}^z - \bar{z})\mathrm{d}z \right|,$$

where z is a complex variable, $\bar{z}$ is its complex conjugate, and C denotes the boundary of a triangle with vertices at the points $i3$, -4 and 0, oriented in counterclockwise direction.

Problem 6 (15%) Let $F(\omega)$ be the Fourier transform of $f(t)$. Compute $\mathfrak{F}(i \cdot t \cdot f(t))$, where $\mathfrak{F}$ stands for the Fourier transform, and $i = \sqrt{-1}$. Write down you answer in terms of $F(\omega)$, and each calculation step is also required.

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科目：控制系統【電機系碩士班乙組】

Problem 1 (20%) A circuit made of a non-inverting amplifier and a device G is displayed in Fig. 1, and the device G has the Nyquist plot in Fig. 2. Assume the op-amp is ideal. Determine R to make the circuit stable with the gain margin of 20 dB.

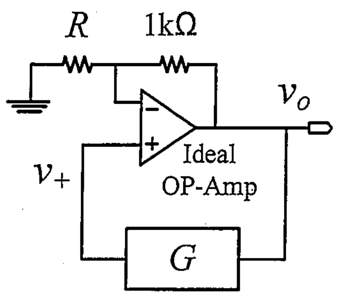

Fig. 1

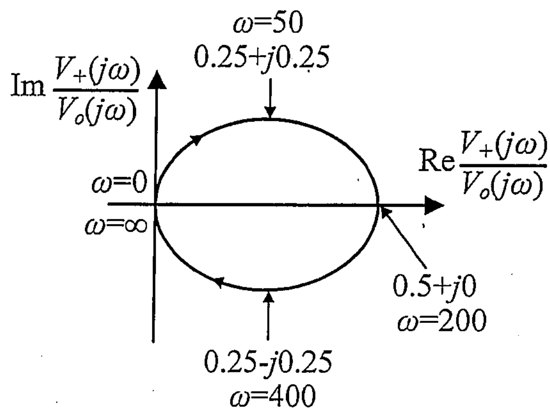

Fig. 2

Problem 2 (45%) Figure 3 shows a circuit to control the angular position θ of the motor. The position θ is measured and converted to the voltage of equal magnitude, and fed back through the circuit. Assume that the op-amp is ideal, and that the frequency response $P(j\omega)$ of the motor from the current i to the position θ is plotted in Fig. 4.

(a) (15%) Roughly estimate the parameters k, a and b of the motor's transfer function $P(s) = k/[(s+a)(s+b)]$, according to the Bode plot in Fig. 4.

(b) (15%) Determine R so that the control system is stable with the phase margin of about 50 degree.

(c) (15%) Determine the steady-state value of θ, given $R = 1\Omega$.

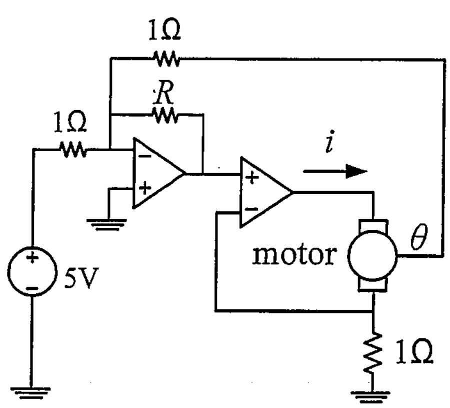

Fig. 3.

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科目：控制系統【電機系碩士班乙組】

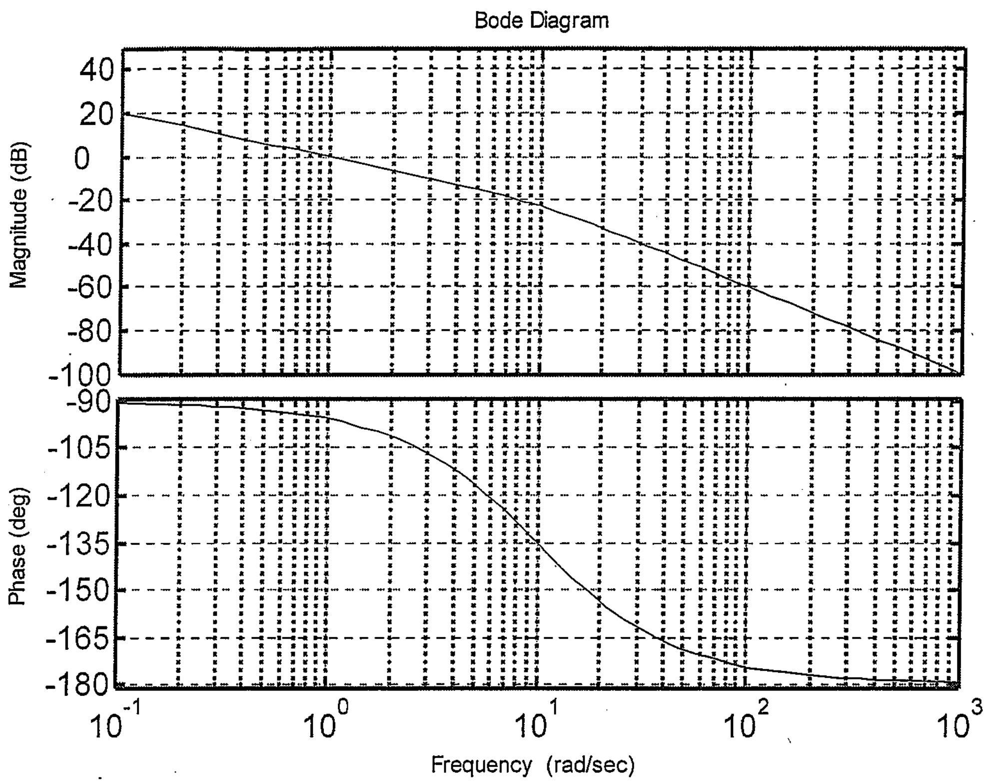


Fig. 4.

Problem 3 (25%) Find if the following controllers would stabilize the plant P in Fig. 5. Note: All answers need justifications or no scores will be given.

(a) (5%) $C(s)=\dfrac{-s+2}{s+2}$, (b) (5%) $C(s)=\dfrac{s+2}{s}$, (c) (5%) $C(s)=\dfrac{4s+2}{s-2}$,

(d) (5%) $C(s)=\dfrac{-0.5s+2}{s}$, (e) (5%) $C(s)=5$.

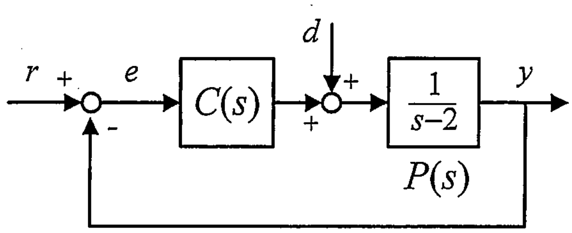


Fig. 5

Problem 4 (10%) A function y is expressed in terms of the following linear convolution. Is it bounded for $t>0$? Justify your answer.

$$y(t)=\int_0^t \sin(\tau)\cos(t-\tau)d\tau .$$

國立中山大學100學年度碩士班招生考試試題

科目：離散數學【電機系碩士班丙組選考】

1. Define or explain the following terms: [20%]
   (a) The well-ordering principle
   (b) The fundamental theorem of arithmetic
   (c) The four color theorem
   (d) A relation on a set

2. For a group with n persons prove that at least two of them know the same number of persons in the group. Assume that if A knows B then B also knows A.
   (Hint: Use a node to represent a person. If two persons know each other, the corresponding nodes are connected with an edge. Consider the situation of the degree of each node.) [10%]

3. Let $A=\{1,2,3,4,5,6,7\}$. For each of the following values of r, determine an equivalence relation $\mathfrak{R}$ on A with $|\mathfrak{R}| = r$, or explain why no such relation exists. [15%]
   (a) $r = 20$ (b) $r = 11$ (c) $r = 31$.

4. Prove that $K_{3,3}$ is nonplanar. [10%]

5. For $A = \{a, b, c, d, e\}$, the Hasse diagram for the poset $(A, \mathfrak{R})$ is shown in the Fig. 1.
   (a) Determine the relation matrix for $\mathfrak{R}$. [5%]
   (b) Topologically sort the poset $(A, \mathfrak{R})$. [5%]

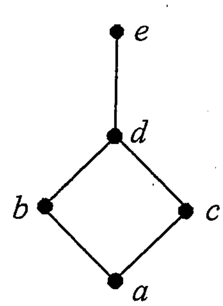


Fig. 1

6. Solve the following recurrence relation. [10%]
   $a_n - 6a_{n-1} + 9a_{n-2} = 0,\ n \geq 2,\ a_0 = 5,\ a_1 = 12$

7. For the graph in Fig. 2,
   (a) Find the chromatic polynomial by decomposition theorem. **(Note: Do not answer only by inspection.)** [10%]
   (b) What is its chromatic number and why? [5%]
   (c) Draw the dual graph of it. [10%]

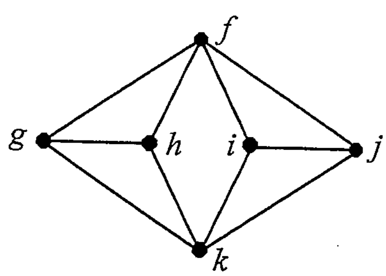


Fig. 2

國立中山大學100學年度碩士班招生考試試題

科目：資料結構【電機系碩士班丙組選考】

1. (a) [5 points] Given an input size n, where n is a positive integer, we assume that a program requires the running time $\Theta(f(n))$. State the *formal* definition of $\Theta(f(n))$.

   (b) [10 points] Given an input size n, where n is a positive integer, we assume that the program requires the running time $T(n)=\Theta(f(n))$, where

$$T(n)=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}.$$

   Derive $f(n)$ in the *simplest* formula.

2. (a) [5 points] The sequence F_n of Fibonacci numbers is defined as follows.

$$F_n=\begin{cases}1 & \text{if } n=0 \text{ or } n=1,\\ F_{n-1}+F_{n-2} & \text{if } n\geq 2.\end{cases}$$

   The FIBONACCI NUMBER PROBLEM is defined as "Given an integer $n\geq 0$, output the n-th Fibonacci number F_n." The following function **`Fib(int n)`** can solve the FIBONACCI NUMBER PROBLEM.

```
int Fib(int n) {
  if ((n==0) || (n==1))
    return 1;
  else
    return Fib(n-1)+Fib(n-2);
}
```

   Prove that the running time $T(n)$ of **`Fib(n)`** is larger than $\left(1+\frac{1}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^{n-2}$.

   (b) [5 points] Now, please tell us whether the FIBONACCI NUMBER PROBLEM is NP-complete? Explain your reasons. (Note that you will get 0 points if you do not present any reasons.)

國立中山大學100學年度碩士班招生考試試題

科目：資料結構【電機系碩士班丙組選考】

3. (a) [4 points] Given an unsorted integer array of size n, does the binary search algorithm perform better than the sequential search algorithm? Use the big-O notation to justify your answer.

(b) [10 points] Given an integer array of size n, show that any comparison-based sorting algorithm requires a running time of $\Omega(n \log n)$ in the worst case.

(c) [10 points] Given an unsorted integer array **A[n]** of size n, the following shows the quick sort algorithm, where we assume that the function **medium(array A)** can return the *medium* from the integer array **A[n]** in $\Theta(n)$ time. Note that given a set of n elements, the median is defined as the $\lceil n/2 \rceil$ largest element in that set.

Derive the worst case running time of **quick_sort(array A)** in terms of Θ notation. (Note that you will get 0 points if you just give the answer directly.)

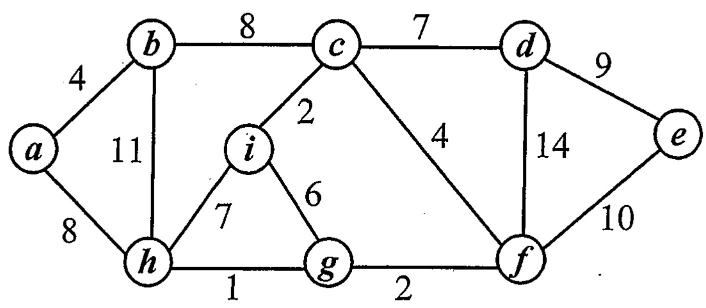
```
quick_sort(array A) {
  int x;
  if ( size[A] == 0 )
    return;
  // That is, if array A contains no element, do nothing.
  x = medium(A);
  S = { y | y∈A and y≤x };
  L = { z | z∈A and z>x };
  quick_sort(S);
  print x;
  quick_sort(L);
}
```

(d) [5 points] Now, suppose that we want to sort an integer array **B[1024]**. Derive the worst case running time of **quick_sort(B)** in terms of Θ notation. (Note that you will get 0 points if you just give the answer directly.)

國立中山大學100學年度碩士班招生考試試題

科目：資料結構【電機系碩士班丙組選考】

4. Consider the following weighted graph.



(a) [5 points] Star from the vertex ***a*** and use Prim's algorithm to find the minimum cost spanning tree. Show the actions step by step.

(b) [5 points] Use Kruskal's algorithm to find the minimum cost spanning tree. Show the actions step by step.

5. [12 points] Complete the following notation translations.

Infix	Prefix	Postfix
`a*(b+c*d)/e-f`		
		`ab/cd+e-*fg-+`
	`*/a-*bc+de-fg`	

6. Insert a sequence of keys {`30, 43, 14, 20, 47, 25, 55, 40, 51, 6, 35`}, in that order, into a data structure which has no keys initially.
(a) [5 points] Construct a binary search tree for that sequence.
(b) [5 points] Construct an AVL tree for that sequence.
(c) [5 points] Construct a heap tree for that sequence. Note that, in the question 6(b), we require that the root must have the maximum key value.

7. Given a hash table of size 11 (assuming that the hash table starts with the index 0), use the function $h(key) = (2 \times key + 5) \bmod 11$ to hash the following keys: **14, 43, 17, 81, 23, 91, 19, 20, 65,** and **8**. Draw the results with two different ways of handling collisions.
(a) [4 points] Collisions are handled by chaining.
(b) [5 points] Collisions are handled by linear probing.

科目：數位電路【電機系碩士班丙組選考、庚組】

[Problem 1] Please use one 4-bit adder, one 2-bit adder, and a few logic gates (AND, OR, or NOT) to implement a one-digit BCD adder with one carry-in bit and one carry-out bit. Please note that the details of the adders are no need to show. (20%)

[Problem 2] In addition to the BCD code, 2-4-2-1 code listed below is also a useful coding to represent decimal digits in self complementing manner. Please design a function F to check if the decimal input encoded by 2-4-2-1 code can be exactly divided by three. In other words, $F = 1$ if and only if the reminder of the division by 3 is zero. Make the truth table of this function and design the two-level NOR-NOR network with minimum number of logic gates and literals. Note that the input code words (a, b, c, d) and their complements can be used directly as fan-in in the logic circuit. (15%)

Table 1

Decimal digit	8-4-2-1 Code (BCD)	2-4-2-1 Code
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
5	0101	1011
6	0110	1100
7	0111	1101
8	1000	1110
9	1001	1111

[Problem 3] The sequential circuit in figure 1 has one input signal X that is synchronized with the clock and one output signal Z.

(a) Complete the state table. (6%)

(b) It is known that this circuit is a sequence detector and is initialized to $AB = 00$ when powered up. What sequence(s) does it detect? (9%)

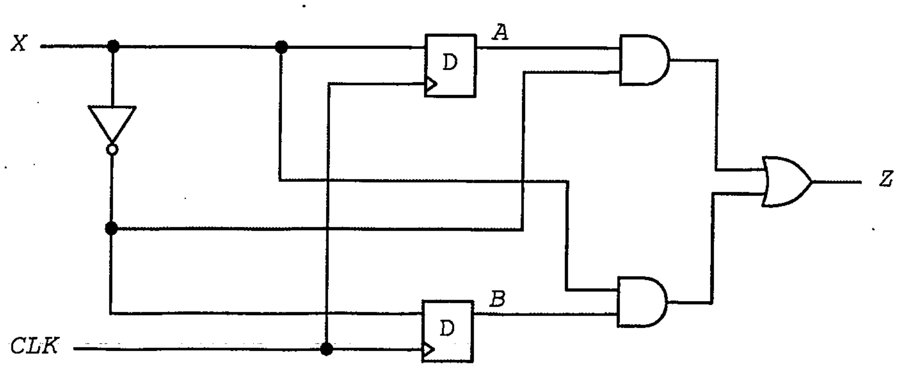


Figure 1

國立中山大學100學年度碩士班招生考試試題

科目：數位電路【電機系碩士班丙組選考、庚組】

[Problem 4] A Moore sequential network has one input and one output. When the input sequence 101 occurs, the output becomes 1 and remains 1 until the sequence 101 occurs again, in which case the output returns to 0. The output then remains 0 until 101 occurs a third time, and so forth. For example, the input sequence

X = 0 1 0 1 1 0 1 0 1 1 0 1 0 0 1 1

has the output

Y = 0 0 0 1 1 1 0 0 1 1 1 0 0 0 0 0

Derive the state diagram with a minimum number of states. Notice that you have to verify that the number of state in your design has been minimized. (15%)

[Problem 5] A timing chart as below is recorded on a Mealy machine with one input X, one output Z, and two JK flip-flops. Construct the state table and draw the sequential circuit. (15%)

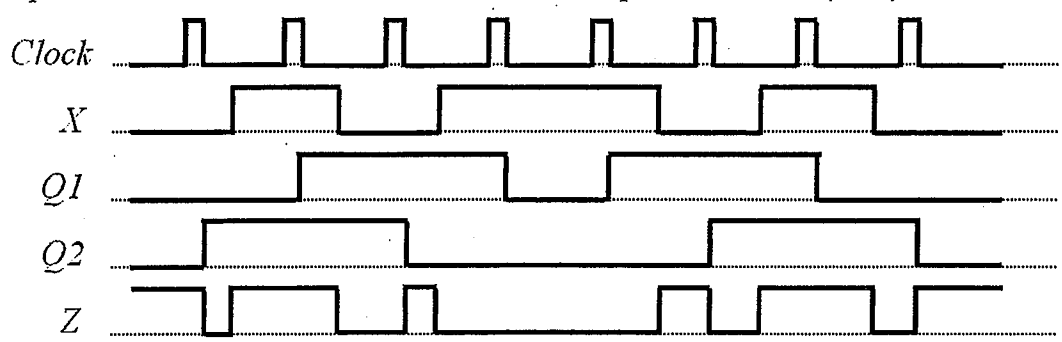


Figure 2

[Problem 6] A switching network has three inputs and two outputs, *a* and *b*, which represent the first and second bits of a binary number (*N*) respectively. Here *N* equals the number of inputs which are 0. For example, if $x = 1$, $y = 0$, and $z = 0$, then $a = 1$, $b = 0$. Please implement this network with two 4-to-1 multiplexers. (20%)

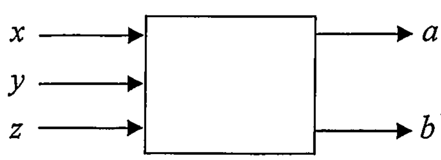


Figure 3

國立中山大學100學年度碩士班招生考試試題

科目：計算機結構【電機系碩士班丙組、庚組】

[Problem 1] Briefly describe what three techniques are possible for I/O operations. (15%)

[Problem 2] (a) Describe the definition of Amdahl's law. (5%)
(b) Suppose we enhance a machine making all floating-point instructions run five times faster. If the execution time of some benchmark before the floating-point enhancement is 60 seconds, what will the speedup be if three-forth of the 60 seconds are spent executing floating-point instructions? (15 %)

[Problem 3] Given a 32-bits fast adder (named as ADD32), a 32-bits 2-to-1 Multiplexor (named as MUX32_2to1), as Figure 1, and the basic gates such as NOT, AND, OR, NAND, NOR and XOR, you are asked to design an ALU, which must match the following requirements: (in function block diagrams) (20%)

(1) Support add, sub, and slt instructions. Their operation selection bits (op_sel) are as follows:
add(00), sub(01), slt(11).
(2) Report the result status in sign, zero, overflow, and carry bits.

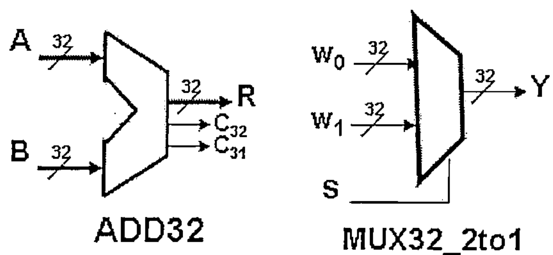


Figure 1

[Problem 4] A set associative cache has a block size of four 32-bit words and a set size of 4. The cache can accommodate a total of 4K words. The main memory size that is cacheable is 256K * 32 bits. Design the cache structure and show how the processor's addresses are interpreted. (20%)

[Problem 5] Use the following code fragment:

```
Loop:   LW      R1, 0(R2)
        ADDI    R1, R1, #1
        SW      0(R2), R1
        ADDI    R2, R2, #4
        SUB     R4, R3, R2
        BNEZ    R4, Loop
```

Assume the initial value of R3 is R2+200. Use the five-stage instruction pipeline (IF, DEC, EXE, MEM, WB) and assume all memory accesses are one cycle operation. Furthermore, branches are resolved in MEM stage.

(a) Show the timing of this instruction sequence for the five-stage instruction pipeline with normal forwarding and bypassing hardware. Assume that branch is handled by predicting it as not taken. How many cycles does this loop take to execute? (15%)
(b) Assuming the five-stage instruction pipeline with a single-cycle delayed branch and normal forwarding and bypassing hardware, schedule the instructions in the loop including the branch-delay slot. You may reorder instructions and modify the individual instruction operands, but do not undertake other loop transformations that change the number of op-code of instructions in the loop. Show a pipeline timing diagram and compute the number of cycles needed to execute the entire loop. (10%)

國立中山大學100學年度碩士班招生考試試題

科目：電路學【電機系碩士班丁組】

1.(10pt)A battery is discharged by a constant current 5A for 5 hours. If the terminal voltage of the battery is (12-0.2*t*) V, where *t* is in hour. Draw output power-time and output energy-time waveforms of the battery for $0<t<5$, respectively.

2.(10pt) For a balanced three-phase system, a-phase voltage and current are described as $v_a(t)=100\cos\omega t$V, $i_a(t)=10\sin(\omega t+60^o)$A. Find power factor, instantaneous power, average power, reactive power, apparent power of this balanced three-phase system.

3.(10pt) The voltage $v(t)=5+3\cos(t+30^o)+\sin(3t+20^o)+\cos(5t+10^o)$V is applied to a 5Ω resistor. Calculate the rms current flowing through the resistor and power consumption on it.

4.(10pt) Find V_1 and V_2 in Fig. 1.

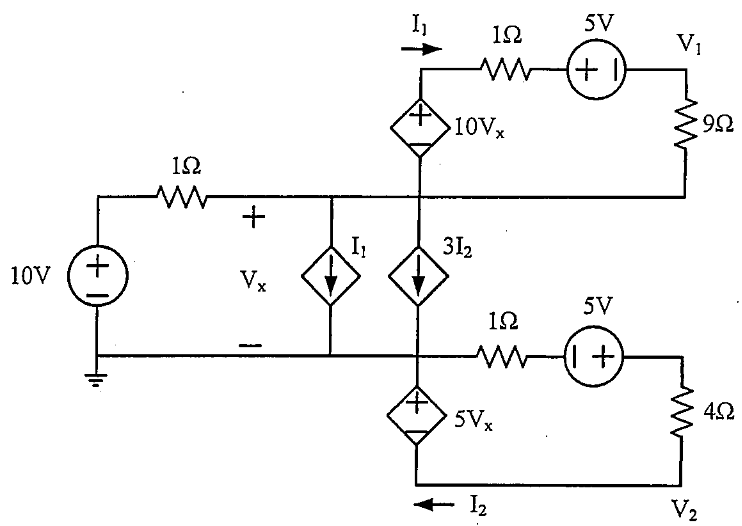


Fig. 1

5.(20pt) Assume the circuit of Fig. 2 is at the steady state before t=0s. The ideal switches S_1 is open at t=0s. Find $i(t)$ and $v(t)$ for t>0 if R=0 and R=4Ω, respectively.

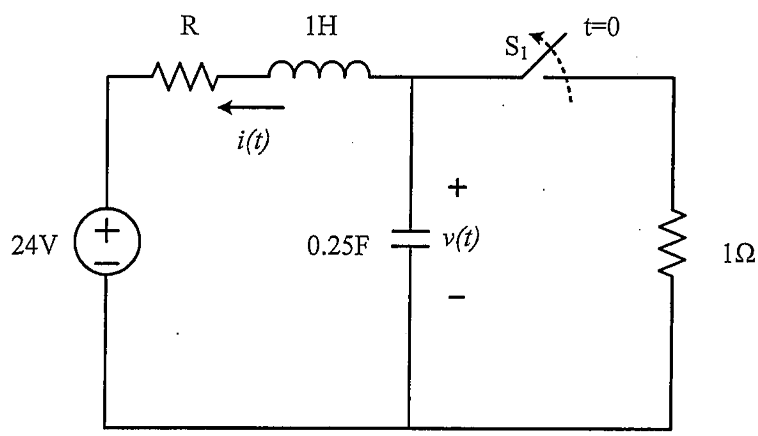


Fig. 2

國立中山大學100學年度碩士班招生考試試題

科目：電路學【電機系碩士班丁組】

6.(20pt) Calculate average power on all resistors and reactive power on all reactive components in the circuit of Fig. 3.

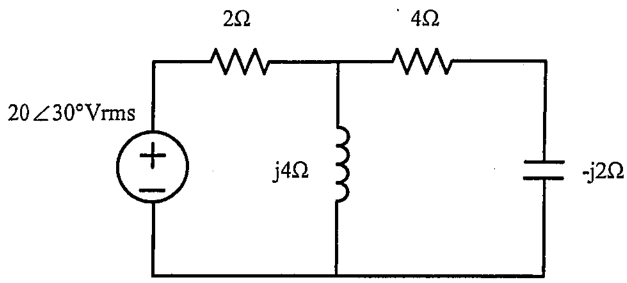


Fig. 3

7.(10pt) In Fig. 4, the turn ratio of the ideal transformer is 1:2. Find V_o and complex power supplied by the source.

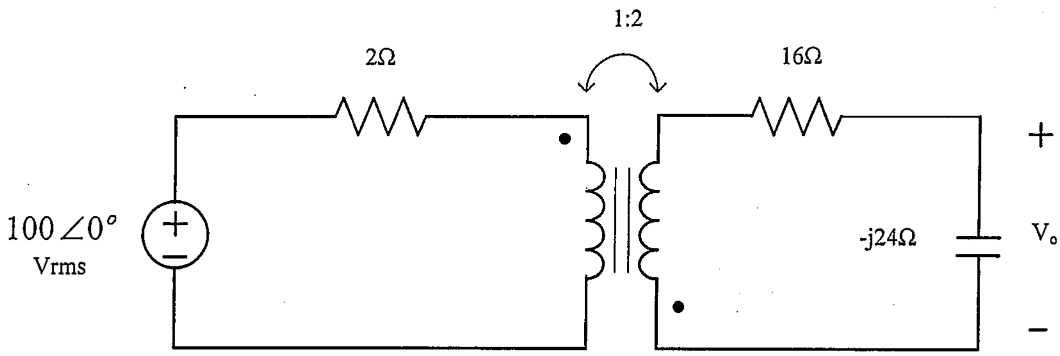


Fig. 4

8.(10pt) Solar energy is becoming increasingly popular recently. Generally, variable power with low dc voltage is produced by the solar panel. So, please describe how to use power electronics converters to deliver the generated power to the utility grid.

國立中山大學100學年度碩士班招生考試試題

科目：電力工程【電機系碩士班丁組】

1. Consider two buses connected by a high voltage transmission line with an impedance $Z = R + jX$, derive the real and reactive power flows on the line in terms of terminal bus voltages. Explain why the real power flow on a transmission line is dependent on phase angle difference, and the reactive power flow is dependent on voltage magnitude difference. (20%)

2. A 50 MVA, 30 kV, three phase 60 Hz synchronous generator has a synchronous reactance of 9Ω per phase and a negligible resistance. The generator is delivering rated power at a 0.8 power factor lagging at the rated terminal voltage to an infinite bus. (a) Determine the excitation voltage per phase and the power angle of the generator. (10%) (b) With the excitation voltage held constant at the voltage found in (a), the driving torque is reduced until the generator is delivering 25 MW. Determine the armature current and the power factor. (10%)

3. (a) Describe the inputs, outputs and applications of a typical power flow program. (10%)
   (b) Describe the solution procedure of a power flow problem solved by a Newton-Raphson technique. (10%)

4. A balanced Δ-connected load consisting of a pure resistance of 18Ω per phase is in parallel with a purely resistive balanced Y-connected load of 12Ω per phase as shown in Figure 1. The combination is connected to a three-phase balanced supply of 346.41-V_{rms} (line-to-line) via a three-phase line having an inductive reactance of j3Ω per phase. Taking the phase voltage V_{an} as reference, determine
   (a) The current, real power and reactive power drawn from the supply. (10%)
   (b) The line-to-neutral and the line-to-line voltage of phase *a* at the combined load terminals. (10%)

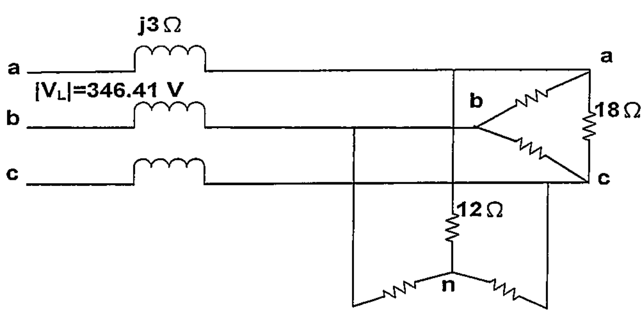


Fig. 1

5. The followings are the production cost functions and operating ranges of three generation units in a power system. Use the equal increment rule to find economic operating point for the three generation units when delivering a total power of 850 MW. (20%)

$C_1(P_1) = 561 + 7.92P_1 + 0.001562P_1^2$ $\quad$ $150MW \leqq P_1 \leqq 600MW$

$C_2(P_2) = 310 + 7.85P_2 + 0.00194P_2^2$ $\quad$ $100MW \leqq P_2 \leqq 400MW$

$C_3(P_3) = 78 + 7.97P_3 + 0.00482P_3^2$ $\quad$ $50MW \leqq P_3 \leqq 200MW$

國立中山大學100學年度碩士班招生考試試題

科目：電磁學【電機系碩士班戊組】

1. Please answer questions about static electric fields. (25%)
   (a) (5%) Under what conditions will the electric field intensity be both solenoidal and irrotational?
   (b) (5%) If the electric potential at a point is zero, dose it follow that the electrical field intensity is also zero at that point? Explain.
   (c) (5%) Why are there no free charges in the interior of a good conductor under static conditions?
   (d) (5%) If $\nabla^2 \mathbf{U} = 0$, why does it not follow that **U** is identically zero?
   (e) (5%) Assume that fixed charges +Q and –Q are deposited on the plates of an isolated parallel-plate capacitor.
      (i) Dose the electric field intensity in the space between the plates depend on the permittivity of the medium?
      (ii) Dose the electric flux density depend on the permittivity of the medium? Explain.

2. Please answer questions about static magnetic fields. (25%)
   (a) (5%) Which postulate of magnetostatics denies the existence of isolated magnetic charges?
   (b) (5%) State the law of conservation of magnetic flux.
   (c) (5%) Does the magnetic field intensity due to a current distribution depend on the properties of the medium?
   (d) (5%) What is meant by the internal inductance of a conductor?
   (e) (5%) What is the relation between the force and the stored magnetic energy in a system of current-carrying circuits under the condition of constant flux linkages?

3. To investigate the electromagnetic coupling of cellular phone antennas and a human head, a phantom head – a plastic container filled with a solution that approximately resembles the dielectric and conductive properties of a human head – is used for measurements. In particular, solutions are made that have the relative permittivity and loss tangent equal to the corresponding average head tissue parameters at two frequency bands allocated for wireless communications in North America: (i) $\varepsilon_r = 44.8$ and $\tan\delta_c = 0.408$ at $f = 835$ MHz and (ii) $\varepsilon_r = 41.9$ and $\tan\delta_c = 0.293$ at $f = 1.9$ GHz. Assume that the phantom solution has the same permeability as that of a vacuum.
   (a) (5%) Find the attenuation constant of a uniform plane wave propagating through the phantom solution.
   (b) (10%) If the rms electric field intensity of the wave at its entry into the solution is $E_0 = 50$ V/m, determine the time-average power absorbed in the first cm of depth into the solution per 1 cm^2 of cross-sectional area, that is, in the first 1 cm × 1 cm × 1 cm of the solution past the interface, at each of the frequencies.

國立中山大學100學年度碩士班招生考試試題

科目：電磁學【電機系碩士班戊組】

4. To determine the frequency, f, and electric-field rms intensity, E_{i0}, of a uniform plane wave traveling in air, a perfectly conducting plate is introduced normally to the wave propagation and electromotive force (emf) induced in a small square wire loop of area 6.25 cm^2 is measured. By varying the orientation and location of the loop, it is found that the rms emf in it has a maximum of 5 mV at a distance of 80 cm from the conducting plate (with the plane of the loop being perpendicular to the magnetic field vector of the wave). It is also found that the first adjacent minimum (zero) of the rms emf is at 60 cm from the conducting plate (for the same orientation of the loop).
(10%) What are f and E_{i0}?

5. RG-402U semi-rigid coaxial cable has an inner conductor of 0.91 mm, and a dielectric diameter (equal to the inner diameter of the outer conductor) of 3.02 mm. Both conductors are copper with conductivity of 5.8×10^7 S/m, and the dielectric material is Teflon with dielectric constant of 2.08 and loss tangent of 0.0004.
(10%) Find the characteristic impedance (in ohm) and the attenuation (in dB/m) of the line at 1 GHz.

6. Consider a TE_{02} mode propagating through an air-filled rectangular metallic waveguide of transverse dimensions a = 38.1 cm and b = 190.05 cm (WR-1500 waveguide).
(a) (5%) Determine the cutoff frequency of this mode.
(b) (10%) Find the power-handling capacity of the waveguide for this mode, i.e., the maximum time-average power that can be carried by the TE_{02} mode prior to an eventual dielectric breakdown at a frequency of f = 1.8 GHz. Note that the dielectric strength of air is $E_{cr0} = 3\times10^6$ V/m).

國立中山大學100學年度碩士班招生考試試題

科目：線性代數【電機系碩士班己組】

單選題 (6x5%=30%):

1. Consider a 3 × 3 matrix

$$A = \begin{bmatrix} 2 & -i & 1+i \\ i & 1 & 0 \\ 1-i & 0 & 1 \end{bmatrix},$$

[i] **A** is an Hermitian matrix

[ii] **A** is positive definite

[iii] The determinant of **A** is 1

[iv] The eigenvalues of **A** are 2, 1 and 1

[v] The trace of **A** is 4

Which statements are correct?

(a) i、ii、iv (b) i、iv、v (c) ii、iv、v

(d) i、ii、iii、v (e) i、v

2. Let $B \in \mathcal{R}^{n\times n}$ be an orthogonal matrix :

[i] The eigenvalues of **B** are 1, 0, or -1

[ii] The determinant of **B** is 1

[iii] The columns of **B** form an orthonormal basis of R^n

[iv] For any $x \in \mathcal{R}^{n\times 1}$, $\|x\| = \|Bx\|$

[v] For any $x, y \in \mathcal{R}^{n\times 1}$, $\langle x,y \rangle = \langle Bx,By \rangle$; where $\langle x,y \rangle = y^T x$ is the inner product of vectors **x** and **y**.

Which statements are always correct?

(a) i、 iv、v (b) ii、iii、iv (c) iii、iv、v

(d) i、ii、v (e) i、ii、iii

3. In the following, which is NOT diagonalizable?

(a) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ (e) $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

國立中山大學100學年度碩士班招生考試試題

科目：線性代數【電機系碩士班己組】

4. Consider three vectors in R^4: $[\,1, 0, 1, 0\,]^T$, $[\,0, 1, -2, 1\,]^T$, $[\,1, -1, 0, 0\,]^T$. Among the following vectors, which form an orthonormal basis of the subspace spanned by three vectors?

   [i] $\left[\ 1/\sqrt{2},\ \ 0,\ \ 1/\sqrt{2},\ \ 0\ \right]^T$

   [ii] $\left[\ 0,\ \ 1/\sqrt{6},\ \ -2/\sqrt{6},\ \ 1/\sqrt{6}\ \right]^T$

   [iii] $\left[\ 1/\sqrt{6},\ \ -2/\sqrt{6},\ \ -1/\sqrt{6},\ \ 0\ \right]^T$

   [iv] $\left[\ 1/\sqrt{2},\ \ -1/\sqrt{2},\ \ 0,\ \ 0\ \right]^T$

   [v] $\left[\ 1/2,\ \ 1/2,\ \ -1/2,\ \ 1/2\ \right]^T$

   (a) i、iii、v　　(b) i、ii、v　　(c) iii、iv、v

   (d) i、ii、iv　　(e) i、iii、iv

5. Given $n \times n$ matrices **A** and **B**, and there is an $n \times n$ invertible matrix **P** such that $\mathbf{B} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$.

   [i] **A** and **B** have the same trace

   [ii] **A** and **B** have the same eigenvectors

   [iii] **A** and **B** have the same determinant

   [iv] **A** and **B** have the same eigenvalues

   [v] **A** and **B** are simultaneously diagonalizable

   Among the above statements, which are not always true?

   (a) ii、iv、v　　(b) i、iv、v　　(c) i、iii、iv

   (d) ii、v　　(e) ii、iii

6. Let **A** be an $n \times n$ matrix with *n* real eigenvalues $\lambda_1 > \lambda_2 > \cdots > \lambda_n > 0$,

   [i] For any nonzero $n \times 1$ vectors **x**, $\lambda_n \leq \frac{\mathbf{x}^H\mathbf{A}\mathbf{x}}{\|\mathbf{x}\|^2} \leq \lambda_1$

   [ii] The determinant of $\mu\mathbf{A}^2$ is $\mu\lambda_1^2\lambda_2^2\cdots\lambda_n^2$

   [iii] The eigenvalues of matrix $(\mathbf{A}+\mathbf{I})^{-1}$ are the same as those of $\mathbf{A}^{-1}$

   [iv] The eigenvectors of matrix $(\mathbf{A}+\mathbf{I})^{-1}$ are the same as those of $\mathbf{A}^{-1}$

   [v] Given a nonzero $n \times 1$ vector **v** and for any nonzero $n \times 1$ vectors **x**, $\frac{|\mathbf{v}^T\mathbf{x}|^2}{\mathbf{x}^T\mathbf{A}\mathbf{x}} \leq \mathbf{v}^T\mathbf{A}^{-1}\mathbf{v}$

   Among above statements, which are not always true?

   (a) ii、iii　　(b) iii、v　　(c) iii、iv、v

   (d) iii、iv　　(e) i、ii

國立中山大學100學年度碩士班招生考試試題

科目：線性代數【電機系碩士班己組】

計算證明題 (70%)

1. Given singular value decomposition of a matrix $\mathbf{H} \in \mathcal{C}^{m\times n}$ as $\mathbf{H} = \mathbf{U}\Sigma\mathbf{V}^H$, where $\mathbf{U}$ and $\mathbf{V}$ are $m\times m$ and $n\times n$ unitary matrices, Σ is an $m\times n$ diagonal matrix composed of nonnegative singular values $\sigma_1, \cdots, \sigma_r, 0, \cdots, 0$, where $r = \text{rank}(\mathbf{H})$.
   (a) Find the eigenvalues of $\mathbf{H}\mathbf{H}^H$. (5%)
   (b) Prove that (5%)

$$\sum_{i=1}^{r}\sigma_i^2 = \sum_{i=1}^{m}\sum_{j=1}^{n}|h_{i,j}|^2 .$$

2. Find the LU decomposition of the following matrix (10%):

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

3. Please use LU–decomposition to solve the following system of linear equations (10%):

$$\begin{cases} -X_1 + 2X_2 - X_3 = 2 \\ X_1 - 4X_2 + 6X_3 = -3 \\ -2X_1 + 6X_2 - 6X_3 = 8 \end{cases}$$

4. Let $B_0 = \left\{\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\right\}$, $B_1 = \left\{\begin{bmatrix}2\\1\end{bmatrix}\begin{bmatrix}3\\-1\end{bmatrix}\right\}$ and $B_2 = \left\{\begin{bmatrix}2\\3\end{bmatrix}\begin{bmatrix}-4\\1\end{bmatrix}\right\}$ be three bases in R^2.
   Let $X = \begin{bmatrix}5\\2\end{bmatrix}$ in B_0
   (a) Write X in term of the vector in B_2. (5%)
   (b) Find the transformation matrix that converts a vector from in terms of Base B_1 to Base B_2. (5%)

5. Consider the vector space R^3 with Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vector $u_1 = (0,1,0)$, $u_2 = (1,1,1)$ and $u_3 = (1,1,2)$ into an orthogonal basis $\{v_1, v_2, v_3\}$; then normalize the orthogonal basis vectors to obtain an orthonomal basis $\{q_1, q_2, q_3\}$ (10%).

國立中山大學100學年度碩士班招生考試試題

科目：線性代數【電機系碩士班己組】

6. Find a singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}$. (10%)

7. Consider the vector space P_3 of polynomials of degree less than 3, and the ordered basis $B = \{x^2, x, 1\}$ for P_3. Let T: $P_3 \rightarrow P_3$ be the linear transformation such that

$$T(ax^2 + bx + c) = (a - c)x^2 - bx + 2c$$

Find the eigenvalues and the eigenvectors for the linear transformation T. (10%)

國立中山大學100學年度碩士班招生考試試題

科目：機率【電機系碩士班己組】

1. (10%) A pair of fair dice is rolled six times. What is the probability that "nine" will not show at all?

2. (10%) Let X be a random variable with exponential distribution, derive and find the mean and variance of $6X$.

3. (10%) Let X be a normal random variable with mean 10 and variance 9. How can we design a random variable Y such that Y is normal distributed with mean 100 and standard deviation 2.

4. (10%) Let X and Y be independent uniform random variables with ranges [0, 2] and [0, 6] respectively. Derive and plot the probability density function of the random variable $Z = X + Y$.

5. (10%) Let X and Y be independent normal random variables with mean 0 and variance 6. Derive and plot the probability density functions of the random variables $Z = \sqrt{X^2 + Y^2}$ and $W = Y/X$

6. (10%) Given a real-valued random variable X with finite second moment. Identify all the true statements:

   (a) $\mathrm{E}\{X^2\} \leq (\mathrm{E}\{X\})^2$;
   (b) $\mathrm{E}\{cX\} \neq c\mathrm{E}\{X\}$, where c is a constant value;
   (c) $\mathrm{E}\{\log(1+X)\} \leq \log(1+\mathrm{E}\{X\})$.

7. (10%) Given a condition so that the fact $\mathrm{E}\{\frac{1}{X}\} = \frac{1}{\mathrm{E}\{X\}}$ is true.

8. (10%) Let X and Y be independent normal random variables with zero mean and unit variance. Find the value of $\mathrm{E}\{X^2Y + XY^2\}$, in which $\mathrm{E}\{\cdot\}$ takes the expectation with respect to X and Y.

9. (10%) Given a real-valued random variable X with finite second moment $\mathrm{E}\{X^2\}$. Show the conditions on c so that the following statement is true:

$$\mathrm{E}\{X^2\} \leq c\mathrm{E}\{X^2\} \text{ if and only if } \mathrm{E}\{X^2\} = 0.$$

10. (10%) Let Y be a binomial distribution with parameters n and p; i.e., the probability distribution function of Y is given by $P(Y = y) = \binom{n}{y} p^y (1-p)^{n-y}$, $y = 0, 1, 2, \cdots, n$. Show the probability generating function of Y.

國立中山大學100學年度碩士班招生考試試題

科目：訊號與系統【電機系碩士班己組】

Fourier Transform formula: $F\{f(t)\} = F(\omega) = \int_{-\infty}^{\infty} f(t)\exp[-j\omega t]dt$

1. (a) Draw the plot (繪圖) for cos(t) and cos(3t). (2%)

   (b) Compute and plot the spectrum (Fourier transformation) for cos(t) and cos(3t) by Euler Theorem: $e^{\pm ju} = \cos u \pm j\sin u$ The computation procedures must be provided. (8%)

2. Prove Coordinate Scaling for Fourier Transform. (10%)

$$F\{f(\alpha t)\} = \frac{1}{|\alpha|}F\left(\frac{\omega}{\alpha}\right)$$

3. By using the above theorem, Please draw the plots of the spectrum for cos(t) and cos(3t) again. (10%)

4. Why the Hibert transform defined by the following equation in the frequency domain for a real function must be a real function too? Please work on the frequency domain. $\hat{F}(\omega) = -j\,\mathrm{sgn}(\omega)$
   Where sgn( ) is the sign function to be 1 or -1 dependent upon positive or negative argument.
   (Please Do not use the inverse Fourier transform). (20%)

5. Explain the two major procedures to transform analog signals to digital signals (data)? (10%)

6. State and explain the Sampling Theorem. (10%)

7. Prove the Sampling Theorem by plots. (15%)

8. Plot the convolution result for the following problem and explain why you have the correct answer. $g(t) = f(t) \otimes h(t)$ (15%)