國立中山大學99學年度碩士班招生考試試題

科目：線性代數【通訊聯招碩士班乙組、電機系碩士班己組、通訊所碩士班甲組】

單選題 (4x5%=20%):

1. Let

$$\mathrm{A} = \begin{bmatrix} 3 & 1-i \\ 1+i & 4 \end{bmatrix}$$

which is not true in the following:

(a) **A** is an Hermitian matrix

(b) **A** is positive definite

(c) Determinant of **A** is 10

(d) Eigenvalues of **A** are -2 and 5

(e) Trace of **A** is 7

2. Given singular value decomposition of a matrix $\mathbf{H} \in \mathcal{C}^{m \times n}$ as $\mathbf{H} = \mathbf{U}\Sigma\mathbf{V}^H$, where $\Sigma = \mathrm{diag}(\sigma_1, \cdots, \sigma_r, 0, 0..)$, and $r = \mathrm{rank}(\mathbf{H})$. In the following, which is false:

(a) The non-zero eigen-values of $\mathbf{H}\mathbf{H}^H$ are identical to that of $\mathbf{H}^H\mathbf{H}$

(b) The non-zero eigen-values of $\mathbf{H}\mathbf{H}^H$ are $\sigma_1^2, \cdots, \sigma_r^2$.

(c) The eigen-vectors of $\mathbf{H}\mathbf{H}^H$ equals to the columns of $\mathbf{V}$

(d) The singular values $\sigma_1, \cdots, \sigma_r$ are all positive

(e) $\sum_{i=1}^{r} \sigma_i^2 = \sum_{i=1}^{M} \sum_{j=1}^{M} |h|_{i,j}^2$

3. Let $\mathrm{A} = \sum_{i=1}^{Q} \mathrm{p}_q \mathrm{p}_q^H$, where $\{\mathrm{p}_q \in \mathcal{C}^{Q\times 1}\}$ is a set of orthonormal vectors. In the following, which is false:

(a) **A** is unitary

(b) **A** is symmetric

(c) $\mathbf{p}_q$ is an eigen-vector of **A**

(d) All eigen-values of **A** equal to 1

(e) Given $\{\alpha_i \neq 0\}$, $\alpha_1\mathbf{p}_1 + \alpha_1\mathbf{p}_2 + \cdots + \alpha_Q\mathbf{p}_Q$ is an eigenvector of **A**

4. Given $m \times m$ matrices **A** and **B**, which statement is not always true in the following.

(a) Trace(**AB**)=Trace(**BA**)

(b) For any $n \times 1$ vectors **x** and **y**, $\mathrm{x}^H\mathrm{A}\mathrm{y} = \mathrm{x}^H\mathrm{B}\mathrm{y} \Leftrightarrow \mathbf{A} = \mathbf{B}$

(c) If **A** is skew-symmetric, then, for any $n \times 1$ vectors **x**, $\mathrm{x}^T\mathrm{A}\mathrm{x} = 0$

(d) If **B** is positive definite and $\mathrm{x}^H\mathrm{B}\mathrm{x} = 0$, **x** must be zero.

(e) If **A** is unitary, |det(**A**)|=1

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科目：線性代數【通訊聯招碩士班乙組、電機系碩士班己組、通訊所碩士班甲組】

計算證明題

1. Consider the following matrix

$$A=\begin{bmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{bmatrix}$$

   (a) Please explain Cayley-Hamilton theorem and give an example to demonstrate it. (10%)

   (b) A^{99} =?(5%)

2. Consider the following matrix

$$A=\begin{bmatrix}-1&0\\1&-1\\1&1\end{bmatrix}$$

   (a) Please find the eigenvalues and eigenvectors of the matrix AA^{T}, where A^{T} is the transport of *A*. (5%)

   (b) Please calculate the singular value decomposition (SVD) of *A*. (5%)

3. Please use LU decomposition to solve the following system of linear equation (10%)

$$\begin{aligned}x_1-2x_2+x_3-3x_4&=20\\-x_1+x_2+x_3+2x_4&=-8\\-2x_1+3x_2+x_3+4x_4&=-21\\3x_1-4x_2-x_3-8x_4&=40\end{aligned}$$

4. Let *A* be an $m \times m$ matrix, please derive the necessary and sufficient condition of *A* being diagonalizable. (15%)

5. Let **U** and **V** be two $m \times m$ positive definite matrices.

   (a) Find a $m \times 1$ complex vector **b**, such that

$$Q=\frac{\mathbf{bUb}^H}{\mathbf{bVb}^H}$$

   is maximized (5%)

   (b) What is the maximum value of Q in (a)? (5%)

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科目：線性代數【通訊聯招碩士班乙組、電機系碩士班己組、通訊所碩士班甲組】

6. Given the following linear equations:

$$\begin{aligned} x+2y &= 2 \\ 3x-y &= 1 \\ x-y &= -3 \\ x+2y &= 10 \end{aligned}$$

(a) Show that the system described above has no solution. (5%)

(b) Find the least-square approximate solution of above system. (5%)

7. (a) Apply the Gram-Schmidt process to the following vectors to form a set of orthonormal bases. (5%)

$$\boldsymbol{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \qquad \boldsymbol{u}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \qquad \boldsymbol{u}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

(b) Find the QR decomposition of (5%)

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

國立中山大學99學年度碩士班招生考試試題

科目：機率【通訊聯招碩士班乙組、電機系碩士班乙組、通訊所碩士班甲組】

1. (Totally, 20 pts) Let X be a Normal random variable with mean 2 and variance 5.
   (a) (10 pts) Derive and find the mean and variance of the random variable $Y = 6X + 8$.
   (b) (10 pts) Derive and find the mean and variance of the random variable $Y = 6X^2$.

2. (Totally, 15 pts) Box 1 contains 2000 components of which 10 percent are defective. Box 2 contains 500 components of which 20 percent are defective. Boxes 3 and 4 contain 1000 each with 10 percent defective. We select at random one of the boxes and we remove at random a single component.
   (a) (8 pts) What is the probability that the selected component is defective?
   (b) (7 pts) What is the probability that this defective component came from Box 2?

3. (Totally, 15 pts) Show that for a random variable X with mean η and variance σ^2, the following inequality holds for any positive number ε:

$$P\{|X-\eta| \geq \varepsilon\} \leq \frac{\sigma^2}{\varepsilon^2}$$

4. (Totally, 15 pts) A firehouse is to be built at some point along a road of length L. A fire is uniformly likely to occur at any point along the road.
   (a) (8 pts) If we build the firehouse at a point at distance a from the left endpoint of the road, what is the expected distance the fire truck will have to travel to the fire?
   (b) (7 pts) Where should the firehouse be located to minimize the expected travel distance to a fire?

5. (Totally, 15 pts) Let p_X be a probability function for a discrete probability distribution. Let

   $x_1 < x_2 < x_3 < \cdots$ be all the values for which $p_X(x_i) > 0$. Let $U_1 \sim \text{Uniform}[0,1]$. Define Y by

$$Y = \min\left\{x_j : \sum_{k=1}^{j} p_X(x_k) \geq U_1\right\}.$$

   Please find the probability function of Y.

6. (Totally, 20 pts) For a Poisson random variable X with parameter λ, show that
   (a) (10 pts)

$$P(0 < X < 2\lambda) > \frac{\lambda - 1}{\lambda};$$

   (b) (10 pts)

$$E[X(X-1)] = \lambda^2, \text{ and } E[X(X-1)(X-2)] = \lambda^3.$$

國立中山大學99學年度碩士班招生考試試題

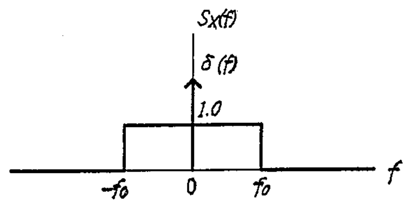
科目：通訊理論【通訊聯招碩士班乙組、電機系碩士班己組、通訊所碩士班甲組】

通訊理論 （Communications Theory）

1. (10 points) Based on the frequency bands, arrange and write the following communication systems in order (**from low to high** frequencies).
   (A) Point-to-point microwave (B) Standard AM broadcast (C) Cellular Mobile Radio
   (D) Worldwide Submarine Communication (E) FM broadcast

2. (10 points) Compare full AM with PAM, emphasizing the similarities and differences, particularly on
   (a) the envelope of the modulated signal
   (b) carrier
   (c) spectrum

3. (15 points) The power spectral density of a random process $X(t)$ is shown below. It consists of a delta function at $f=0$ and a rectangular component.



   (a) (6 points) Determine and sketch the autocorrelation function $R_X(\tau)$ of $X(t)$.
   (b) (3 points) What is the DC power contained in the $X(t)$?
   (c) (3 points) What is the AC power contained in the $X(t)$?
   (d) (3 points) If $X(t)$ is sampled, determine the lower bound of sampling frequency so that $X(t)$ is uniquely determined by its samples.

4. (15 points) Let a message signal $m(t)$ be transmitted using single-sideband modulation. The power spectral density of $m(t)$ is

$$S_M(f)=\begin{cases}2\dfrac{|f|}{W}, & |f|\le W\\ 0, & otherwise\end{cases}$$

   where W is a constant. White Gaussian noise of zero mean and power spectral density $N_0/2$ is added to the SSB modulated wave at the receiver input.
   (a) (6 points) Determine the average signal power.
   (b) (9 points) Assume that a modulated wave is expressed as $s(t)=\frac{1}{2}A_c\cos(2\pi f_c t)m(t)+\frac{1}{2}A_c\sin(2\pi f_c t)\hat{m}(t)$, where $\hat{m}(t)$ is the Hilbert transform of the message signal $m(t)$. Find the output signal-to-noise ratio of the SSB receiver.

5. (15 points) Nyquist pulse-shaping criterion (Nyquist condition for zero ISI)
   (a) (10 points) Show that the necessary and sufficient condition for $x(t)$ to satisfy $x(nT)=\begin{cases}1 & (n=0)\\ 0 & (n\neq 0)\end{cases}$ is that its Fourier transform $X(f)$ satisfy $\sum_{m=-\infty}^{\infty} X(f+m/T)=T$.
   (b) (5 points) Suppose that the signal has a bandwidth of W. Determine $X(f)$ for the case of $T=1/2W$.

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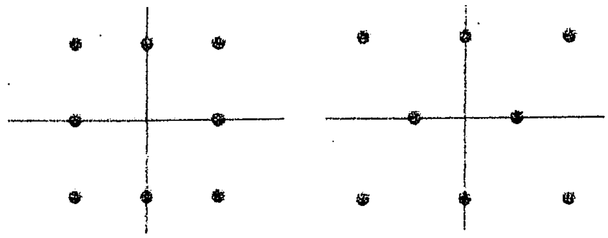
6. (10 points) Consider the two 8-point QAM signal constellations shown in the following figure. The minimum distance between adjacent points is $2A$.
   (a) (5 points) Determine the average transmitted power for each constellation, assuming that the signal points are equally probable.
   (b) (5 points) Which constellation is more power-efficient?



(a) (b)

7. (15 points) M-ary PAM signals are represented geometrically as M one-dimensional signal points with value

$$s_m = \sqrt{\frac{1}{2}\varepsilon_g} A_m, \quad m = 1, 2, \dots, M$$

where ε_g is the energy of the basic signal pulse $g(t)$. The amplitude values may be expressed as

$$A_m = (2m-1-M)d, \quad m = 1, 2, \dots, M$$

where the Euclidean distance between adjacent signal points is $d\sqrt{2\varepsilon_g}$. Assuming equally probable signals:

   (a) (5 points) Find the average energy.
   (b) (5 points) Calculate the average probability of a symbol error.
   (c) (5 points) Find the probability of a symbol error for rectangular M-ary QAM. ($M = 2^k$, k is even)

   Hint: $\sum_{m=1}^{M} m = \frac{M(M+1)}{2}$; $\sum_{m=1}^{M} m^2 = \frac{M(M+1)(2M+1)}{6}$.

8. (10 points) Fourier Transform
   (a) (5 points) Show that the spectrum of a real-valued signal exhibits conjugate symmetry, i.e., the amplitude spectrum is an even function of f and the phase spectrum is an odd function of f.
   (b) (5 points) Given $G(f) = \int_{-\infty}^{\infty} g(t)\exp(-j2\pi ft)\,dt$, i.e., $G(f)$ is the Fourier transform of $g(t)$.

   Show that $\int_{-\infty}^{t} g(\tau)\,d\tau \rightleftharpoons \frac{1}{j2\pi f}G(f) + \frac{G(0)}{2}\delta(f)$.

國立中山大學99學年度碩士班招生考試試題

科目：工程數學甲【通訊聯招碩士班甲組、電機系碩士班甲組、丙組選考、丁組、戊組、庚組、通訊所碩士班乙組】

1. (10%) Find the solution to the following differential equation:

$$\dot{x}_1(t) = -2x_1(t) + x_2(t) \quad x_1(0) = 1$$
$$\dot{x}_2(t) = x_1(t) - 2x_2(t) \quad x_2(0) = -1$$

2. (15%) Consider a two-dimensional flow governed by the vector field $F(x,y) = (x^2 + y^2)\mathbf{i}$, and a domain D enclosed by curves C_1 and C_2 as illustrated in Figure 1. Define the flow into D as positive flow and out of D as negative flow. Calculate the net flow across the boundary of D.

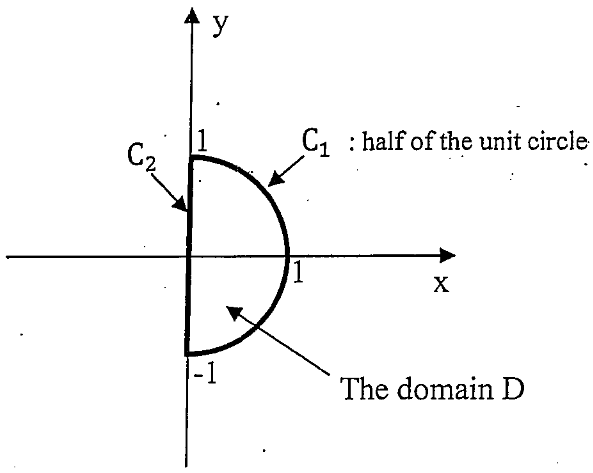


Figure 1: The domain D.

3. (a) (15%) Find the solution to the following wave equation:

$$\frac{\partial^2 w}{\partial t^2}(x,t) = \frac{\partial^2 w}{\partial x^2}(x,t) \qquad \forall 0 < x < 1, t > 0$$
$$w(0,t) = w(1,t) = 0 \qquad \forall t > 0$$
$$w(x,0) = 0 \qquad \forall 0 < x < 1$$
$$\frac{\partial w}{\partial t}(x,0) = \sin 3\pi x + \sin 6\pi x \qquad \forall 0 < x < 1$$

(b) (5%) Besides the boundary points x=0 and x=1, what are the stationary points of the solution to the above wave equation; i.e., at what x's w(x,t) is equal to zero for all t>0?

4. (15%) Let $\mathbf{x}$ and $\mathbf{y}$ be two vectors in the vector space V and let S be a set of vectors in V. Denote by $LC(\mathbf{x},S)$ the set of all linear combinations of $\mathbf{x}$ and any vector $\mathbf{s}$ in S, i.e.,

$$LC(\mathbf{x},S) := \{ \mathbf{v} = \alpha\mathbf{x} + \beta\mathbf{s} \mid \alpha, \beta \in \mathbb{R} \text{ and } \mathbf{s} \in S \}.$$

(a) Show that if $LC(\mathbf{x},S) \subset LC(\mathbf{y},S)$, then $\mathbf{x} \in LC(\mathbf{y},S)$. (5%)
(b) Under what condition on set S will the implication "$\mathbf{x} \in LC(\mathbf{y},S) \Rightarrow LC(\mathbf{x},S) \subset LC(\mathbf{y},S)$" hold? Give the condition and show your answer. (10%)

5. (10%) Let M and N be two subspaces of $\mathbb{R}^n$ such that $\mathbb{R}^n = M \oplus N$ and let P be the projection matrix that projects vectors of $\mathbb{R}^n$ onto M along N.
(a) Show that P is an idempotent matrix, i.e. $P^2 = P$. (5%)
(b) Let λ be an eigenvalue of matrix P. Find all possible values of λ. (5%)

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科目：工程數學甲【通訊聯招碩士班甲組、電機系碩士班甲組、丙組選考、丁組、戊組、庚組、通訊所碩士班乙組】

6. (15%) Evaluate the following integral

$$\oint_C z^{n-1} e^{1/z} dz$$

where z is a complex variable, and C is the circle $|z|=1$ in counterclockwise direction.
Hint: Find the Laurent series representation of $e^{1/z}$ first.

7. (15%) Let $F(\omega)$ and $G(\omega)$ be the Fourier transforms of two continuous signals $f(t)$ and $g(t)$ respectively. Prove that

$$\mathscr{F}\big(f(t)g(t)\big) = \frac{1}{2\pi} F(\omega) * G(\omega),$$

where $\mathscr{F}$ stands for Fourier transform, and "$*$" is the convolution operator.

國立中山大學99學年度碩士班招生考試試題

科目：電子學【通訊聯招碩士班甲組、電機系碩士班甲組、乙組、戊組、通訊所碩士班乙組】

1. (15%) Calculate the built-in voltage of a junction in which the *p* and *n* regions are doped with 10^{15} atoms/cm^3 and 10^{16} atoms/cm^3, respectively. Assume $n_i \approx 10^{10}$/cm^3. With no external voltage applied, (i) (6%) what is the width of the depletion region, and (ii) (3%) how far does it extend into the *p* and *n* regions? If the cross-sectional area of the junction is 100 μm^2, (iii) (3%) find the magnitude of the charge stored on either side of the junction, and (iv) (3%) calculate the junction capacitance C_j.

2. (15%) The NMOS and PMOS transistors in the circuit of Figure 1 are matched with $k'_n(W_n/L_n) = k'_p(W_p/L_p) = 1$ mA/V^2 and $V_{tn} = -V_{tp} = 0.7$ V. (where k' is the process transconductance parameter, W/L is the ratio of the channel width to the channel length, V_t is the threshold voltage). Assuming process-technology parameter $\lambda = 0$ for both devices (neglect the effect of channel-length modulation), find the drain currents i_{DN} and i_{DP} and the voltage v_O for $v_I = 0$ V, +2.5 V, and -2.5 V.

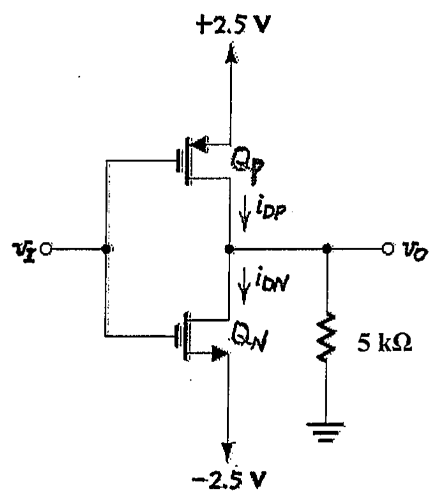

Figure 1

3. (20%) To analyze the circuit in Figure 2, (i) (15%) to determine the voltages at all nodes (A, B, C, D, and E) and the currents through all branches (I_{B1}, I_{C1}, I_{E1}, I_{B2}, I_{C2} and I_{E2}), (ii) (5%) find the total current drawn from the power supply. Hence find the power dissipated in the circuit.

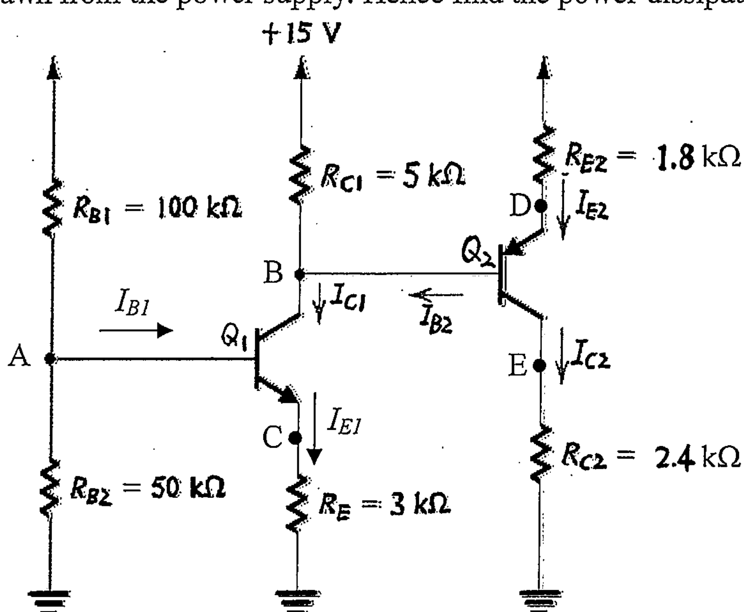

Figure 2

國立中山大學99學年度碩士班招生考試試題

科目：電子學【通訊聯招碩士班甲組、電機系碩士班甲組、乙組、戊組、通訊所碩士班乙組】

4. (15%) To analyze the instrumentation amplifier circuit shown in Figure 3, (i) (5%) to determine v_o as a function of v_1 and v_2, (ii) (5%) to determine the differential gain $[v_o/(v_2 - v_1)]$ and (iii) (5%) to find the input resistance.

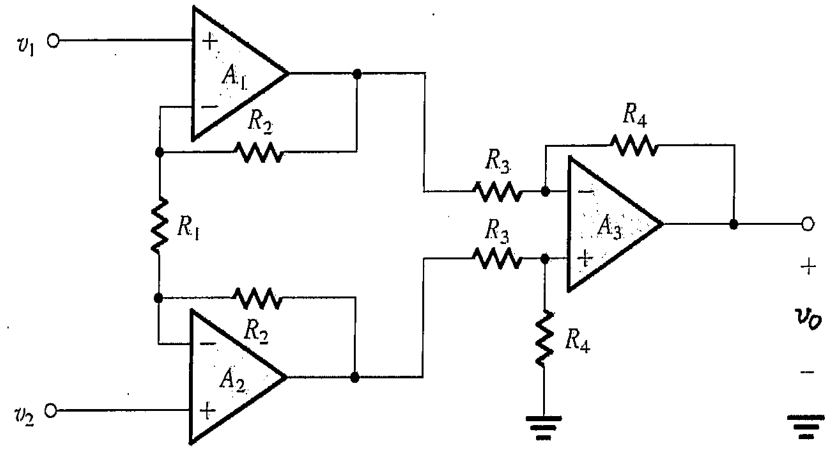

Figure 3

5. (20%) The differential amplifier in Figure 4 uses transistors with $\beta = 100$. Evaluate the following: (i) (5%) The input differential resistance R_{id}. (ii) (5%) The overall differential voltage gain v_O/v_{sig} (neglect the effect of r_O). (iii) (5%) The worst-case common-mode gain if the two collector resistances are accurate to within ±1%. (iv) (5%) The common-mode rejection ration (CMRR), in dB.

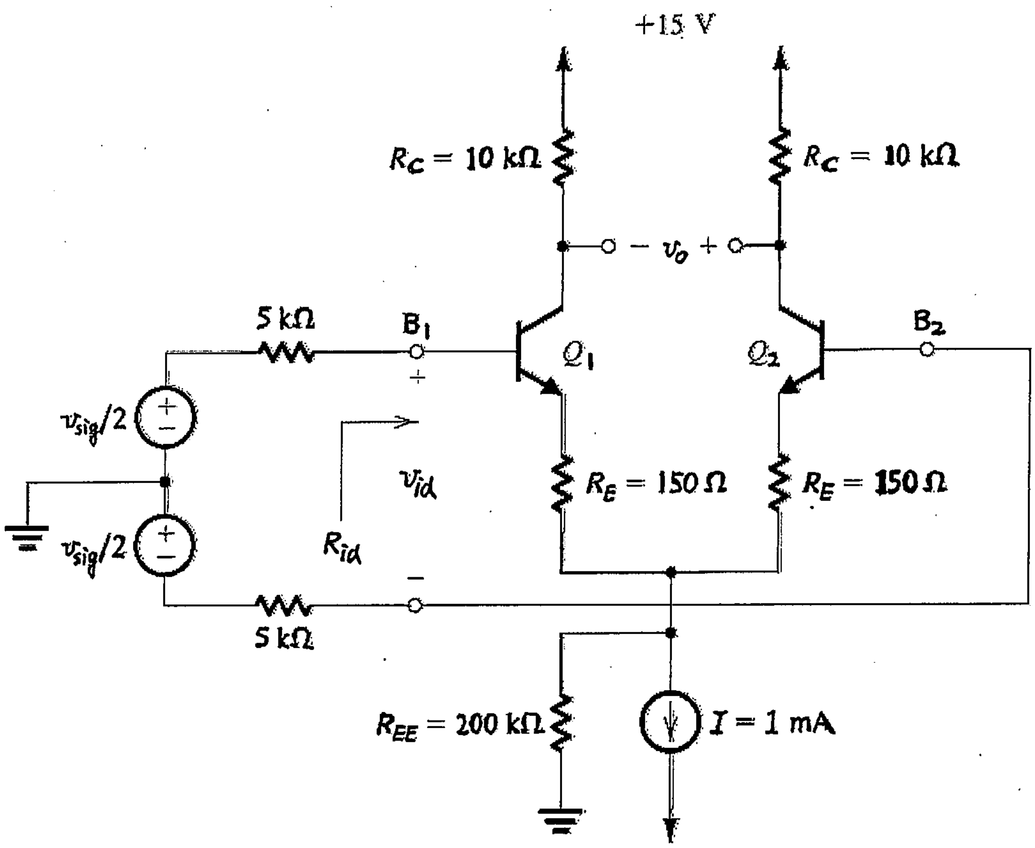

Figure 4

6. (15%) (i) (9%) Sketch a CMOS logic circuit that realize the function: $Y = \overline{A+B(C+D)}$

   (ii) (6%) Sketch a CMOS logic circuit that realize the function: $Y = ABC+\bar{A}\bar{B}\bar{C}$

國立中山大學99學年度碩士班招生考試試題

科目：電磁學【通訊聯招碩士班甲組、電機系碩士班戊組、通訊所碩士班乙組】

1. A spherical conducting shell of radius a, centered at the origin, is maintained at a potential V_0 (zero potential at infinity) in air. Denote $r=\sqrt{x^2+y^2+z^2}$
   (a) Determine potential function $V(r)$ for $r < a$ and $r > a$. (10%)
   (b) Determine the electric field intensity **E** for $r < a$ and $r > a$. (10%)
   (c) Find the energy stored in the electric field. (5%)

2. A very long, straight wire is along the z-axis. The tips of a triangular loop are located at $(d, 0, 0)$, $(d+b, 0, 0)$, and $(d, 0, d+b)$. Find the mutual inductance between the straight wire and the loop. (15%)

3. The three regions shown in Fig.P3 contain perfect dielectrics. For a wave in medium 1 incident normally upon the boundary at $z = -d$, what combination of ε_{r2} and d produces no reflection? Express the answer in terms of ε_{r1}, ε_{r3} and f of the wave. (20%)

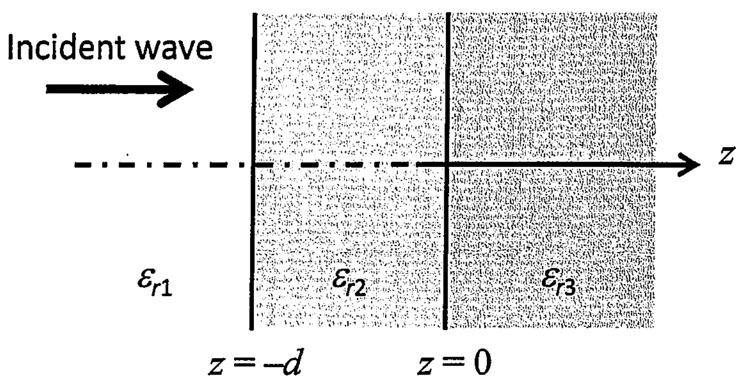

Fig.P3

4. Fig.P4 shows an open-circuited transmission line connected to a source with internal resistance of 50Ω and source voltage

$$V_g(t) = V_0\cos(3f_0t)\cos(f_0t)$$

with $\ell = \lambda/4$ at $f = f_0$. Find the root-mean-square (rms) values of the line voltage and current at $z = 0$, $z = -\ell/2$ and $z = \ell$. (20%)

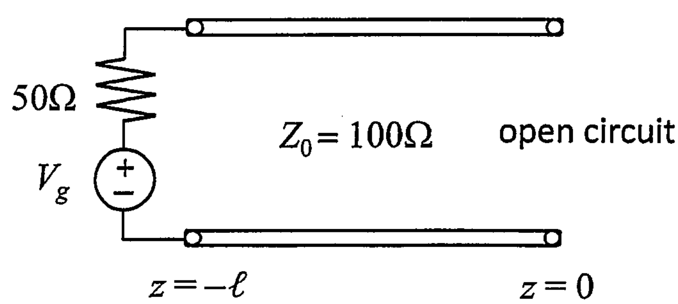

Fig.P4

國立中山大學99學年度碩士班招生考試試題

科目：電磁學【通訊聯招碩士班甲組、電機系碩士班戊組、通訊所碩士班乙組】

5. A rectangular air-filled waveguide has a cross-section of 45×90 mm. Find
   (a) cutoff wavelength λ_c for the dominant mode,
   (b) relative phase velocity u_p/c in the guide at a frequency of $1.6 f_c$,
   (c) cutoff wavelength if the guide is filled with a dielectric of $\varepsilon_r = 1.7$, and
   (d) relative phase velocity u_p/c with the dielectric at $1.6 f_c$.

(20%)