1) (Totally, 15pts) The probability density function (pdf) of a Gaussian random variable $X$ is given by

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

Consider another random variable defined as $Y = e^X$.

(a) (5 pts) Find the pdf for $Y$.

(b) (5 pts) Find the mean, median, and variance of $Y$. Examine the mean and median for $Y$ and find out which one is larger.

(c) (5 pts) Define the complementary cumulative distribution function for zero mean and unit variance Gaussian random variable as

$$Q(a) = \frac{1}{\sqrt{2\pi}} \int_a^\infty e^{-t^2/2} dt$$

Express $P[Y > y]$ using $Q(\cdot)$ function for $y > 0$.

2) (Totally, 15pts) Let $X$ and $Y$ be independently exponentially distributed random variables. The densities of $X$ and $Y$ are

$$f_X(x) = \alpha e^{-\alpha x}, x > 0,$$
$$f_Y(y) = \beta e^{-\beta y}, y > 0$$

respectively.

(a) (5 pts) Calculate $P[X < Y]$.

(b) (10 pts) Let $Z = \min(X, Y)$, what is $P[Z < z]$, for $z > 0$?

3) (Totally, 20pts) Let $X$ be a random variable such that, for $k = 0, 1, 2, \cdots$,

$$E\left(X^{2k}\right) = \frac{(2k)!}{k!}, \text{ and } E\left(X^{2k+1}\right) = 0.$$

(a) (10 pts) Find the moment generation function and its characteristic function.

(b) (10 pts) Find the distribution of $X$.

4) (Totally, 10pts) Prove the inequality of Bienayme, that for an arbitrary random variable $X$, two arbitrary numbers $a$ and $n$, and $\varepsilon > 0$, we have

$$P\{|X-a|^n \geq \varepsilon^n\} \leq \frac{E\{|X-a|^n\}}{\varepsilon^n}$$

5) (Totally, 20pts) Let $V$ be a finite dimensional abstract inner product space, let $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$ be an orthonormal basis for $V$, and let $T : V \to \mathbb{R}$ be a linear transformation.

(a) (10 pts) Show that the vector $\boldsymbol{x} = T(\boldsymbol{v}_1)\boldsymbol{v}_1 + \cdots + T(\boldsymbol{v}_n)\boldsymbol{v}_n$ satisfies the equation

$$< \boldsymbol{x}, \boldsymbol{y} >= T(\boldsymbol{y}) \text{ for all } \boldsymbol{y} \in V.$$

(b) (10 pts) Show that such a vector $\boldsymbol{x}$ from part (a) is unique.

6) (Totally, 20pts) Consider the following matrix:

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

(a) (5 pts) Find the eigenvalues and eigenvectors of matrix $\mathbf{A}\mathbf{A}^T$, where $\mathbf{A}^T$ is the transpose of A.

(a) (10 pts) Compute the Singular Value Decomposition (SVD) of A.

(b) (5 pts) Compute the rank 1 approximation of A.

# 國立中山大學95學年度碩士班招生考試試題

科目：通訊系統【通訊所碩士班甲組】 

1. [10%] Show that the magnitude response of a real function $g(t)$ is an even function.
2. [10%] Show that an AM signal can be demodulated using coherent demodulation. Why is coherent demodulation of AM not used in practice?
3. [10%] Find the PSD of a polar signal.
4. [20%] In a coherent FSK system, the signals $s_1(t)$ and $s_2(t)$ representing symbols 1 and 0, respectively, are defined by

$$s_1(t), s_2(t) = A\cos\left[2\pi\left(f_c \pm \frac{\Delta f}{2}\right)t\right], \quad 0 \le t \le T_b \text{ and } f_c > \Delta f .$$

   What is the minimum value of frequency deviation $\Delta f$ for which the signals $s_1(t)$ and $s_2(t)$ are orthogonal? Find the bit error rate of this system in an AWGN channel.
5. [20%] $M$-ary PAM signals are represented geometrically as $M$ one-dimensional signal points with value

$$s_m = \sqrt{\frac{1}{2}\varepsilon_g}A_m, \quad m = 1, 2, \ldots, M$$

   where $\varepsilon_g$ is the energy of the basic signal pulse $g(t)$. The amplitude values may be expressed as

$$A_m = (2m - 1 - M)d, \quad m = 1, 2, \ldots, M$$

   where the Euclidean distance between adjacent signal points is $d\sqrt{2\varepsilon_g}$. Assuming equally probable signals:

   A. [5%] Find the average energy.

   B. [10%] Calculate the average probability of a symbol error.

   C. [5%] Find the probability of a symbol error for rectangular $M$-ary QAM. ($M = 2^k$, $k$ is even)

   Hint: $\sum_{m=1}^{M} m = \frac{M(M+1)}{2}$; $\sum_{m=1}^{M} m^2 = \frac{M(M+1)(2M+1)}{6}$.
6. [10%] For the QAM signal constellaiton shown in Figure 1, determine the optimum decision boundaries for the detector, assuming that the SNR is sufficiently high so that errors only occur between adjacent points.

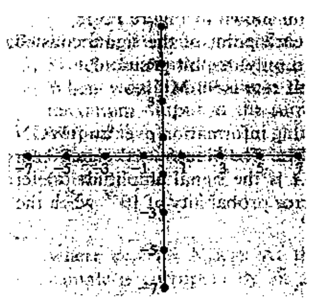

Figure 1

7. [20%] Consider the octal signal point constellations in Figure 2.
   A. [3%] The nearest-neighbor signal points in the 8-QAM signal constellation are separated in distance by $A$ units. Determine the radii $a$ and $b$ of the inner and outer circles.
   B. [2%] The adjacent signal points in the 8-PSK are separated by a distance of $A$ units. Determine the radius $r$ of the circle.
   C. [5%] Determine the average transmitter powers for the two signal constellaitons and compare the two powers. (Assume that all signal points are equally probable.)
   D. [10%] Consider the 8-QAM signal constellation, is it possible to assign three data bits to each point of the signal constellation such that nearest (adjacent) points differ in only one bit position? <u>Please explain your answers.</u>

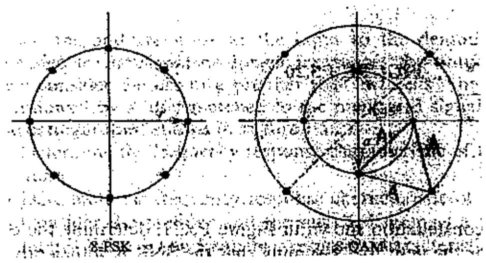


Figure 2

# 國立中山大學９５學年度碩士班招生考試試題

科目：電磁學【通訊所碩士班乙組】 

1. As illustrated in Fig. 1, the charge density in the cylindrical region surrounding the $z$ axis is $\rho = \rho_0$ for $r < a$ and $\rho = 0$ for $r > a$. The electric potential is zero at $r = b$ ($b$ > a). Assume that the region for $0 < r < b$ has a constant permittivity $\varepsilon_0$ everywhere.
   (a) (10%) Find the electric potential in the cylindrical region for $r < a$.
   (b) (10%) Find the electric potential in the cylindrical region for $a < r < b$.

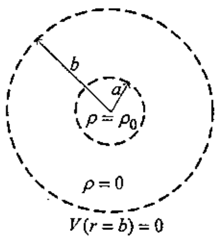

Fig. 1

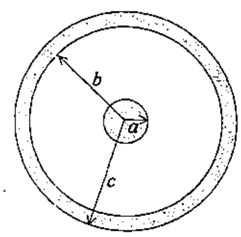

Fig. 2

2. As illustrated in Fig. 2, a coaxial line consists of a solid inner conductor of radius $a$ and a finitely thick outer conductor of inner radius $b$ and outer radius $c$. The conductor has permeability $\mu_c$ and the region between the inner and the outer conductors has permeability $\mu$. Assume that the current is uniformly distributed over the cross section of the inner and outer conductors.
   (a) (10%) Find the external inductance between the inner and the outer conductors.
   (b) (10%) Find the internal inductance due to the inner and outer conductors.

3. (a) (10%) What is the Lorentz condition for potentials?
   (b) (10%) Prove that the Lorentz condition for potentials is consistent with the equation of continuity.

4. A plane TEM wave with the electric field $\bar{E}_i = \hat{a}_y E_0 \exp(-j\beta_0 z)$ is normally incident from free space ($z < 0$) onto a good conductor ($z > 0$). The good conductor is characterized by $\varepsilon_0$, $\mu_0$, $\sigma_c$, with $\sigma_c >> \omega\varepsilon_0$.
   (a) (10%) Find the transmission coefficient.
   (b) (10%) Find the skin depth and the group velocity for the TEM wave propagating in the good conductor.

5. For a lossy transmission line of length $l$, let $Z_{sc}$ be the input impedance when one end is short-circuited, and let $Z_{oc}$ be the input impedance when one end is open-circuited.
   (a) (10%) Derive the expressions in terms of $l$, $Z_{sc}$ and $Z_{oc}$ for the characteristic impedance ( $Z_0$ ) and the complex propagation constant ( $\gamma$ ).
   (b) (10%) Derive the expressions in terms of $l$, $Z_{sc}$ and $Z_{oc}$ for the distributed circuit parameters ($R$, $L$, $G$, $C$).

填空題，計分僅以最後答案為準，不考慮計算過程。答案請寫在答案卷上計算題部份，註明小題號，由(1)、(2)、...、依序列出。

In the following $\vec{r}=\vec{a}_x x+\vec{a}_y y+\vec{a}_z z$ is the position vector, $r=|\vec{r}|$, and $\vec{a}_\alpha$ denotes the unit vector in the corresponding $\alpha$ direction. Denote the spherical coordinates as $(r,\theta,\phi)$.

1. (10pts) A charge distributes over a sphere of radius 2 with surface charge density $\rho_s=\sin(\theta/2)\cos^2\phi$ C/m$^2$. Determine the amount of charge $Q_T$=__(1)__C above ($z>0$) and $Q_B$=__(2)__C below ($z<0$) the $xy$ plane, respectively.

2. (20pts) A vector field is given as $\vec{E}=\vec{r}/r^4$. The rectangular coordinates for several points are given as: $P_1(1,0,0)$, $P_2(1,2,2)$ and $P_3(\frac{1}{2},1,1)$.

   (a) Find the line integral $\int_{P_1}^{P_2}\vec{E}\cdot d\vec{l}$ =__(3)__ along the straight line from $P_1$ to $P_2$.

   (b) Compute $\nabla\times\vec{E}$=__(4)__ at point $P_2$.

   (c) If there exists a scalar $V$ such that $\vec{E}=\nabla V$, find $V(P_2)-V(P_3)$=__(5)__ (answer NaN if $V$ does not exist).

   (d) Compute $\nabla\cdot\vec{E}$=__(6)__ at point $P_2$.

3. (20pts) Given $\theta=\tan^{-1}(\sqrt{x^2+y^2}/z)$ and point $P$ at the rectangular coordinates $(1,\sqrt{3},2)$, find

   (a) the direction of $\nabla\theta$ at $P$ in terms of rectangular coordinates. Answer=__(7)__.

   (b) the directional derivative $d\theta/dl$=__(8)__ along the $\vec{a}_r$ direction at $P$.

   (c) the maximum directional derivative $d\theta/dl$=__(9)__ at $P$.

   (d) the surface integral $\left|\int_S\nabla\theta\cdot d\vec{s}\right|$=__(10)__ over the unit circle centered at the origin on the $xy$ plane.

4. (20pts) Consider the following partial differential equation

$$\frac{\partial^2 w}{\partial z^2}-\frac{1}{4}\frac{\partial^2 w}{\partial t^2}=0.$$

(a) Make the transformation $u=z-4t$, $v=z+4t$. Express $\frac{\partial w}{\partial z}=c_1\frac{\partial w}{\partial u}+c_2\frac{\partial w}{\partial v}$ and $\frac{\partial w}{\partial t}=d_1\frac{\partial w}{\partial u}+d_2\frac{\partial w}{\partial v}$. Then $c_1c_2=$ __(11)__ and $d_1d_2=$ __(12)__.

(c) If we use the following transformation $u=z-c_3t$, $v=z+c_3t$, find $|c_3|=$ __(13)__ such that $\frac{\partial^2 w}{\partial z^2}-\frac{1}{4}\frac{\partial^2 w}{\partial t^2}=c_4\frac{\partial^2 w}{\partial u\partial v}$. What is $c_4$? Answer= __(14)__.

5. (20pts) The differential equation $y'(t)+5y(t)=5x(t)$ has the initial condition $y(0^-)=-2$. The forcing function $x(t)=(3/5)e^{-2t}u(t)$, where $u(t)$ is the unit step function. We use the Laplace transform (LT) to solve this problem.

(a) Find the LT $X(s)=$ __(15)__ of $x(t)$.

(b) Let $Y(s)$ be the LT of $y(t)$. Evaluate $Y(1)=$ __(16)__.

(c) Expand $Y(s)=\frac{c_1}{s+d_1}+\frac{c_2}{s+d_2}$. Compute $c_1+c_2+d_1+d_2=$ __(17)__.

(d) Use the inverse LT to solve for $y(t)$. Then $y(0^+)=$ __(18)__.

6. (10pts) Solve $y'(t)=2y/t+1$ with the initial condition $y(1)=0$. Calculate (a) $y(2)=$ __(19)__, (b) $y''(2)=$ __(20)__.