Problem 1. (Totally, 20 points)

(1) (10 points) Let $f(t) = c \cdot e^{-t^2}$ be a probability density function and c is a normalization constant. Calculate the value of c.

(2) (10 points) Let X be a Gaussian random variable with mean μ and variance σ^2. Calculate the third moment of X.

Problem 2. (Totally, 20 points)

Let M be a 2×2 random matrix in which elements are independent, identically distributed random variables. In addition, each matrix element is uniformly distributed in $[0, 2]$. (Note that each matrix element is a continuous random variable rather than a discrete random variable.)

(1) (10 points) Calculate the probability that the trace of M is smaller than $\frac{4}{3}$.

(2) (10 points) Calculate the probability that $(\log_2[M]_{1,1}) + (\log_2[M]_{1,2})$ is smaller than -1.

Problem 3. (Totally 10 points)

Let $U_1, U_2, U_3, .., U_k$ be a sequence of independent, identically distributed random variables. Each U_i, where $i \in \{1, 2, 3, .., k\}$, is uniformly distributed in $\{1, 2, 3, .., n\}$. Calculate the probability that $\prod_{i=1}^{k-1} \prod_{j=i+1}^{k} (U_j - U_i) \neq 0$.

Problem 4. (Totally, 20 points)

Let

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 4 \end{bmatrix}$$

(1) (10 points) Calculate the eigenvalues of A and the corresponding multiplicity of the eigenvalues.

(2) (10 points) Let I be the 3×3 identity matrix. Calculate the value of B, where

$$B = (A - 3I)^3 \cdot A^{100} - (A - 3I)^2 \cdot A^{100} + A^2 - 6A + 9I \qquad (1)$$

Problem 5. (Totally, 30 points)

Let $V = P(R)$ be an inner product space with inner product

$$< f(t), g(t) > = \int_{-1}^{1} f(t) \cdot g(t) dt \qquad (2)$$

Recall that $P(R)$ is the vector space composed of all polynomials with real coefficients. Similarly, $P_n(R)$ is the vector space composed of all polynomials with real coefficients and degree smaller than or equal to n.

Now, consider the subspace $P_2(R)$ with the standard ordered basis $\beta = \{1, x, x^2\}$. Use the Gram-Schmidt process to replace β by an orthonormal basis $\{v_1, v_2, v_3\}$ for $P_2(R)$.

國立中山大學九十四學年度碩士班招生考試試題

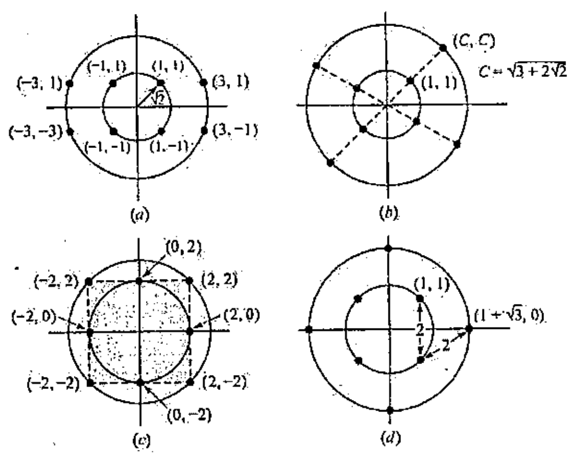
科目：通訊系統【通訊所碩士班甲組】

1. [30%]
   (a) What are the advantages and disadvantages of digital communications?
   (b) What is the sampling theorem?
   (c) What is the meaning of the terms "white" and "Gaussian" in an AWGN channel?
   (d) What is the purpose of matched filters in a receiver?
   (e) Explain capture effect and threshold effect of frequency modulation.
   (f) Draw a block diagram of a superheterodyne AM receiver.
2. [20%] Derive the bit error rate of a noncoherent ASK demodulator in an AWGN channel.
3. Consider a decision rule based on the computation of the posterior probabilities defined as

$$P(\mathbf{s}_m \mid \mathbf{r}) \equiv P(\text{signal } \mathbf{s}_m \text{ was transmitted} \mid \mathbf{r}), \quad m = 1, 2, \ldots, M.$$

This decision criterion, called the *maximum a posterior probability* (MAP) criterion, is based on selecting the signal corresponding to the maximum of the set of posterior probabilities $\{P(\mathbf{s}_m \mid \mathbf{r})\}$. On the other hand, the decision criterion based on the maximum of $P(\mathbf{r} \mid \mathbf{s}_m)$ over the M signals is called *maximum-likelihood* (ML) criterion. Consider the case of binary PAM signals in which the two possible signal points are $s_1 = -s_2 = \sqrt{\varepsilon_b}$, where ε_b is the energy per bit. The priori probabilities are $P(s_1) = p$ and $P(s_2) = 1 - p$. If the signals are sent over an additive white Gaussian noise channel with variance $\sigma_n^2 = \frac{1}{2} N_0$:

   (a) [10%] Determine the metrics, i.e. decision boundary of the signal constellation, for the optimum ML detector.
   (b) [15%] Determine the metrics for the optimum MAP detector.
   (c) [10%] When will these two decision criteria reach the same result?
4. [15%] Among the four 8-QAM modulation schemes shown in the following figure, which one results in the smallest error probability for a given transmitted power? Explain your answers.



Vector Analysis and Differential Equations

2005

1. (a) Please define and explain the following operations for scalar A or vector $\bar{F}$: (12%)
   i. ∇A
   ii. $\nabla \cdot \bar{F}$
   iii. $\nabla \times \bar{F}$

   (b) Using the previous definition, please derive the expression of $\nabla \cdot \bar{F}$ in cylindrical coordinates. (10%)

2. (a) Please define and explain "Irrotational Field" and "Solenoidal Field". (8%)

   (b) Given a vector function $\bar{F} = \hat{a}_x(x+c_1 z)+\hat{a}_y(c_2 x-3z)+\hat{a}_z(x+c_3 y+c_4 z)$ (15%)

   i. Determine the constants c_1, c_2, and c_3 if $\bar{F}$ is irrotational.

   ii. Determine the constant c_4 if $\bar{F}$ is solenoidal.

   iii. Determine the scalar potential function V whose negative gradient equals $\bar{F}$.

3. (a) Please define and explain the divergence theorem for a vector field $\bar{F}$.. (5%)

   (b) Please prove $\nabla \cdot (A\bar{F}) = A\nabla \cdot \bar{F} + \bar{F} \cdot \nabla A$ in Cartesian Coordinates. (10%)

   (c) According to (a) and (b), please prove (15%)

$$\int_v (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dv = \oint_s (\phi \nabla \psi - \psi \nabla \phi) \cdot d\bar{s}$$

   , where v is an arbitrary volume and s is its surface.

4. Please find the complete solution of the equation $y'' + y = \sec x$. (15%)

5. Please find a complete solution of the equation $x^2 y'' + xy' + 9y = 0$. (10%)

國立中山大學九十四學年度碩士班招生考試試題

科目：電磁學【通訊所碩士班乙組】

1. (a) Determine the capacitance per unit length between two long, parallel, circular conducting wires of radius a. The axes of the wires are separated by a distance D. Assume that all the space is filled with air. (10%)
   (b) When the above conducting wires are treated as a two-wire transmission line, find the characteristic impedance. (10%)

2. Consider a very long coaxial cable. The inner conductor has a radius a and is maintained at a potential V_0. The outer conductor has an inner radius b and is grounded. Assume that the dielectric filled between the conductors has an intrinsic impedance of η.
   (a) Determine the potential distribution in the space between the conductors. (10%)
   (b) Assume that only the dominant mode is propagating inside the coaxial cable. Find the electric and magnetic field in the space between the conductors. (10%)

3. (a) What is a solenoid inductor? (5%)
   (b) What is a spiral inductor? (5%)
   (c) How do you define the quality factor of an inductor? (5%)
   (d) Find the stored magnetic energy for a current I flowing in a single inductor with inductance L. (5%)

4. Consider a right-hand circularly polarized plane wave normally incident from free-space ($z < 0$) onto a good conductor ($z > 0$) with conductance and skin depth equal to σ and δ, respectively. Let the incident electric field be of the form

$$\vec{E} = E_0(\hat{x} - j\hat{y})e^{-jk_0 z}.$$

   (a) Find the electric and magnetic field of the reflected wave. What is the polarization of the reflected wave? (10%)
   (b) Compute the Poynting vectors for $z < 0$ and $z > 0$, and show that complex power is conserved. (10%)

5. Consider a Teflon ($\varepsilon_r = 2.08$, $\tan\delta = 0.0004$) filled copper ($\sigma = 5.8\times10^7$ S/m) K-band waveguide, having dimensions $a = 1.07$ cm and $b = 0.43$ cm.
   (a) Find the cutoff frequencies of the first five propagating modes. (10%)
   (b) If the operating frequency is 15 GHz, find the attenuation in dB/m due to dielectric and conductor losses. (10%)