國立中山大學 111 學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：線性代數【通訊所碩士班甲組】

—作答注意事項—

考試時間：100 分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

一、選擇題 （單選，一題五分，請於答案卡作答）

1. (5%) Consider a complex 3 × 3 matrix

$$A = \begin{bmatrix} 1 & 1-i & i \\ 1+i & 2 & -i \\ -i & i & 1 \end{bmatrix}.$$

Which is not true in the following?

(A) **A** is Hermitian symmetric

(B) $Trace(\boldsymbol{A}) = 4$

(C) **A** is positive definite

(D) Columns of the matrix **A** are linearly independent

(E) **A** is non-singular

2. (5%) Given $m \times m$ matrices **A** and **B**, which statement is not always true in the following?

(A) If **A** is unitary, $|det(\boldsymbol{A})| = 1$

(B) If **B** is positive semi-definite and $\mathrm{x}^H \mathrm{Bx} = 0$, **x** must be zero.

(C) If **A** is skew-symmetric, then, for any $m \times 1$ vector **x**, $\mathbf{x}^T \mathbf{A} \mathbf{x} = 0$

(D) $Trace(\boldsymbol{AB}) = Trace(\boldsymbol{BA})$

(E) For any $m \times 1$ vectors **x** and **y**, $\mathrm{x}^H \mathrm{Ay} = \mathrm{x}^H \mathrm{By} \Leftrightarrow \mathbf{A} = \mathbf{B}$

3. (5%) Given eigen-decomposition of a complex $m \times m$ matrix $\mathbf{A} = \mathbf{P\Sigma P}^{-1}$, where the matrix $\boldsymbol{\Sigma} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_m)$. Which is not always true in the following?

(A) $\lambda_1, \lambda_2, \ldots,$ and λ_m are real

(B) $Trace(\mathbf{A}) = \lambda_1 + \lambda_2 + \cdots + \lambda_m$

(C) $det(\mathbf{A}) = \lambda_1 \times \lambda_2 \times \ldots \times \lambda_m$

(D) If **A** is a Hermitian matrix, **P** is unitary

(E) If **A** is a Unitary matrix, $|\lambda_i| = 1, \forall\, i = 1,2, \ldots, m$.

4. (5%) Which of the following matrix is not diagonalizable?

(A) $\begin{bmatrix} 3 & 1+i \\ 1-i & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}$ (E) $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & 2 \\ 0 & 2 & -1 \end{bmatrix}$

5. (5%) Consider a matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

What is the orthogonal projection of a vector $\mathbf{x} = [2 \quad 1 \quad 2 \quad -1]^T$ on the column space of **A**?

(A) $[2 \quad 0 \quad 2 \quad 0]^T$ (B) $[0 \quad 1 \quad 0 \quad -1]^T$ (C) $[1/2 \quad 1 \quad 1/2 \quad 2]^T$

(D) $[0 \quad 1 \quad 0 \quad 1]^T$ (E) $[2 \quad 1 \quad 2 \quad -1]^T$

6. (5%) Given a matrix $\mathbf{A} = \mathbf{u}\mathbf{u}^T$, where $\mathbf{u}$ is a real $m \times 1$ vector and $\|\mathbf{u}\| = 1$. Which of the following statement is not true?
   (A) The rank of $\mathbf{A}$ is one
   (B) The non-zero eigenvalue of $\mathbf{A}$ equals to one
   (C) $Trace(\mathbf{A}) = 1$
   (D) $det(\mathbf{A}) = 1$
   (E) $\mathbf{A}^2 = \mathbf{A}$

7. (5%) Let $\mathbf{A} \in \mathbb{R}^{m \times m}$ be an orthogonal matrix, which of the following statement is not true?
   (A) The inverse matrix of $\mathbf{A}$ is $\mathbf{A}^T$
   (B) For any vector $\mathbf{x} \in \mathbb{R}^{m \times 1}$, $\|\mathbf{x}\| = \|\mathbf{A}\mathbf{x}\|$
   (C) $det(\mathbf{A}) = \pm 1$
   (D) The eigenvalues of $\mathbf{A}$ are always ± 1
   (E) $\mathbf{A}$ is diagonalizable

8. (5%) For a matrix

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

   What is the value of $det\ (\mathbf{A})$?
   (A) 11　(B) 5　(C) 8　(D) 7　(E) 1

9. (5%) Let (u, v, x, y) be the solution of the following equations:

$$\begin{aligned} 2u - v \quad\quad\quad &= 0 \\ -u + 2v - x \quad &= 0 \\ -v + 2x - y &= 0 \\ -x + 2y &= 5 \end{aligned}$$

   Which of the following answer is true?
   (A) $u = 2$　(B) $v = 2$　(C) $x = 1$　(D) $y = 3$　(E) $u = 4$

10. (5%) Let $\mathbf{A} \in C^{m \times m}$ be a positive definite matrix, which of the following statement is not always true?
   (A) Diagonal entries of $\mathbf{A}$ are all positive
   (B) The eigenvalues of $\mathbf{A}$ are all positive
   (C) If $c \geq 0$, $\mathbf{A} + c\mathbf{I}$ is also positive definite
   (D) $\mathbf{A}$ is non-singular
   (E) The entries of any eigenvector of $\mathbf{A}$ are all positive

科目名稱：線性代數【通訊所碩士班甲組】　　題號：437006

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　

二、問答計算題（請於答案卷作答）

1. (25%) Consider three vectors:

$$\mathbf{u}_1 = \begin{bmatrix}1\\1\\0\\0\\0\end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix}1\\0\\1\\0\\0\end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix}0\\1\\0\\1\\0\end{bmatrix}.$$

Answer the following questions:

(A) (5%) Apply the Gram-Schmidt process to $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ to form a set of orthonormal bases.

(B) (5%) Find the orthogonal projection of a vector $\mathbf{b} = [1 \quad -1 \quad 2 \quad 0 \quad 3]^T$ on the space spanned by $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$.

(C) (5%) Find the QR decomposition of

$$U = \begin{bmatrix}1 & 1 & 0\\1 & 0 & 1\\0 & 1 & 0\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix}$$

(D) (10%) Find a solution of $\mathbf{x} = [x_1 \quad x_2 \quad x_3]^T$, such that $\|\mathbf{U}\mathbf{x} - \mathbf{b}\|^2$ is minimized.

2. (25%) Consider a 3×3 matrix

$$A = \begin{bmatrix}4 & 0 & -1+i\\0 & 4 & -1-i\\-1-i & -1+i & 4\end{bmatrix}.$$

Answer the following Questions

(A) (12%) Find the eigen-decomposition of the matrix $\mathbf{A}$.

(B) (5%) Find a vector $\mathbf{x} \in \mathbb{C}^{3\times 1}$ with $\|\mathbf{x}\| = 1$, such that $\mathbf{x}^H\mathbf{A}\mathbf{x}$ is maximized. What is the maximal value?

(C) (8%) Find the inverse matrix of $\mathbf{A}$.

國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】　　題號：437005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　共3頁第1頁

一、選擇題(單選，計分方式:不倒扣，答對得該題全部分數，答錯及未作答得零分)

1. (5%) Let A and B be events with probabilities $\Pr(A) = 3/4$ and $\Pr(B) = 1/3$. Which of the following is not possible?
   (A) $\Pr(A \cap B) = 1/2$
   (B) $\Pr(A \cup B) = 11/12$
   (C) $\Pr(A \cap B) = 1/4$
   (D) $\Pr(A \cap B) = 1/11$
   (E) None of these

2. (5%) Bucket A contains 4 green, x black and 5 red balls and the probability of getting one black ball is 2/5. Bucket B contains $x + 2$ black, $x + 3$ pink and $x - 3$ red balls. Find the probability of getting at least one is pink ball if two balls are taken from bucket B.
   (A) 1/19
   (B) 11/38
   (C) 27/38
   (D) 25/38
   (E) None of these

3. (5%) A factory produces four different types of products, A, B, C and D. The probability that a random piece of products A, B, C and D is found to be defective are 0.2, 0.3, 0.05 and 0.1, respectively. During an inspection, one piece of each product is randomly selected. What is the probability that exactly three of them are found to be defective?
   (A) 0.005
   (B) 0.0103
   (C) 0.0124
   (D) 0.01
   (E) None of these

4. (5%) Let the continuous random variables X and Y have the joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} 1/x & \text{if } 0 < y < x < 1; \\ 0 & \text{otherwise.} \end{cases}$$

   Which of the following is incorrect?
   (A) $E[X] = 1/2$
   (B) $E[Y] = 1/4$
   (C) $E[Y|X = x] = 1/x$
   (D) $E[XY] = 1/6$
   (E) None of these

5. (5%) A continuous random variable X has the cumulative distribution function

$$F_X(x) = \begin{cases} a & \text{for } x \le 0, \\ x^2 & \text{for } 0 < x < 1, \\ b & \text{for } x \ge 1. \end{cases}$$

   Which of the following is incorrect?
   (A) $a = 0$
   (B) $b = 1$
   (C) $E[X] = 2/3$
   (D) $Var[x] = 1/18$
   (E) None of these

6. (5%) Let $\mathbb{R}$ denote the set of all real numbers. Which of the following is not a cumulative distribution function?

(A) $F(x) = \begin{cases} 1 - e^{-x^2}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$

(B) $F(x) = \begin{cases} e^{-1/x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$

(C) $F(x) = \frac{e^x}{e^x + e^{-x}},\ x \in \mathbb{R}.$

(D) $F(x) = e^{-x^2} + \frac{e^x}{e^x + e^{-x}},\ x \in \mathbb{R}.$

(E) None of these

7. (5%) Let X and Y be independent random variables, each taking the values -1 or 1 with probability 1/2. Let $Z = XY$. Which of the following is correct?

(A) $\Pr(X = 1, Z = 1) = 1/2$

(B) Y and Z are dependent

(C) Y and Z are correlated

(D) X, Y and Z are dependent

(E) None of these

8. (5%) Suppose that the moment-generating function $M_X(t)$ of the continuous random variable X has the property $M_X(t) = e^t M_X(-t)$ for all t. What is $E[X]$?

(A) 1/2

(B) 1

(C) 2

(D) 4

(E) None of these

9. (5%) The joint probability density function of the random variables X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{10} e^{-\frac{1}{2}(y+3-x)} & \text{for } 5 < x < 10, y > x - 3; \\ 0 & \text{otherwise.} \end{cases}$$

What is $\Pr(X < Y)$?

(A) e^{-1}

(B) $e^{-\frac{3}{2}}$

(C) e^{-2}

(D) $e^{-\frac{5}{2}}$

(E) None of these

10. (5%) Let X be a continuous random variable, uniformly distributed on $[-1,1]$. Find the probability density function of $Y = \sin^{-1} X$.

(A) $f_Y(y) = \frac{2}{\pi} \sin^{-1} y$, for $0 \leq y \leq 1$

(B) $f_Y(y) = \cos y$, for $0 \leq y \leq \frac{\pi}{2}$

(C) $f_Y(y) = \frac{1}{2} \cos y$, for $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

(D) $f_Y(y) = 1/\pi$, for $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

(E) None of these

二、問答計算題:

1. (10%) Define or describe the following terminologies.
   - (a) (5%) Central limit theorem.
   - (b) (5%) Bayes' rule.

2. (5%) Let X and Y be two DEPENDENT random variables. Does the relationship $E[X+Y] = E[X] + E[Y]$ hold? Provide your justification.

3. (15%) Let X_1, X_2,... be independent normal distributions. Define $Y_n = \sum_{i=1}^{n} X_i$, $n = 1,2,\ldots$.
   - (a) (10%) Compute the mean and the variance of Y_n.
   - (b) (5%) Compute the covariance of Y_m and Y_n, $m \le n$.

4. (20%) Let X and Y be independent and identically distributed random variables, and the probability density function (PDF) of X is given by

$$f_X(x) = \begin{cases} \exp(-x), & x \ge 0 \\ 0, & x < 0. \end{cases}$$

   Assume $U = X+Y$ and $V = X/(X+Y)$.
   - (a). (10%) Find the joint PDF of U and V.
   - (b). (10%) Are U and V independent? Provide your justification.

國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組、電波聯合選考：電機系碩士班戊組、通訊所碩士班乙組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組、電波聯合選考：電機系碩士班戊組、通訊所碩士班乙組】題號：437002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）

1. (10%) Please explain the following concepts as detail as possible:
   (a) (2%) Describe the conditions for a random process to be wide-sense stationary (WSS).
   (b) (2%) What is an Ergodic process?
   (c) (2%) Describe the Wiener-Khinchin Theorem.
   (d) (2%) What is an Additive White Gaussian Noise?
   (e) (2%) What is a Gaussian Process?

2. (12%) Find the Fourier transform of $x(t) = sinc^3(t)$. (Hint: The answer is a piecewise function which consists of five intervals.)

3. (15%) The characteristic function of a random variable X is defined as the statistical average

$$E\left(e^{jvX}\right) \equiv \psi(jvX) = \int_{-\infty}^{\infty} e^{jvx} p(x) dx.$$

   (a) (10%) Find the characteristic function of a Gaussian random variable.
   (b) (5%) Show that the variable Y, which is defined as the sum of N independent and identically distributed (i.i.d.) Gaussian random variables X_i, $i = 1,2, \dots, N$, is a Gaussian random variable.

4. (10%) An AM signal $s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$ is considered in the following systems:
   (a) (5%) If $s(t)$ is used as the input to a square-law detector which has a transfer characteristic defined as $v_O(t) = a_1 v_I(t) + a_2 v_I^2(t)$, where a_1 and a_2 are constants, $v_I(t)$ denotes the input, and $v_O(t)$ denotes the output. Find the conditions for which the message signal $m(t)$ can be recovered from $v_O(t)$.
   (b) (5%) Let $r(t)$ denote the recovered signal in (a). Suppose that we use an ideal sampling with a sampling interval of T_s to sample $r(t)$ and obtain the sampled signal $r_\delta(t)$, please find the Fourier transform of $r_\delta(t)$.

5. (15%) Please answer the following questions.
   (a) (5%) For a quaternary communication system, the possible transmitted signals are

$$s_k(t) = A\cos\left(\frac{20\pi}{T}t - \frac{(k-1)}{2}\pi\right), 0 \le t \le T, k = 1, \cdots, 4.$$

   Assume $T = 40ms, A = 100mV, P(s_k(t)) = \frac{1}{4}, \forall k$, and the noise PSD $S_n(f) = 20\ \mu\text{W/Hz}$. Please calculate the error probability P_e.

$$\left(Hint\text{: } P_e = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) - Q^2\left(\sqrt{\frac{E_s}{N_0}}\right)\right)$$

   (b) (5%) If T changes to $1ms$, in order to maintain the same P_e obtained in (a), please calculate the required amplitude value A.
   (c) (5%) Please show the orthonormal basis functions for the signal constellation $s_k(t)$.

科目名稱：通訊理論【通訊所碩士班甲組、電波聯合選考：電機系碩士班戊組、通訊所碩士班乙組】題號：437002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）

6. (20%) The definition of entropy is the expected value of the self information:

$$H(X) \triangleq E[I(x_k)] = \sum_{k=0}^{K-1} p_k \log_2\left(\frac{1}{p_k}\right) = -\sum_{x\in X} P(x)\log_2\big(P(x)\big).$$

Let variables X, Y have the joint probability

$$P(X,Y) = \begin{bmatrix} P(x_1,y_1) & P(x_1,y_2) \\ P(x_2,y_1) & P(x_2,y_2) \end{bmatrix} = \begin{bmatrix} 0.54 & 0.06 \\ 0.06 & 0.34 \end{bmatrix}.$$

Please find the following quantities:

(a) (4%) $P(Y|X)$ and $P(X|Y)$

(b) (4%) $H(X)$ and $H(Y)$

(c) (6%) Calculate $H(X|Y)$ and describe the physical meaning of $H(X|Y)$.

(d) (6%) Calculate $I(X;Y)$ and describe the physical meaning of $I(X;Y)$.

7. (18%) Consider the encoder for a binary (3,1,2) convolutional code shown in Fig. 1. There are one input message u, two registers and three outputs v_1, v_2 and v_3.

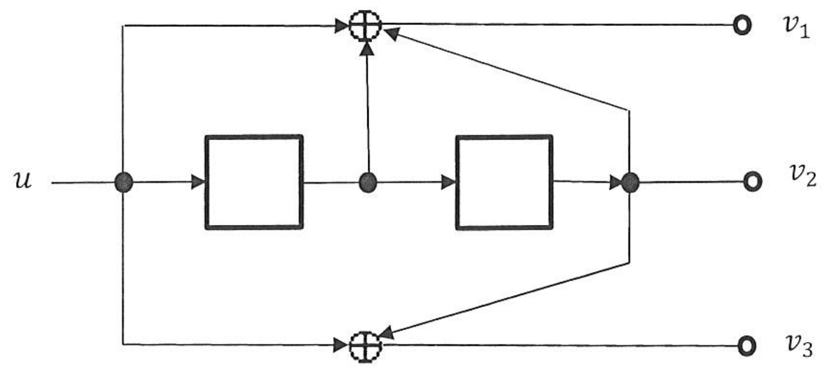

Figure 1

(a) (3%) Find the codeword **v** corresponding to the information sequence **u** = (1 1 1 0 1 0 0).

(b) (5%) Draw the state diagram of this encoder.

(c) (10%) Please use Viterbi algorithm to decode the received sequence (110 010 111 100 101 001), assuming that a binary symmetric channel with a crossover probability $p < 1/2$ is considered.

國立中山大學 111 學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學甲【電機系碩士班甲組、己組、庚組、電波聯合選考：電機系碩士班戊組、通訊所碩士班乙組】

—作答注意事項—

考試時間：100 分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學甲【電機系碩士班甲組、己組、庚組、電波聯合選考：電機系碩士班戊組、通訊所碩士班乙組】題號：431002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）

下面 1-15 題為單選題，總分 45 分。每題答對 3 分，答錯扣 4 分，未作答者以 0 分計。總分低於 0 分者以 0 分計算。

1. Consider the autonomous differential equation $y' = (2/\pi)y - \sin y$. Which of the following is INCORRECT?
   (A) There are three critical points.
   (B) One of critical point is semi-stable.
   (C) Two of critical points are unstable.
   (D) One of the critical points is 0.

2. If $y = e^{3x}\cos x$ is the solution to $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + ky = 0$, what is the value of k?
   (A) 3 (B) -2 (C) 10 (D) 8

3. The differential equation $e^x\frac{dy}{dx} + 3y = x^2y$ is linear and separable.
   (A) True (B) False

4. The improved Euler's method is what type of Runge-Kutta method?
   (A) First order (B) Second order (C) Third order (D) Fourth order

5. Consider $y(x)$ is the solution to the initial-value problem $x^2y'' - 2xy' + 2y = 0$ where $x > 0$, $y(1) = 4$, and $y'(1) = 9$, use Euler's method to compute $y(1.2)$. Given $h = 0.1$, which of the following is correct?
   (A) The general solution is $y = C_1x - C_2x^2$, where $C_1 + C_2 = 6$.
   (B) The general solution is $y = C_1x + C_2x^2$, where $C_1 + C_2 = 6$.
   (C) $y(1.2) = 5.9$.
   (D) $y(1.2) = 6$.

6. Given the three vectors (1, 0, 3, 1), (0, 1, -6, -1) and (0, 2, 1, 0) in R^4, they are linearly dependent.
   (A) True (B) False

7. Provided the system below, the rank is
$$X_1 - X_3 + 2X_4 + X_5 + 6X_6 = -3$$
$$X_2 + 2X_3 + 3X_4 + 2X_5 + 4X_6 = 1$$
$$X_1 - 4X_2 + 3X_3 + X_4 + 2X_6 = 0$$
   (A) 1 (B) 2 (C) 3 (D) 4

8. Which one of the following is correct regarding Fourier series?
   (A) $e^{-|x|}$ is odd function.
   (B) f' must be continuous on the interval $[a, b]$ to ensure that the Fourier series of f on $[a, b]$ converges to f.
   (C) $f(x) = |x|$ is continuous on $[-\pi, \pi]$.
   (D) The Fourier series of $f(x) = x^2 + 1$, where $0 < x < 3$, converges to 0 at $x = 0$.

9. Expand $f(x) = 2x^2 - 1, -1 < x < 1$ in a Fourier series and yield $f(x) = A + \sum \frac{B}{n^2\pi^2} C$. Which of the following is correct?
(A) $A = -2/3$　(B) $B = 4$　(C) $C = (-1)^n \cos n\pi x$　(D) None of the above

10. If $y_1(x) = x$ is one of the solutions of the following differential equation, what is the other linear independent solution $y_2(x)$?
$$y'' - \frac{2x}{1+x^2} y' + \frac{2}{1+x^2} y = 0$$
(A) $y_2(x) = 2x^2 + 1$　(B) $y_2(x) = \frac{x^2-1}{x}$　(C) $y_2(x) = \frac{1}{x} - 1$　(D) $y_2(x) = x^2 - 1$

11. Use the Laplace transform to solve the following initial-value problem. If the solution is $y = A + Be^{-t} + Ce^{3t} + De^{4t}$, which of the following is true?
$$y'' - 4y' = 6e^{3t} - 3e^{-t}, y(0) = 1,\ y'(0) = -1$$
(A) $A + B + C + D = 1$.
(B) $B = -2$
(C) $A + B + D = 2$
(D) All of the above

12. The Laplace transform of a function f is denoted by $\mathcal{L}\{f\}$. If $\mathcal{L}\{f(t)\} = F(s)$ and $\mathcal{L}\{g(t)\} = G(s)$, then $\mathcal{L}^{-1}\{F(s)G(s)\} = f(t)g(t)$.
(A) True　(B) False

13. If $\mathcal{L}\{f(t)\}$ represents the Laplace transform of a function $f(t)$. Let $f(t) = \begin{cases} 3 & , if\ 0 \le t \le 2 \\ 5-t, & if\ t > 2 \end{cases}$, then $\mathcal{L}\{f(t)\}$ is
(A) $\frac{3}{s^2} + \frac{e^{2s}}{s^2}$　(B) $\frac{3}{s} + \frac{e^{-2s}}{s^2}$　(C) $\frac{3}{s} - \frac{e^{-2s}}{s^2}$　(D) $\frac{3}{s^2} - \frac{e^{-2s}}{s^2}$

14. Provided the differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which of the following is true?
(A) first order, linear, non-homogeneous
(B) second order, nonlinear
(C) second order, linear, non-homogeneous
(D) second order, linear, homogeneous

15. The Fourier transform of a function f is denoted by $\Im\{f\}$. Suppose $\Im\{f(t)\} = F(\omega), \Im\{g(t)\} = g(\omega)$, which of the following is INCORRECT?
(A) $\int_{-\infty}^{\infty} f(\tau)g(t-\tau)\,d\tau = \Im^{-1}\{F(\omega)G(\omega)\}$
(B) $\int_{-\infty}^{\infty} f(t-\tau)g(\tau)\,d\tau = \Im^{-1}\{F(\omega)G(\omega)\}$
(C) $\Im\{f(t-\tau)\} = F(\omega)e^{-i\omega\tau}$
(D) None of the above

科目名稱：工程數學甲【電機系碩士班甲組、己組、庚組、電波聯合選考：電機系碩士班戊組、通訊所碩士班乙組】題號：431002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　共4頁第3頁

下面 **16-21** 題為複選題，每題 **5** 分，總分 **30** 分，每題有五個選項，其中至少有一個是正確答案，答錯 **1** 個選項者，得 **3** 分，答錯 **2** 個選項者，得 **1** 分，答錯多於 **2** 個選項或未作答者，該題以零分計算。

16. Let $\mathbf{A}$ and $\mathbf{B}$ be matrices in $\mathbb{R}^{n\times n}$. Which of the following statements are true?
(A) $\det(-\mathbf{A}) = -\det(\mathbf{A})$.
(B) If $\mathbf{A}\mathbf{A}^T = \mathbf{I}$, then $\det(\mathbf{A}) = 1$.
(C) If $\mathbf{A}\mathbf{A}^T = \mathbf{I}$, then $\mathrm{trace}(\mathbf{A}) = n$.
(D) If two rows of $\mathbf{A}$ are equal, then $\det(\mathbf{A}) = 0$.
(E) If $\det(\mathbf{A}) = \det(\mathbf{B})$, then $\mathbf{A}$ and $\mathbf{B}$ have the same rank.

17. Let $\mathbf{A} \in \mathbb{R}^{3\times 3}$ and its eigenvalues are λ_1, λ_1, and λ_2, where λ_1 and λ_2 are distinct eigenvalues. Suppose the dimension of $N(\mathbf{A} - \lambda_1\mathbf{I})$ is 1, where $N(\mathbf{A})$ denotes the null space of $\mathbf{A}$. Which of the following statements are true?
(A) λ_1 must be a real number (not a complex number).
(B) λ_2 must be a real number (not a complex number).
(C) The dimension of $N(\mathbf{A} - \lambda_2\mathbf{I})$ equals 1.
(D) $\mathbf{A}$ is diagonalizable.
(E) $\mathbf{A}$ has two linearly independent eigenvectors corresponding to λ_1.

18. Let $\mathbf{A} \in \mathbb{R}^{m\times n}$. Consider the linear equation $\mathbf{Ax} = \mathbf{b}$ or the homogeneous linear equation $\mathbf{Ax} = \mathbf{0}$. Which of the following statements are true?
(A) If $\mathrm{rank}(\mathbf{A}) = m$, then $\mathbf{Ax} = \mathbf{b}$ has at least one solution for any $\mathbf{b} \in \mathbb{R}^m$.
(B) If $\mathrm{rank}(\mathbf{A}) = m$, then $\mathbf{Ax} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$.
(C) If $\mathrm{rank}(\mathbf{A}) = n$, then $\mathbf{Ax} = \mathbf{b}$ has at most one solution for any $\mathbf{b} \in \mathbb{R}^m$.
(D) If $\mathrm{rank}(\mathbf{A}) = n$ and $m > n$, then $\mathbf{Ax} = \mathbf{0}$ has infinitely many solutions.
(E) If $\mathrm{rank}(\mathbf{A}) = m$ and $n > m$, then $\mathbf{Ax} = \mathbf{0}$ has infinitely many solutions.

19. Let $\mathbf{A}$ and $\mathbf{B}$ be square matrices. Suppose that $\mathbf{A}$ is similar to $\mathbf{B}$, that is, $\mathbf{B} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ for some nonsingular matrix $\mathbf{P}$. Which of the following statements are true?
(A) If $\mathbf{x}$ is an eigenvector of $\mathbf{B}$, then $\mathbf{x}$ is also an eigenvector of $\mathbf{A}$.
(B) If $\mathbf{y}$ is in the column space of $\mathbf{B}$, then $\mathbf{y}$ is also in the column space of $\mathbf{A}$.
(C) $\mathrm{trace}(\mathbf{A}) = \mathrm{trace}(\mathbf{B})$.
(D) $\mathbf{A} - \mathbf{I}$ is similar to $\mathbf{B} - \mathbf{I}$.
(E) $\mathbf{A}^5$ is similar to $\mathbf{B}^5$.

20. Let $\mathbf{A} \in \mathbb{R}^{m\times n}$, $R(\mathbf{A})$ denotes the column space of $\mathbf{A}$, $N(\mathbf{A})$ denotes the null space of $\mathbf{A}$, and $\dim(S)$ denotes the dimension of a subspace S. Which of the following statements are true?
(A) If $\mathbf{y} \in R(\mathbf{A})$, then $\mathbf{y} \in R(\mathbf{A}\mathbf{A}^T)$.
(B) If $\mathbf{x} \in N(\mathbf{A})$, then $\mathbf{x} \in N(\mathbf{A}\mathbf{A}^T)$.
(C) $\mathrm{rank}(\mathbf{A}) + \dim\big(N(\mathbf{A})\big) = \mathrm{rank}(\mathbf{A}^T) + \dim(N(\mathbf{A}^T))$.
(D) It is possible for a matrix $\mathbf{A}$ to have $[2,1,-1]^T$ in $N(\mathbf{A})$ and $[1,-2,3]^T$ in $R(\mathbf{A}^T)$.
(E) Let $\mathbf{y} \in \mathbb{R}^m$. If $\mathbf{y} = \mathbf{u}_1 + \mathbf{v}_1 = \mathbf{u}_2 + \mathbf{v}_2$, where $\mathbf{u}_1, \mathbf{u}_2 \in R(\mathbf{A})$ and $\mathbf{v}_1, \mathbf{v}_2 \in N(\mathbf{A}^T)$, then $\mathbf{u}_1 = \mathbf{u}_2$ and $\mathbf{v}_1 = \mathbf{v}_2$.

21. Let

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 3 & 2 \\ 1 & 4 & 3 & 3 \\ -1 & 11 & 6 & 7 \end{bmatrix}.$$

Which of the following vectors are in the column space of **A**?

(A) $[3,1,2]^T$

(B) $[1,0,-1]^T$

(C) $[0,1,3]^T$

(D) $[2,1,1]^T$

(E) $[4,2,-1]^T$

以下第 22 題到第 23 題需要詳明推導計算過程。如推導計算過程錯誤，將酌扣分數或不給分。

22. (10分) 求出以下複平面上之路徑積分值，z 為複數。

$\int_C \frac{z^5}{1-z^3}dz$，其中 C 為沿著 $\{z : |z| = 2\}$ 正向旋轉一周之封閉路徑。

23. (15分) 利用餘值（residues）求取以下瑕積分，其中參數 $a > 0$。

$$\int_0^{\infty} \frac{\cos ax}{x^2+1}dx$$

國立中山大學 111 學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：電子學【電波聯合碩士班選考、通訊所碩士班乙組選考、電機系碩士班戊組選考】

—作答注意事項—

考試時間：100 分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：電子學【電波聯合碩士班選考、通訊所碩士班乙組選考、電機系碩士班戊組選考】題號：482003

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. (30%) For the common-base circuit in Fig. 1, assuming the bias current to be about 1 mA, β = 100, C_μ = 0.8 pF, r_e = 25 Ω, and f_T = 800 MHz:
   (a) Estimate the midband gain V_o/V_s. (10%)
   (b) Use the short-circuit time-constants method to estimate the lower 3-dB frequency, f_L. (10%) (*Hint:* In determining the resistance seen by C_1, the effect of the 47-kΩ resistor must be taken into account.)
   (c) Find the high-frequency poles, and estimate the upper 3-dB frequency, f_H. (10%)

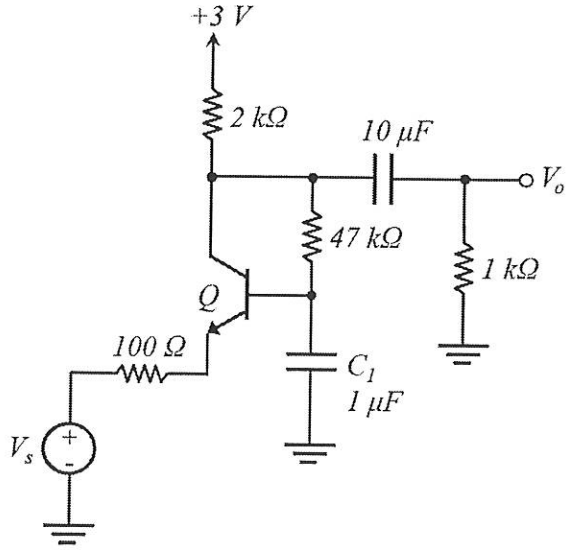

Fig. 1

2. (25%) The current-steering circuit of Fig. 2 is fabricated in a CMOS technology for which k_n' = 90 µA/V², k_p' = 30 µA/V², V_{tn} = 0.8 V, and V_{tp} = -0.9 V. If all devices have L = 2 µm, design the circuit so that I_{REF} = 20 µA, I_2 = 100 µA, and I_5 = 40 µA. Use the minimum width of 2 µm for as many of the devices as possible. (a) Give the required width for each transistor and the value of R required. (10%) (b) What is the highest voltage possible at the drain of Q_2? (5%) (c) What is the lowest voltage possible at the drain of Q_5? If $|V_{Ap}|$ = 16 L, where L is in µm and V_{Ap} is in volts, (5%) (d) find the output resistance of the current source Q_2. (5%)

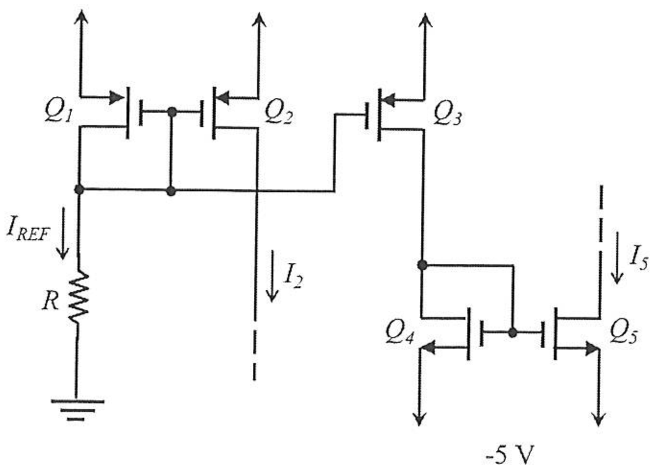

Fig. 2

國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：電子學【電波聯合碩士班選考、通訊所碩士班乙組選考、電機系碩士班戊組選考】題號：482003

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）

3. (15%) A second-order filter has its poles at $s = -(1/8) \pm j(\sqrt{63}/8)$. The transmission is zero at ω = 5 rad/s and is unity at dc (ω = 0). Find the transfer function.

4. (30%) For the emitter-follower circuit shown in Fig. 3 the BJT used is specified to have β values in the range of 20 to 200. For the two extreme values of β = 20 and β = 200, find :
(a) I_E, V_E, and V_B. (10%)
(b) the input resistance R_i. (10%)
(c) the voltage gain v_o/v_s. (10%)

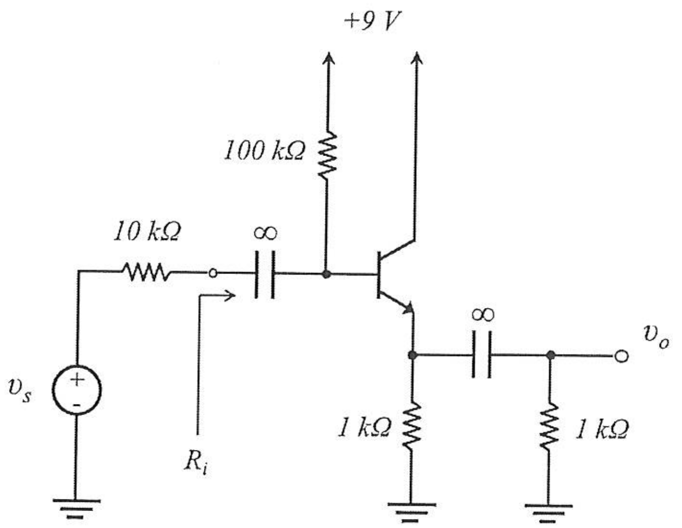

Fig. 3

國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：電磁學【電波聯合碩士班、通訊所碩士班乙組、電機系碩士班戊組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：電磁學【電波聯合碩士班、通訊所碩士班乙組、電機系碩士班戊組】題號：482004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　共 2 頁第 1 頁

1. (3%) (a) At any point (x_0, y_0, z_0) in the domain of a scalar function $V(x, y, z)$, we take a path a_ℓ along $V = c_i$, where c_i is a constant, or take a path a_n along ∇V. Tell me about the main characteristic (主要特徵) of these two paths a_ℓ and a_n, and also what is $a_\ell \cdot a_n$, the dot product of a_ℓ and a_n?

   (3%) (b) () = $\nabla V \cdot (a_\ell)\, d\ell$, where V is a scalar function, $d\ell$為任意方向 a_ℓ 之小路徑。() 裏應填什麼？

   (3%) (c) 利用 Divergence theorem for $\nabla \cdot$E 寫下 E 和 Q (真空中有一 charge Q)的關係。

   (3%) (d) 利用 Stokes' theorem for $\nabla \times$B 寫下 B 和 I (真空中有一 current I) 的關係。

   (3%) (e) 在運算 Divergence $\nabla \cdot$A 或 Curl $\nabla \times$ A時 (A 為一向量場)，我們選擇的體積或面積在大小和形狀各有何限制？

2. (5%) Using the *Method of Image*, write down the potential distribution, $V(x, y, z)$, for a point P(x, y, z) in the space, Fig. 1, the dielectric constant of the space is ε_0. Q is a positive point charge of Q 庫侖 *Coul.*

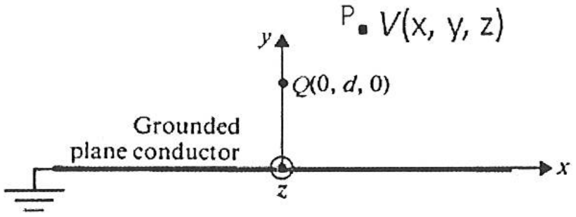

Fig. 1. A point charge Q distance d above the Ground.

3. (3%) (a) 在 dielectric constant 為 ε_r (=1+X_e) dielectric 之內部電場為 E (V/m)，請問 Polarization vector P 為何？

   (3%) (b) 在 relative permeability 為 μ_r (=1+X_m)的一 ferromagnetic material 外面線圈通電流，在其內部產生磁場 H (A/m)，請問 Magnetization vector M 為何？

   (4%) 銅的導電性很好，(c) 它的 permittivityε 和 permeability μ 各為何？請簡單提供你的理由。

4. For a coaxial transmission line shown in Fig. 2, the capacitance per unit length is $c = \frac{2\pi\varepsilon_0}{ln\frac{b}{a}} \left[\frac{\text{F}}{\text{m}}\right]$, and the inductance per unit length is $\ell = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi}\ln\frac{b}{a} \left[\frac{\text{H}}{\text{m}}\right]$. At high frequencies, the internal inductance drops off (that is, approaching 0, and you should know which term is the internal inductance).

   (2%) (a) Find the characteristic impedance of the coaxial line $Z_c=(l/c)^{0.5}$ at high frequencies 請務必寫 Z_c 之單位。

   (2%) (b) 請問在地 (Ground, 即半徑 b 粗體部分) 之外的 magnetic flux density B 值為何？

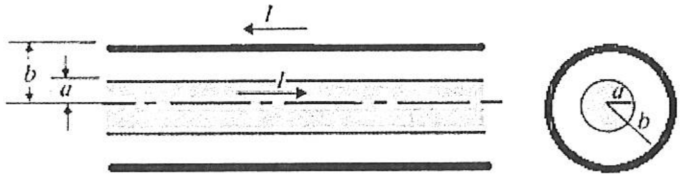

Fig. 2. Coaxial cable side and cross-sectional views, 粗體線代表地 Ground

The capacitance of a line charge of radius a over a ground 0, as shown in Left of Fig. 3 $C = \frac{2\pi\varepsilon_0}{ln\frac{2h}{a}} \left[\frac{\text{F}}{\text{m}}\right]$.

(3%) (c) Find the external inductance L for such a transmission system in air using a quasi-TEM property $L \cdot C = \mu_0 \cdot \varepsilon_0$.

(3%) (d) Obtain the internal inductance from the inductance formula l, also the external inductance found in c), write down the per unit length internal & external inductance for the conductor system shown in the Right of Fig. 3.

試題請隨卷繳回，請留意背面是否有題

國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：電磁學【電波聯合碩士班、通訊所碩士班乙組、電機系碩士班戊組】題號：482004
※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　共2頁第2頁

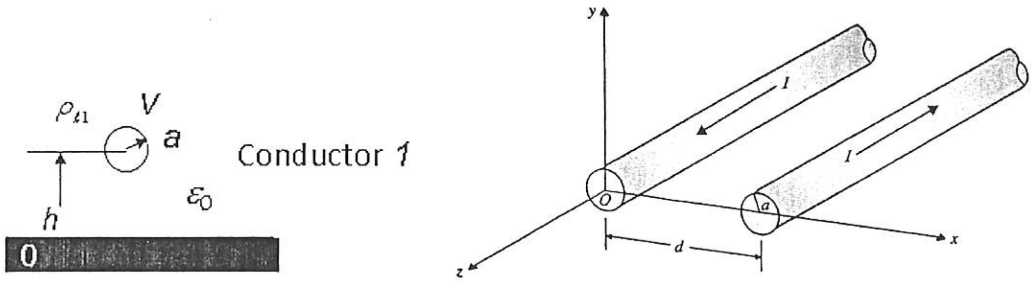

Fig. 3. A single conductor above a Ground (Left); A two-conductor system with currents flow in opposite direction (Right); the two conductors both with radius *a* and *d* distance apart.

5. (5%) (a) A position vector R = $a_x(x-x') + a_y(y-y') + a_z(z-z')$, $R = [(x-x')^2+(y-y')^2+(z-z')^2]^{1/2}$, $a_R = R/R$, where $P(x, y, z)$ is an observation point, and $P'(x', y', z')$ is a source point. Show that $\nabla'(\frac{1}{R})=(a_R\frac{1}{R^2})$, $\nabla' f$ is the gradient operator with respect to the source coordinates, that is, $\nabla' f = a_x\frac{\partial f}{\partial x'} + a_y\frac{\partial f}{\partial y'} + a_z\frac{\partial f}{\partial z'}$.
(5%) (b) As shown in Fig. 4, determine B at $P(0, 0, z^*)$?

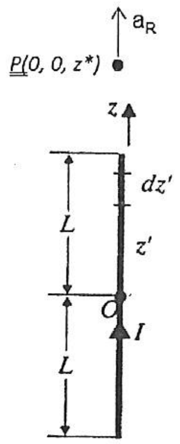

Fig. 4.

6. (10%) (a) Derive the electromagnetic wave equation in free space.
(5%) (b) Explain the traveling-wave factor.

7. A uniform plane wave ($\mathbf{E}_i$, $\mathbf{H}_i$) of an angular frequency ω is incident from air (medium 1) on a very large, perfectly conducting wall (medium 2) at an angle of incidence θ_i with perpendicular polarization. Find
(10%) (a) **E** and **H** in medium 1.
(5%) (b) **E** and **H** in t medium 2.
(5%) (c) the current induced on the wall surface, and
(5%) (d) the time-average Poynting vector in medium 1.

8. (10%) As shown in Fig. 5 with $Z_1 = \sqrt{Z_0 Z_L}$ and $l = \lambda/4$, please explain how the circuit works to achieve impedance matching between Z_0 at Z_L, and obtain the bandwidth with the maximum Γ of Γ_m.

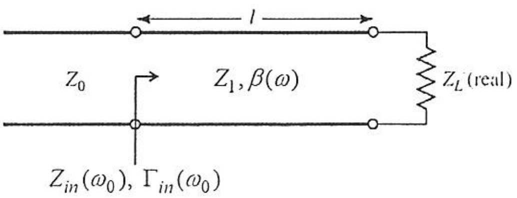

Fig. 5.

試題請隨卷繳回，請留意背面是否有題