國立中山大學110學年度
碩士暨碩士專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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國立中山大學110學年度碩士暨碩士專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】 題號：437005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題） 共3頁第1頁

一、選擇題(單選，一題五分，計分方式:不倒扣，答對得該題全部分數，答錯及未作答得零分)

1. Two coins are tossed simultaneously. If one of them turned head, what is the probability that the other one also turns head?
   (A) 0.1
   (B) 0.25
   (C) 0.5
   (D) 0.75
   (E) None of these

2. Let A_1, A_2, A_3 be events in a sample space $\mathcal{S}$. Let $P(A_1) = 0.4,\ P(A_2) = 0.2,\ P(A_3) = 0.4$, and $A_i \cap A_j = \emptyset$ if $i \neq j$. Let B be another event with $P(B|A_1) = 0.1,\ P(B|A_2) = 0.2,\ P(B|A_3) = 0.05$. What is $P(A_1 \cup A_2|B)$?
   (A) 0.8
   (B) 0.4
   (C) 0.1
   (D) 0.08
   (E) None of these

3. Consider two random variables X and Y with joint pdf
$$f(x,y) = \begin{cases} x+y & 0 \leq x,y \leq 1 \\ 0 & otherwise \end{cases}$$
   Which of the following statements is correct?
   (A) $\mathbf{E}[X] = 1/2$
   (B) $P(X+Y \leq 1) = 1/2$
   (C) $\mathbf{E}[XY] = 1/3$
   (D) X and Y are uncorrelated
   (E) X and Y are independent

4. Let U be uniformly distributed over $[0,1]$. Which of the following statements is wrong?
   (A) $\mathbf{E}[U] = 1/2$
   (B) $Var(U) = 1/12$
   (C) Let $Y = -\ln(U)$. The pdf of Y is $f(y) = e^{-y}$, for $y > 0$.
   (D) Let $Z = e^U$. The pdf of Z is $f(z) = 1/z$, for $1 \leq z \leq e$.
   (E) The MGF of U is $M_U(t)$=e^t/s.

5. Consider two independent random variables X_1 and X_2 with Poisson distribution:
$$P(X_i = k) = \frac{\lambda_i^k e^{-\lambda}}{k!}, \qquad k = 0,1,2,3, ...$$
   Which of the following statements is wrong?
   (A) The mean of X_i is λ_i
   (B) The variance of X_i is $1/\lambda_i$
   (C) The moment generating function of X_i is $M_{X_i}(t) = e^{\lambda_i(e^t-1)}$
   (D) $X_1 + X_2$ is Poisson distributed
   (E) None of these

6. Suppose U_1 and U_2 are independent uniform random variables on $[0,1]$. Let $X = \min\{U_1, U_2\}$. What is $\mathbf{E}[X]$?
   (A) 1/4
   (B) 1/3
   (C) 1/2
   (D) 2/3
   (E) 3/4

7. If the pdf of a continuous random variable is given as
$$f(x) = \begin{cases} x/2, & 0 \le x \le 2, \\ 0, & \text{otherwise}, \end{cases}$$
   what is the value of $P(X = 1)$?
   (A) 0
   (B) 1/4
   (C) 1/3
   (D) 1/2
   (E) 1

8. David invests 100 dollars in a stock. Each year his investment doubles in value with probability 0.4 and decreases in value by 50% with probability 0.6. What is the expected value of David's investment after 20 years?
   (A) 90
   (B) 110
   (C) 259
   (D) 673
   (E) 1745

9. Let X be a random variable with a moment generating function of the form
$$M_X(t) = \frac{1}{1 - 2t^2}, \quad |t| < \frac{1}{\sqrt{2}}.$$
   What is the variance of X ?
   (A) 0
   (B) 1
   (C) 2
   (D) 3
   (E) 4

10. Let X and Y be the outcomes of two independent dies and each die takes value of $\{1,2,3,4,5,6\}$ equally. Define $W = X + Y$. What is $P(X = 3|W = 8)$?
   (A) 1/2
   (B) 1/3
   (C) 1/4
   (D) 1/5
   (E) 1/6

科目名稱：機率【通訊所碩士班甲組】 題號：437005

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二、問答計算題:

1. (25%) Consider two continuous random variables X and Y with joint pdf:

$$f(x,y) = \begin{cases} ce^{-x} & x \geq 0, |y| < x \\ 0 & otherwise \end{cases}$$

   (a) (5%) Find the constant c
   (b) (5%) Find the conditional pdf $f_{X|Y(x|y)}$
   (c) (5%) Find the conditional pdf $f_{Y|X(y|x)}$
   (d) (5%) Find the conditional mean $\mathrm{E}[Y|X=x]$, for $x \geq 0$
   (e) (5%) Find the conditional variance $\mathrm{Var}(Y|X=x)$, for $x \geq 0$

2. (10%) Let X and Y be two i.i.d. Geometric random variables with pmf

$$P(X=k) = P(Y=k) = (1-p)^{k-1}p,$$

   for $0 < p < 1$ and $k = 1,2,3, \ldots$. Let $W = \max\{x, y\}$. Find the distribution of W.

3. (15%) Let X be an exponential random variable with the following pdf

$$f(x) = \lambda e^{-\lambda x}, \; x \geq 0.$$

   (a) (5%) Find the moment generating function (MGF) of X . (Please identify the domain of function)
   (b) (5%) Using MGF, find $\mathbf{E}[X^n]$
   (c) (5%) Find the conditional mean $\mathbf{E}[X|X>1]$

國立中山大學110學年度
碩士暨碩士專班招生考試試題

科目名稱：線性代數【通訊所碩士班甲組】

—作答注意事項—

考試時間：100分鐘

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國立中山大學 110 學年度碩士暨碩士專班招生考試試題

科目名稱：線性代數【通訊所碩士班甲組】　　題號：437006

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

In the following, boldface capital and lower-case letters denote matrices and vectors, respectively. For all questions, please provide both answers and justifications.

1. (10%) Consider the set $S = \{0,1\}$ with the operations:

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

A binary matrix $\mathbf{H}_{2\times5}$ with the form of $[\mathbf{P^T}_{2\times3} \quad \mathbf{I}_{2\times2}]$ can be generated from another binary matrix $\mathbf{G}_{3\times5}$ such that $\mathbf{G}\cdot\mathbf{H}^{\mathrm{T}} = \mathbf{0}$. $\mathbf{I}_{2\times2}$ denotes an identity matrix. Suppose we have a matrix $\mathbf{G}'$ given by

$$\mathbf{G}' = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix},$$

Please use $\mathbf{G}'$ to find binary matrices $\mathbf{G}$ and $\mathbf{H}$. (Hint: use reduced row echelon form)

2. (8%) Let $\mathbf{A} = \begin{bmatrix} 6 & 5 \\ 4 & 8 \end{bmatrix}$ and $f(s) = s^4 - 13s^3 + 17s^2 - 13s + 87$. Please find $f(\mathbf{A})$.

3. (12%) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by $T(\mathbf{x}) = [x_1 + x_2, x_1 - 2x_2, x_2]^{\mathrm{T}}$. If we represent any vector $\boldsymbol{\alpha}$ in R^2 as $\boldsymbol{\alpha} = \alpha_1 \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix} + \alpha_2 \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and any vector $\boldsymbol{\beta}$ in R^3as

$\boldsymbol{\beta} = \beta_1 \cdot \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + \beta_2 \cdot \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} + \beta_3 \cdot \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$, please find the corresponding matrix $\mathbf{M}$ such that

$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \mathbf{M} \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$.

4. (8%) Consider the matrix $\mathbf{A} = \begin{bmatrix} 0 & -1 & 4 \\ -1 & 3 & a \\ 4 & a & 0 \end{bmatrix}$. Diagonalization of matrix $\mathbf{C}$ is denoted by $\mathbf{Q}^{\mathrm{T}}\mathbf{A}\mathbf{Q}$, where $\mathbf{Q}$ is an orthogonal matrix. If $\frac{1}{\sqrt{6}}[1 \quad 2 \quad 1]^{\mathrm{T}}$ is the first column of $\mathbf{Q}$, please find the value of a.

5. (12%) Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be any vectors in $\mathbb{R}^n$. Let $\mathbf{w}_1 = 3\mathbf{v}_1 + 2\mathbf{v}_2$, $\mathbf{w}_2 = \mathbf{v}_2 + 3\mathbf{v}_3$, and $\mathbf{w}_3 = -\mathbf{v}_1 + 2\mathbf{v}_3$. Determine whether $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3$ are independent or not and explain specifically.

6. (8%) Suppose $\mathbf{A} \in \mathrm{F}^{nxn}$ is a normal matrix. Prove that the eigenvectors of $\mathbf{A}$ corresponding to different eigenvalues are orthogonal.

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7. (12%) Suppose matrix $\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 \\ 8 & 2 & 8 \\ 1 & 0 & 2 \end{bmatrix}$. Find a matrix $\mathbf{B}$ such that $\mathbf{B} \cdot \mathbf{B} = \mathbf{A}$.

8. (10%) Let $\mathbf{A} = \begin{bmatrix} a & b & b & \cdots & b \\ b & a & b & \cdots & b \\ b & b & a & \cdots & b \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b & b & b & \cdots & a \end{bmatrix}$ be an $n \times n$ matrix and denote the adjugate of $\mathbf{A}$ as $\operatorname{adj}(\mathbf{A})$.
Please answer the following questions using parameters a and b.

(A) (3%) Compute $\det\left(\operatorname{adj}(\mathbf{A})\right)$.

(B) (3%) Compute $\left(\operatorname{adj}(\mathbf{A})\right)^{-1}$.

(C) (4%) Prove that $\det\left(\operatorname{adj}(\mathbf{A})\right) \cdot \left(\operatorname{adj}(\mathbf{A})\right)^{-1} = \operatorname{adj}\left(\operatorname{adj}(\mathbf{A})\right)$.

9. (12%) Let $= \begin{bmatrix} 1 & -1 & 0 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & -4 \end{bmatrix}$. Please compute the $\mathbf{QR}$ decomposition of $\mathbf{A}$, where $\mathbf{Q}$ is an orthonormal matrix and $\mathbf{R}$ is an upper-triangular matrix.

10. (8%) Find a matrix representation $\mathbf{A}$ for the reflection of the plane in the line $y = mx$. (Hint : Assume that $\mathbf{b}_1$ is a vector which lies along the line $y = mx$, and $\mathbf{b}_2$ is a vector which is orthogonal to $\mathbf{b}_1$.)

國立中山大學110學年度
碩士暨碩士專班招生考試試題

科目名稱：電磁學【電波聯合碩士班、通訊所碩士班乙組、電機系碩士班戊組】

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國立中山大學110學年度碩士暨碩士專班招生考試試題

科目名稱：電磁學【電波聯合碩士班、通訊所碩士班乙組、電機系碩士班戊組】題號：482004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題） 共2頁第1頁

1. (4%) (a) At any point (x_0, y_0, z_0) in the domain of a scalar function $V(x, y, z)$, we take a path a_ℓ along $V = c_i$, where c_i is a constant, or take a path a_n along ∇V. Tell me about the main characteristic (主要特徵) of these two paths a_ℓ and a_n, and also what is $a_\ell \cdot a_n$, the dot product of a_ℓ and a_n?

   (4%) (b) () = $\nabla V \cdot (a_\ell) d\ell$, where V is a scalar function, $d\ell$為任意方向 a_ℓ 之小路徑。() 裏應填什麼？

   (4%) (c) 利用 Divergence theorem for $\nabla \cdot$E 寫下 E 和 Q (真空中有一 charge Q)的關係，and

   (4%) (d) 利用 Stokes' theorem for $\nabla \times$B 寫下 B 和 I (真空中有一 current I)的關係。

   (4%) (e) 在運算 Divergence $\nabla \cdot$A 或 Curl $\nabla \times$ A時 (A 為一向量場)，我們選擇的體積或面積在大小和形狀各有何限制？

2. (5%) Using the *Method of Image*, write down the potential distribution, $V(x, y, z)$, for a point P(x, y, z) in the space, Fig. 1, the dielectric constant of the space is ε_0. Q is a positive point charge of Q 庫侖 *Coul*.

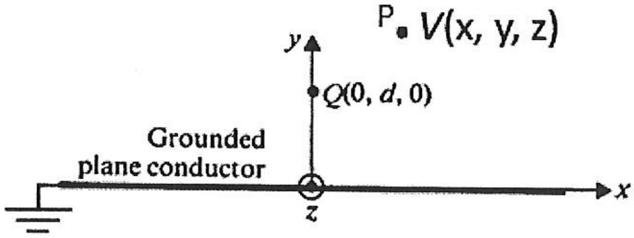

Fig. 1. A point charge Q distance d above the Ground.

3. 在 dielectric constant 為 ε_r (=1+X_e) 的 dielectric 內部之電場為 E (V/m)

   (3%) (a) 請問 Polarization vector P 為何？在 relative permeability 為 μ_r (=1+X_m)的一 ferromagnetic material 外面線圈通電流，在其內部產生磁場為 H (A/m)，

   (3%) (b) 請問 Magnetization vector M 為何？

   (3%) (c) 銅的導電性很好，它的 permittivity ε 和 permeability μ 各為何？請簡單提供你的理由。

4. For a coaxial transmission line shown in Fig. 2, the capacitance per unit length is $C = \dfrac{2\pi\varepsilon_0}{ln\frac{b}{a}}$ $\left[\dfrac{\text{F}}{\text{m}}\right]$, and the inductance per unit length is $\ell = \dfrac{\mu_0}{8\pi} + \dfrac{\mu_0}{2\pi}\ln\dfrac{b}{a}$ $\left[\dfrac{\text{H}}{\text{m}}\right]$. At high frequencies, the internal inductance drops off (that is, approaching 0, and you should know which term is the internal inductance).

   (4%) (a) Find the characteristic impedance of the coaxial line $Z_c = \sqrt{\dfrac{\ell}{c}}$ at high frequencies 請務必將 internal inductance 拿掉，並寫 Z_c 之單位 (即簡化$\sqrt{\text{H/F}}$)。

   (4%) (b) 請問在地(Ground, 即半徑 b 粗體部分) 之外的 magnetic flux density B 值為何？

   The capacitance of a line charge of radius a over a ground 0, as shown in Left of Fig. 3 $C = \dfrac{2\pi\varepsilon_0}{ln\frac{2h}{a}}$ $\left[\dfrac{\text{F}}{\text{m}}\right]$.

(4%) (c) Find the external inductance L for such a transmission system in air using a quasi-TEM property $L \cdot C = \varepsilon_0 \cdot \mu_0$.

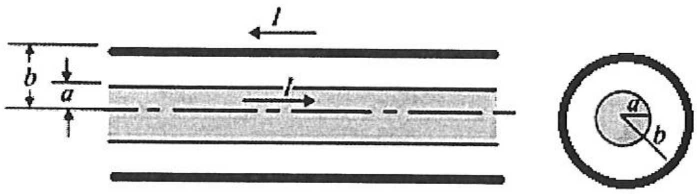

Fig. 2. Coaxial cable side and cross-sectional views, 粗體線代表地 Ground

(4%) (d) Using the result from a), along with the per unit length external inductance found in c), write down the internal & external inductance for the conductor system shown in the Right of Fig. 3.

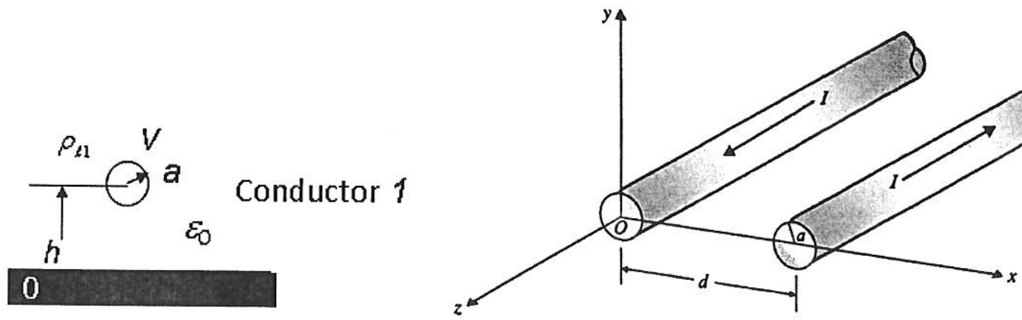

Fig. 3. A single conductor above a Ground (Left); A two-conductor system with currents flow in opposite direction (Right); the two conductors both with radius a and d distance apart.

5. (10%) A right-hand circularly polarized wave represented by the phasor $\boldsymbol{E}(z) = E_0(\boldsymbol{a}_x - j\boldsymbol{a}_y)e^{-j\beta z}$ is incident normally from air onto a perfect conductor at $z = 0$. Find the induced current on the conducting wall.

6. (15%) An air-filled $a \times b$ ($b < a < 2b$) rectangular waveguide is to be constructed to operate at 5.8 GHz in the dominant mode. We desire the operating frequency to be at least 25% higher than the cut-off frequency of the dominant mode and also at least 30% below the frequency of the next higher-order mode. Give a typical design for the dimensions a and b.

7. The open-circuited and short-circuited impedances measured at the input terminals of a lossless transmission line of length 1.5 (m), which is less than a quarter wavelength, are $-j54.6$ (Ω) and $j103$ (Ω), respectively.
(15%) (a) Determine Z_0, α, and β of the line.
(10%) (b) Determine R, L, G, and C of the line.

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科目名稱：通訊理論【通訊所碩士班電波聯合碩士班選考、通訊所碩士班甲組、乙組選考、電機系碩士班戊組選考】

—作答注意事項—

考試時間：100 分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，不得另攜帶紙張，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，其後果由考生自行負擔。
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- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學110學年度碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班電波聯合碩士班選考、通訊所碩士班甲組、乙組選考、電機系碩士班戊組選考】題號：437002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　共2頁第1頁

1. (30%) Short answer questions.

(a). (10%) Explain the sampling theory.

(b). (10%) What is the Nyquist criterion?

(c). (5%) What is color noise?

(d). (5%) Explain why the matched filter is optimum in the additive white Gaussian noise (AWGN) channel.

2. (25%) Consider a signal $x(t)$ whose Fourier transform is expressed as

$$X(jw) = \delta(w) + \delta(w-\pi) + \delta(w-2),$$

and assume

$$h(t) = u(t) - u(t-2),$$

where $u(t)$ is the unit step function.

(a). (10%) Is $x(t)$ a periodic signal? Justify your answer.

(b). (5%) Compute the Fourier transform of $h(t)$.

(c). (10%) Is $x(t)*h(t)$ a periodic signal? Provide your reason.

3. (5%) Consider a signal $x(t) = u(t+T_0) - u(t-T_0)$, where $u(t)$ is the unit step function. Performing the impulse-train sampling on $x(t)$, we can have $x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t-nT)$. What is the sampling period so that $x(t)$ can be perfectly reconstructed by $x_p(t)$?

4. (10%) Assume that we have a complex-value Gaussian random variable $Z = X + jY$, where X and Y are independent identically distributed random variables with zero mean and variance σ^2. Suppose $W = Ze^{j\phi}$ with a fixed value of ϕ. Show that W and Z have the same joint probability density function (PDF).

5. (10%) Assume that $p_1(t)$ and $p_2(t)$ are two orthogonal complex waveforms.

(a). (5%) Let $\phi_1(t) = p_1(t)e^{j2\pi f_c t}$ and $\phi_2(t) = p_2(t)e^{j2\pi f_c t}$. Prove that $\phi_1(t)$ and $\phi_2(t)$ are orthogonal for any f_c.

(b). (5%) Let $p_2(t) = p_1(t-T)$. Show that $\phi_2(t) = \phi_1(t-T)$ as f_c is an integer multiple of $1/T$.

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※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

6. (20%) Consider a simple communication system that contains a single antenna at the transmitter and M antennas at the receiver. The received signal at the mth antenna with a binary transmitted signal can thus be expressed as

$$y_m = uh_m + w_m,$$

where $u = \pm a$ is the transmitted signal, h_m is the gain of antenna m, and w_m is the associated AWGN with zero mean and variance σ^2. Herein, u, $w_1,\ldots,$ w_M are mutually independent. Collecting $y_1,\ldots,$ y_M into a vector, we have

$$\mathbf{y} = u\mathbf{h} + \mathbf{w},$$

where $\mathbf{y} = \begin{bmatrix} y_1 & \cdots & y_M \end{bmatrix}^T$, $\mathbf{h} = \begin{bmatrix} h_1 & \cdots & h_M \end{bmatrix}^T$ and $\mathbf{w} = \begin{bmatrix} w_1 & \cdots & w_M \end{bmatrix}^T$.

(a). (5%) Combine all y_k's with the coefficients c_k, $k = 1,\ldots,M$. We have $Y = \sum_{m=1}^{M} c_m y_m = \mathbf{c}^T\mathbf{y}$. Decide the maximum likelihood detector for u given the observation Y.

(b). (5%) What is the error probability P_e of the detector in (a)?

Hint: You can use Q-function, where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{u^2}{2}\right)du$.

(c). (5%) Explain why changing $\alpha\mathbf{c}$ to $\mathbf{c}$ for some nonzero scalar does not change P_e.

(d). (5%) Compute $\mathbf{c}$ so that P_e is minimized.

國立中山大學110學年度
碩士暨碩士專班招生考試試題

科目名稱：工程數學甲【電機系碩士班電波聯合碩士班選考、電機系碩士班甲組、通訊所碩士班乙組選考、電機系碩士班戊組選考、己組、庚組】

—作答注意事項—

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國立中山大學110學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學甲【電機系碩士班電波聯合碩士班選考、電機系碩士班甲組、通訊所碩士班乙組選考、電機系碩士班戊組選考、己組、庚組】題號：431002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）

下面 **1-10** 題為複選題，每題 **5** 分，總分 **50** 分。每題有 **5** 個選項，僅答錯 **1** 個選項者，得 **3** 分；答錯 **2** 個選項者，得 **1** 分；答錯多於 **2** 個選項或未作答者，該題以零分計算。

1. Let $\mathbf{x}$ and $\mathbf{y}$ be nonzero vectors in $\mathbb{R}^n$, $n \geq 2$. Suppose $\mathbf{x}$ and $\mathbf{y}$ are not orthogonal, i.e. $\mathbf{x}^T\mathbf{y} \neq 0$. Let $\mathbf{A} = \mathbf{x}\mathbf{y}^T$. Which of the following statements are true?
   (A) 0 is an eigenvalue of $\mathbf{A}$.
   (B) $\mathbf{x}^T\mathbf{y}$ is an eigenvalue of $\mathbf{A}$.
   (C) $\mathbf{x}$ is an eigenvector of $\mathbf{A}$.
   (D) The number of linearly independent eigenvectors associated with eigenvalue 0 is $n-1$.
   (E) $\mathbf{A}$ is diagonalizable.

2. Let $\mathbf{A}$ and $\mathbf{B}$ be square matrices and α be a real number. Which of the following statements are true?
   (A) $\det(\mathbf{A}^2) = \det(\mathbf{A}^T\mathbf{A})$
   (B) $\det(\mathbf{B}^T\mathbf{A}) = \det(\mathbf{A}^T\mathbf{B}^T)$
   (C) $\det(\alpha\mathbf{A}) = \alpha \cdot \det(\mathbf{A})$
   (D) $\det(\mathbf{A}+\mathbf{B}) = \det(\mathbf{A}) + \det(\mathbf{B})$
   (E) Swapping two rows of $\mathbf{A}$ will not change its determinant.

3. Consider the linear function $L: \mathbb{R}^3 \to \mathbb{R}^3$. Suppose $L(\mathbf{x}_i) = \mathbf{y}_i$ for $i = 1,2,3,4$, where
$$\mathbf{x}_1 = \begin{bmatrix}1\\0\\-1\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}1\\2\\0\end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix}2\\-1\\1\end{bmatrix}, \mathbf{x}_4 = \begin{bmatrix}3\\3\\2\end{bmatrix}, \mathbf{y}_1 = \begin{bmatrix}1\\1\\0\end{bmatrix}, \mathbf{y}_2 = \begin{bmatrix}0\\1\\-1\end{bmatrix}, \mathbf{y}_3 = \begin{bmatrix}1\\0\\-1\end{bmatrix}.$$
   Suppose $L(\mathbf{x}_4) = [c_1, c_2, c_3]^T$. Which of the following statements are true?
   (A) $c_1 = 1$ (B) $c_1 = 2$ (C) $c_2 = 2$ (D) $c_2 = 3$ (E) $c_3 = -3$

4. Let $\mathbf{A} \in \mathbb{R}^{m\times n}$, $\mathrm{R}(\mathbf{A})$ denote the column space of $\mathbf{A}$, and $\mathrm{N}(\mathbf{A})$ denote the null space of $\mathbf{A}$. Which of the following statements are true?
   (A) There exists a nonzero vector $\mathbf{y} \in \mathbb{R}^m$ such that $\mathbf{A}^T\mathbf{y} = \mathbf{0}$ and $\mathbf{y} = \mathbf{A}\mathbf{x}$ for some $\mathbf{x} \in \mathbb{R}^n$.
   (B) For every $\mathbf{y} \in \mathrm{R}(\mathbf{A})$, we can always find some $\mathbf{x} \in \mathrm{R}(\mathbf{A}^T)$ such that $\mathbf{y} = \mathbf{A}\mathbf{x}$.
   (C) Let $\mathbf{p}$ be the orthogonal projection of $\mathbf{y} \in \mathbb{R}^m$ onto $\mathrm{R}(\mathbf{A})$. Then, $\mathbf{y} - \mathbf{p} \in \mathrm{N}(\mathbf{A}^T)$.
   (D) Let $\mathbf{y}$ be a vector in $\mathbb{R}^m$. If $\mathbf{y} \notin \mathrm{R}(\mathbf{A})$, then $\mathrm{y} \in \mathrm{N}(\mathbf{A}^T)$.
   (E) The intersection of $\mathrm{R}(\mathbf{A})$ and $\mathrm{N}(\mathbf{A}^T)$ is the empty set.

5. Let $E = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis for a three-dimensional subspace S of an inner product space V. Define $\|\mathbf{v}\| = \sqrt{\langle \mathbf{v}, \mathbf{v}\rangle}$. Suppose E is an orthogonal set with $\|\mathbf{v}_1\| = \sqrt{2}$, $\|\mathbf{v}_2\| = \sqrt{3}$, $\|\mathbf{v}_3\| = 1$. Let $\mathbf{x} = 5\mathbf{v}_1 + 1\mathbf{v}_2 - 3\mathbf{v}_3$ and $\mathbf{y} = 1\mathbf{v}_1 - 3\mathbf{v}_2 - 2\mathbf{v}_3$. Let $\mathbf{p}$ be the orthogonal projection of $\mathbf{x}$ onto $\mathbf{y}$, and suppose $\mathbf{p} = \alpha\mathbf{y}$, where α is a scalar. Which of the following statements are true?
   (A) $\langle \mathbf{x}, \mathbf{y}\rangle = 7$
   (B) $\langle \mathbf{x}, \mathbf{y}\rangle = 8$
   (C) $\alpha = 7/33$
   (D) $\alpha = 8/14$
   (E) $\|\mathbf{x}\| = 62$

6. Let $\mathbf{A} \in \mathbb{R}^{5\times3}$ and nullity$(\mathbf{A})$ denote the dimension of N$(\mathbf{A})$. Suppose nullity$(\mathbf{A}) = 2$. Which of the following statements are true?
(A) rank$(\mathbf{A}^T) = 3$
(B) nullity$(\mathbf{A}^T) = 4$
(C) Let $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3]$, where $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ are column vectors of $\mathbf{A}$. Then $\mathbf{a}_1$ and $\mathbf{a}_2$ are linearly dependent.
(D) The linear equation $\mathbf{Ax} = \mathbf{0}$ has infinitely many solutions.
(E) The linear equation $\mathbf{A}^T\mathbf{y} = \mathbf{d}$ cannot have a unique solution for any $\mathbf{d} \in \mathbb{R}^3$.

7. Suppose that function f has period of 7, the Laplace transform of f is denoted by $\mathcal{L}[f](s)$, and

$$f(t) = \begin{cases} 5 & \text{for } 0 < t \le 3 \\ 0 & \text{for } 3 < t \le 7 \end{cases}, \quad \mathcal{L}[f](s) = \frac{k_1(1-e^{k_2 s})}{s(1-e^{k_3 s})}$$

Which of the following statements are true?
(A) $k_1 + k_2 = 2$ (B) $k_1 - k_2 = 3$ (C) $k_1 + k_3 = 12$ (D) $k_2 + k_3 = -10$ (E) $k_2 - k_3 = 5$

8. Suppose that $f(t) = te^{-2t}\cos(3t)$, the Laplace transform of f is denoted by $\mathcal{L}[f](s)$, and

$$\mathcal{L}[f](s) = \frac{(s+k_1)^2+k_2}{((s+k_3)^2+k_4)^2}$$

Which of the following statements are true?
(A) $k_1 = 1$ (B) $k_2 = 9$ (C) $k_3 = 3$ (D) $k_4 = 9$ (E) $k_1 + k_2 + k_3 + k_4 = 4$

9. Suppose that $f(t)$ satisfies the integral equation:

$$f(t) = -1 + \int_0^t f(t-\tau)e^{-3\tau}d\tau.$$

Solving the integral equation, we obtain $f(t) = k_1 e^{k_2 t} + k_3$. Which of the following statements are true?
(A) $k_1 + k_3 = -1$ (B) $k_2 + k_3 = 1$ (C) $k_1 - k_3 = 2$ (D) $k_2 - k_3 = 3$ (E) $k_2 = -2$

10. Suppose the Fourier transform of f is defined by $\mathcal{F}[f](\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$, and

$$f(t) = 5(H(t-3) - H(t-11)), \ \mathcal{F}[f](\omega) = \frac{k_1}{\omega}e^{k_2 j\omega}\sin(k_3\omega),$$

where $H(t)$ is the Heaviside function (unit step function). Which of the following statements are true?
(A) $k_1 = 11$ (B) $k_2 = -7$ (C) $k_3 = 3$ (D) $k_1 + k_2 = -5$ (E) $k_2 + k_3 = 5$

下面11-15題為單選題，每題2分，總分10分。答錯或未作答者該題以0分計。

以下第11題到15題，考慮非線性微分方程式：$\ddot{x}-\mu(1-x^2)\dot{x}+\sin(x)=0$。

11. 令 $(y_1, y_2)=(x, \dot{x})$。方程式的平衡點發生在

(A) $(y_1, y_2)=(0,0)$ (B) $(y_1, y_2)=(\pi,0)$ (C) 以上皆是。

12. 若 $\mu>0$，則初值在平衡點 $(y_1, y_2)=(0,0)$ 附近的解會隨時間增加而

(A) 收斂到 $(0,0)$ (B) 遠離 $(0,0)$ (C) 不一定，跟初值有關。

13. 若 $\mu<0$，則初值在平衡點 $(y_1, y_2)=(0,0)$ 附近的解會隨時間增加而

(A) 收斂到 $(0,0)$ (B) 遠離 $(0,0)$ (C) 不一定，跟初值有關。

14. 考慮對 $(y_1, y_2)=(\pi,0)$ 作線性化之線性方程式。此時，若 $\mu<0$，則該線性方程式之非零初值解會隨時間增加而

(A) 收斂到 $(0,0)$ (B) 遠離 $(0,0)$ (C) 不一定，跟初值有關。

15. 考慮上題所述之線性方程式。此時，若 $\mu>0$，則該線性方程式之非零初值解會隨時間增加而

(A) 收斂到 $(0,0)$ (B) 遠離 $(0,0)$ (C) 不一定，跟初值有關。

下面16-20題為單選題，每題1分，總分5分。答錯或未作答者該題以0分計。

16. 考慮複函數 $f(z)=\cos z$。

(A) $|f(z)|$ 隨 z 的虛部增大而發散 (B) $|f(z)|$ 隨 z 的實部增大而發散 (C) $|f(z)|=1, \forall z$。

17. 考慮複函數 $f(z)=\sin\left(\dfrac{1}{z}\right)$。

(A) 原點是 $f(z)$ 的唯一一個極點（pole）。
(B) $f(z)$在原點之外的所有複平面上皆為解析（analytic）。
(C) 以上皆對。

18. 考慮平面場函數 $F(x,y)=(y^2+5)\mathbf{i}+(2xy-8)\mathbf{j}$，$\mathbf{i}$ 與 $\mathbf{j}$ 分別為 x 軸與 y 軸的單位向量。令 C_1 為從原點沿 x 軸走到 $(x,y)=(1,0)$ 之路徑。C_2 為從原點沿 $x=y$ 直線走到 $(x,y)=(1,1)$，再沿 $x=1$ 直線走到 $(x,y)=(1,0)$ 之路徑。

(A) F 沿 C_1 之路徑積分之值為 5 (B) F 沿 C_2 之路徑積分之值為 5 (C) 以上皆對。

19. 考慮18題中的平面場函數 $F(x,y)$。

(A) F 沿任意封閉路徑之路徑積分值為 0。
(B) F 只有沿以原點為圓心之圓形路徑的路徑積分值才會為 0。
(C) F 沿任意封閉路徑之路徑積分值皆不為 0。

20. 考慮18題中的平面場函數 $F(x,y)$。

(A) F 不是保守場 (B) F 是保守場，對應的位能函數為 $\varphi(x,y)=xy^2+5x-8y$ (C) 以上皆非

以下第**21**題到第**22**題需要詳明推導計算過程。如推導計算過程錯誤，將酌扣分數或不給分。

21. (共20分) 令 $Y=\begin{bmatrix}y_1\\y_2\end{bmatrix}$，$A=\begin{bmatrix}-2&1\\1&-2\end{bmatrix}$，$B=\begin{bmatrix}0\\1\end{bmatrix}$。考慮微分方程式：$\ddot{Y}=AY+Bu$。

(a). (10分) 令 $u\equiv 0$，$Y(0)=\begin{bmatrix}1\\-1\end{bmatrix}$, $\dot{Y}(0)=\begin{bmatrix}0\\0\end{bmatrix}$，請找出微分方程式之解。

(b). (5分) 請問 (a) 小題的解是否滿足 $\lim_{t\to\infty}Y(t)=\begin{bmatrix}0\\0\end{bmatrix}$?

(c). (5分) 令 $Y(0)=\dot{Y}(0)=\begin{bmatrix}0\\0\end{bmatrix}$，$u$ 為單位步階函數；i.e., $u(t)=\begin{cases}1 & t\geq 0\\ 0 & t<0\end{cases}$。請問此時微分方程式的解是否滿足 $\lim_{t\to\infty}Y(t)=-A^{-1}B$。

22. (共15分) 求出下面複平面上之路徑積分值。以下 z 為複數，i 代表$\sqrt{-1}$。

(a). (5分) $\int_C \frac{z^2+1}{z^2+2z+5}dz$，其中 C 為沿著頂點為± 2與$\pm 2i$之正方形四邊正向旋轉一周之封閉路徑。

(b). (10分) $\int_C z^2 e^{1/z}dz$，其中 C 為沿著 $\{z:|z|=3\}$ 正向旋轉一周之封閉路徑。

國立中山大學110學年度
碩士暨碩士專班招生考試試題

科目名稱：電子學【電波聯合碩士班選考、通訊所碩士班乙組選考、電機系碩士班戊組選考】

—作答注意事項—

考試時間：100分鐘

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- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學110學年度碩士暨碩士專班招生考試試題

科目名稱：電子學【電波聯合碩士班選考、通訊所碩士班乙組選考、電機系碩士班戊組選考】題號：482003

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　共2頁第1頁

1. (15%) A second-order filter has its poles at $s = -(1/4) \pm j(\sqrt{15}/4)$. The transmission is zero at $\omega = 3$ rad/s and is unity at dc ($\omega = 0$). Find the transfer function. (15%*1)

2. (20%) In the circuit of Fig. 1, the NMOS transistor has $|V_t| = 0.9$ V, and $V_A = 50$ V, and operates with $V_D = V_{GS} = 2$ V. (a) What is the voltage gain v_o/v_i ? (10%) (b) What do V_D and (c) the gain become for I increased to 1 mA? (5%*2)

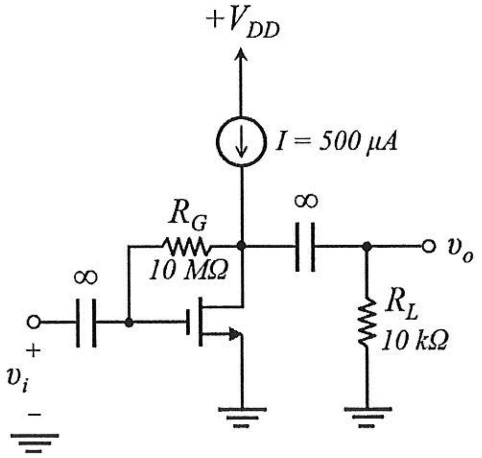

Fig. 1

3. (30%) For the common-base circuit in Fig. 2, assuming the bias current to be about 1 mA, $\beta = 100$, $C_\mu = 0.8$ pF, $r_e = 25\ \Omega$, and $f_T = 500$ MHz:
   (a) Estimate the midband gain V_o/V_s.
   (b) Use the short-circuit time-constants method to estimate the lower 3-dB frequency, f_L. (*Hint:* In determining the resistance seen by C_1, the effect of the 47-kΩ resistor must be taken into account.)
   (c) Find the high-frequency poles, and estimate the upper 3-dB frequency, f_H. (10%*3)

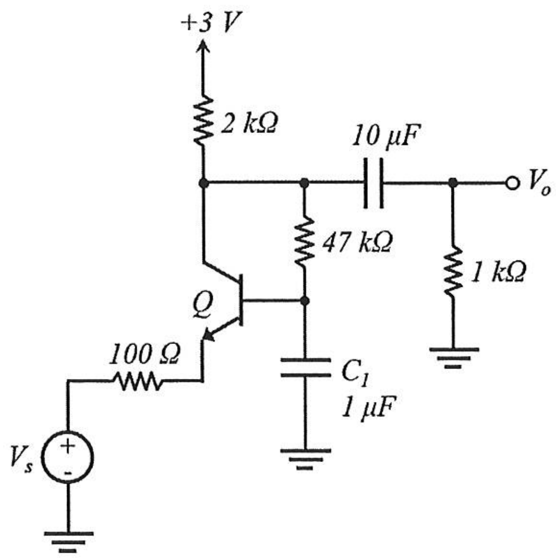

Fig. 2

試題請隨卷繳回，請留意背面是否有題

國立中山大學 110 學年度碩士暨碩士專班招生考試試題

科目名稱：電子學【電波聯合碩士班選考、通訊所碩士班乙組選考、電機系碩士班戊組選考】題號：482003

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　共 2 頁第 2 頁

4. (35%) The amplifier of Fig. 3 consists of two identical common-emitter amplifiers connected in cascade. Observe that the input resistance of the second stage, R_{in2}, constitutes the load resistance of the first stage.
   (a) For V_{CC} = 15 V, V_T = 0.025 V, R_1 = 100 kΩ, R_2 = 47 kΩ, R_E = 3.9 kΩ, R_C = 6.8 kΩ, and β = 100, determine the dc collector current of each transistor. (5%)
   (b) Draw the small-signal equivalent circuit of the entire amplifier and give the values of all its components. Neglect r_{o1} and r_{o2}. (10%)
   (c) Find R_{in1} and v_{b1}/v_s for R_s = 5 kΩ.
   (d) Find R_{in2} and v_{b2}/v_{b1}.
   (e) For R_L = 2 kΩ, find v_o/v_{b2}.
   (f) Find the overall voltage gain v_o/v_s. (5%*4)

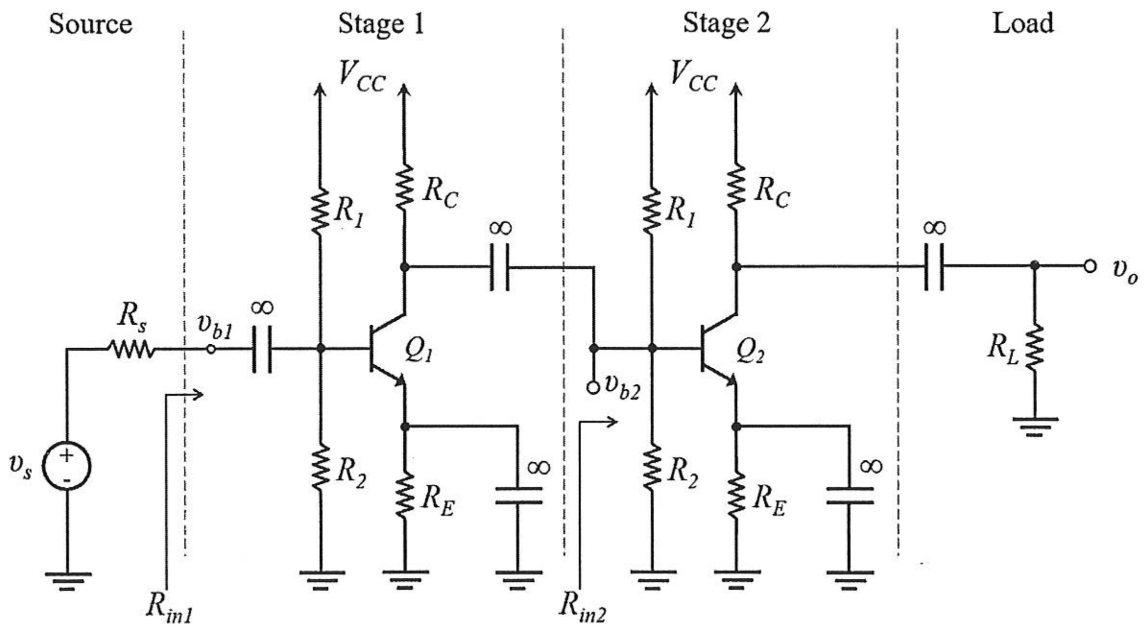

Fig. 3

試題請隨卷繳回，請留意背面是否有題

國立中山大學110學年度
碩士暨碩士專班招生考試試題

科目名稱：工程數學甲【電機系碩士班電波聯合碩士班選考、電機系碩士班甲組、通訊所碩士班乙組選考、電機系碩士班戊組選考、己組、庚組】

—作答注意事項—

考試時間：100分鐘

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- **違規者依本校招生考試試場規則及違規處理辦法處理。**

下面 **1-10** 題為複選題，每題 **5** 分，總分 **50** 分。每題有 **5** 個選項，僅答錯 **1** 個選項者，得 **3** 分；答錯 **2** 個選項者，得 **1** 分；答錯多於 **2** 個選項或未作答者，該題以零分計算。

1. Let $\mathbf{x}$ and $\mathbf{y}$ be nonzero vectors in $\mathbb{R}^n$, $n \geq 2$. Suppose $\mathbf{x}$ and $\mathbf{y}$ are not orthogonal, i.e. $\mathbf{x}^T\mathbf{y} \neq 0$. Let $\mathbf{A} = \mathbf{x}\mathbf{y}^T$. Which of the following statements are true?
(A) 0 is an eigenvalue of $\mathbf{A}$.
(B) $\mathbf{x}^T\mathbf{y}$ is an eigenvalue of $\mathbf{A}$.
(C) $\mathbf{x}$ is an eigenvector of $\mathbf{A}$.
(D) The number of linearly independent eigenvectors associated with eigenvalue 0 is $n-1$.
(E) $\mathbf{A}$ is diagonalizable.

2. Let $\mathbf{A}$ and $\mathbf{B}$ be square matrices and α be a real number. Which of the following statements are true?
(A) $\det(\mathbf{A}^2) = \det(\mathbf{A}^T\mathbf{A})$
(B) $\det(\mathbf{B}^T\mathbf{A}) = \det(\mathbf{A}^T\mathbf{B}^T)$
(C) $\det(\alpha\mathbf{A}) = \alpha \cdot \det(\mathbf{A})$
(D) $\det(\mathbf{A}+\mathbf{B}) = \det(\mathbf{A}) + \det(\mathbf{B})$
(E) Swapping two rows of $\mathbf{A}$ will not change its determinant.

3. Consider the linear function $L: \mathbb{R}^3 \to \mathbb{R}^3$. Suppose $L(\mathbf{x}_\mathrm{i}) = \mathbf{y}_\mathrm{i}$ for i = 1,2,3,4, where
$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \mathbf{x}_4 = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}, \mathbf{y}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{y}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \mathbf{y}_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$
Suppose $L(\mathbf{x}_4) = [c_1, c_2, c_3]^T$. Which of the following statements are true?
(A) $c_1 = 1$ (B) $c_1 = 2$ (C) $c_2 = 2$ (D) $c_2 = 3$ (E) $c_3 = -3$

4. Let $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathrm{R}(\mathbf{A})$ denote the column space of $\mathbf{A}$, and $\mathrm{N}(\mathbf{A})$ denote the null space of $\mathbf{A}$. Which of the following statements are true?
(A) There exists a nonzero vector $\mathbf{y} \in \mathbb{R}^m$ such that $\mathbf{A}^T\mathbf{y} = \mathbf{0}$ and $\mathbf{y} = \mathbf{A}\mathbf{x}$ for some $\mathbf{x} \in \mathbb{R}^n$.
(B) For every $\mathbf{y} \in \mathrm{R}(\mathbf{A})$, we can always find some $\mathbf{x} \in \mathrm{R}(\mathbf{A}^T)$ such that $\mathbf{y} = \mathbf{A}\mathbf{x}$.
(C) Let $\mathbf{p}$ be the orthogonal projection of $\mathbf{y} \in \mathbb{R}^\mathrm{m}$ onto $\mathrm{R}(\mathbf{A})$. Then, $\mathbf{y} - \mathbf{p} \in \mathrm{N}(\mathbf{A}^T)$.
(D) Let $\mathbf{y}$ be a vector in $\mathbb{R}^m$. If $\mathbf{y} \notin \mathrm{R}(\mathbf{A})$, then $\mathrm{y} \in \mathrm{N}(\mathbf{A}^T)$.
(E) The intersection of $\mathrm{R}(\mathbf{A})$ and $\mathrm{N}(\mathbf{A}^T)$ is the empty set.

5. Let $E = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis for a three-dimensional subspace S of an inner product space V. Define $\|\mathbf{v}\| = \sqrt{\langle \mathbf{v}, \mathbf{v} \rangle}$. Suppose E is an orthogonal set with $\|\mathbf{v}_1\| = \sqrt{2}$, $\|\mathbf{v}_2\| = \sqrt{3}$, $\|\mathbf{v}_3\| = 1$. Let $\mathbf{x} = 5\mathbf{v}_1 + 1\mathbf{v}_2 - 3\mathbf{v}_3$ and $\mathbf{y} = 1\mathbf{v}_1 - 3\mathbf{v}_2 - 2\mathbf{v}_3$. Let $\mathbf{p}$ be the orthogonal projection of $\mathbf{x}$ onto $\mathbf{y}$, and suppose $\mathbf{p} = \alpha\mathbf{y}$, where α is a scalar. Which of the following statements are true?
(A) $\langle \mathbf{x}, \mathbf{y} \rangle = 7$
(B) $\langle \mathbf{x}, \mathbf{y} \rangle = 8$
(C) $\alpha = 7/33$
(D) $\alpha = 8/14$
(E) $\|\mathbf{x}\| = 62$

6. Let $\mathbf{A} \in \mathbb{R}^{5\times 3}$ and nullity$(\mathbf{A})$ denote the dimension of $\mathrm{N}(\mathbf{A})$. Suppose nullity$(\mathbf{A}) = 2$. Which of the following statements are true?
(A) $\mathrm{rank}(\mathbf{A}^T) = 3$
(B) $\mathrm{nullity}(\mathbf{A}^T) = 4$
(C) Let $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3]$, where $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ are column vectors of $\mathbf{A}$.
Then $\mathbf{a}_1$ and $\mathbf{a}_2$ are linearly dependent.
(D) The linear equation $\mathbf{Ax} = \mathbf{0}$ has infinitely many solutions.
(E) The linear equation $\mathbf{A}^T\mathbf{y} = \mathbf{d}$ cannot have a unique solution for any $\mathbf{d} \in \mathbb{R}^3$.

7. Suppose that function f has period of 7, the Laplace transform of f is denoted by $\mathcal{L}[f](s)$, and

$$f(t) = \begin{cases} 5 & \text{for } 0 < t \le 3 \\ 0 & \text{for } 3 < t \le 7 \end{cases}, \quad \mathcal{L}[f](s) = \frac{k_1(1-e^{k_2 s})}{s(1-e^{k_3 s})}$$

Which of the following statements are true?
(A) $k_1 + k_2 = 2$ (B) $k_1 - k_2 = 3$ (C) $k_1 + k_3 = 12$ (D) $k_2 + k_3 = -10$ (E) $k_2 - k_3 = 5$

8. Suppose that $f(t) = te^{-2t}\cos(3t)$, the Laplace transform of f is denoted by $\mathcal{L}[f](s)$, and

$$\mathcal{L}[f](s) = \frac{(s+k_1)^2 + k_2}{((s+k_3)^2 + k_4)^2}$$

Which of the following statements are true?
(A) $k_1 = 1$ (B) $k_2 = 9$ (C) $k_3 = 3$ (D) $k_4 = 9$ (E) $k_1 + k_2 + k_3 + k_4 = 4$

9. Suppose that $f(t)$ satisfies the integral equation:

$$f(t) = -1 + \int_0^t f(t-\tau)e^{-3\tau}d\tau.$$

Solving the integral equation, we obtain $f(t) = k_1 e^{k_2 t} + k_3$. Which of the following statements are true?
(A) $k_1 + k_3 = -1$ (B) $k_2 + k_3 = 1$ (C) $k_1 - k_3 = 2$ (D) $k_2 - k_3 = 3$ (E) $k_2 = -2$

10. Suppose the Fourier transform of f is defined by $\mathcal{F}[f](\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$, and

$$f(t) = 5(H(t-3) - H(t-11)),\ \mathcal{F}[f](\omega) = \frac{k_1}{\omega}e^{k_2 j\omega}\sin(k_3\omega),$$

where $H(t)$ is the Heaviside function (unit step function). Which of the following statements are true?
(A) $k_1 = 11$ (B) $k_2 = -7$ (C) $k_3 = 3$ (D) $k_1 + k_2 = -5$ (E) $k_2 + k_3 = 5$

下面 11-15 題為單選題，每題 2 分，總分 10 分。答錯或未作答者該題以 0 分計。

以下第 11 題到 15 題，考慮非線性微分方程式： $\ddot{x} - \mu(1-x^2)\dot{x} + \sin(x) = 0$。

11. 令 $(y_1, y_2) = (x, \dot{x})$。方程式的平衡點發生在

(A) $(y_1, y_2) = (0, 0)$ (B) $(y_1, y_2) = (\pi, 0)$ (C) 以上皆是。

12. 若 $\mu > 0$，則初值在平衡點 $(y_1, y_2) = (0, 0)$ 附近的解會隨時間增加而

(A) 收斂到 $(0,0)$ (B) 遠離 $(0,0)$ (C) 不一定，跟初值有關。

13. 若 $\mu < 0$，則初值在平衡點 $(y_1, y_2) = (0, 0)$ 附近的解會隨時間增加而

(A) 收斂到 $(0,0)$ (B) 遠離 $(0,0)$ (C) 不一定，跟初值有關。

14. 考慮對 $(y_1, y_2) = (\pi, 0)$ 作線性化之線性方程式。此時，若 $\mu < 0$，則該線性方程式之非零初值解會隨時間增加而

(A) 收斂到 $(0,0)$ (B) 遠離 $(0,0)$ (C) 不一定，跟初值有關。

15. 考慮上題所述之線性方程式。此時，若 $\mu > 0$，則該線性方程式之非零初值解會隨時間增加而

(A) 收斂到 $(0,0)$ (B) 遠離 $(0,0)$ (C) 不一定，跟初值有關。

下面 16-20 題為單選題，每題 1 分，總分 5 分。答錯或未作答者該題以 0 分計。

16. 考慮複函數 $f(z) = \cos z$。

(A) $|f(z)|$ 隨 z 的虛部增大而發散 (B) $|f(z)|$ 隨 z 的實部增大而發散 (C) $|f(z)| = 1, \forall z$ 。

17. 考慮複函數 $f(z) = \sin\left(\dfrac{1}{z}\right)$。

(A) 原點是 $f(z)$ 的唯一一個極點（pole）。
(B) $f(z)$ 在原點之外的所有複平面上皆為解析（analytic）。
(C) 以上皆對。

18. 考慮平面場函數 $F(x,y) = (y^2+5)\mathbf{i} + (2xy-8)\mathbf{j}$ ，$\mathbf{i}$ 與 $\mathbf{j}$ 分別為 x 軸與 y 軸的單位向量。令 C_1 為從原點沿 x 軸走到 $(x,y) = (1,0)$ 之路徑。C_2 為從原點沿 $x = y$ 直線走到 $(x,y) = (1,1)$，再沿 $x = 1$ 直線走到 $(x,y) = (1,0)$ 之路徑。

(A) F 沿 C_1 之路徑積分之值為 5 (B) F 沿 C_2 之路徑積分之值為 5 (C) 以上皆對。

19. 考慮 18 題中的平面場函數 $F(x,y)$。

(A) F 沿任意封閉路徑之路徑積分值為 0。
(B) F 只有沿以原點為圓心之圓形路徑的路徑積分值才會為 0。
(C) F 沿任意封閉路徑之路徑積分值皆不為 0。

20. 考慮 18 題中的平面場函數 $F(x,y)$。

(A) F 不是保守場 (B) F 是保守場，對應的位能函數為 $\varphi(x,y) = xy^2 + 5x - 8y$ (C) 以上皆非

以下第 **21** 題到第 **22** 題需要詳明推導計算過程。如推導計算過程錯誤，將酌扣分數或不給分。

21. (共 20 分) 令 $Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$，$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$，$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$。考慮微分方程式：$\ddot{Y} = AY + Bu$。

(a). (10 分) 令 $u \equiv 0$，$Y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\dot{Y}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$，請找出微分方程式之解。

(b). (5 分) 請問 (a) 小題的解是否滿足 $\lim_{t\to\infty} Y(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$?

(c). (5 分) 令 $Y(0) = \dot{Y}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$，$u$ 為單位步階函數；i.e., $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$。請問此時微分方程式的解是否滿足 $\lim_{t\to\infty} Y(t) = -A^{-1}B$。

22. (共 15 分) 求出下面複平面上之路徑積分值。以下 z 為複數，i 代表$\sqrt{-1}$。

(a). (5 分) $\int_C \frac{z^2+1}{z^2+2z+5} dz$，其中 C 為沿著頂點為± 2 與$\pm 2i$ 之正方形四邊正向旋轉一周之封閉路徑。

(b). (10 分) $\int_C z^2 e^{1/z} dz$，其中 C 為沿著 $\{z : |z| = 3\}$ 正向旋轉一周之封閉路徑。