國立中山大學109學年度
碩士暨碩士專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】

—作答注意事項—

考試時間：100分鐘

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國立中山大學109學年度碩士暨碩士專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】 題號：437005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題） 共3頁第1頁

一、選擇題(單選，計分方式:不倒扣，答對得該題全部分數，答錯及未作答得零分)

1. (5%) Let E^c denote the complement of an event E. Which of the following pairs of events, A and B, can be disjoint?
   (A) $\Pr(A) = \frac{1}{3}$ and $\Pr(B^c) = \frac{1}{4}$
   (B) $\Pr(A) = \frac{1}{4}$ and $\Pr(B^c) = \frac{1}{2}$
   (C) $\Pr(A) = \frac{1}{2}$ and $\Pr(B) = \frac{2}{3}$
   (D) $\Pr(A) = \frac{1}{2}$ and $\Pr(B) = \frac{4}{5}$
   (E) None of these

2. (5%) The random variable X has the probability density function
   $$f_X(x) = \begin{cases} cx^{-2} & \text{if } 1 \le x \le 2; \\ 0 & \text{otherwise.} \end{cases}$$
   Let A be the event $\{X > 3/2\}$ and $Y = X^2$. What is the conditional variance of Y given A?
   (A) 1
   (B) 1/2
   (C) 1/4
   (D) 2
   (E) None of these

3. (5%) Let the joint probability density function of X and Y be given by
   $$f(x,y) = \begin{cases} \sin(x)\sin(y) & \text{if } 0 \le x \le \pi/2, \quad 0 \le y \le \pi/2; \\ 0 & \text{elsewhere.} \end{cases}$$
   Which of the following is the correlation coefficient of X and Y?
   (A) 0
   (B) 1
   (C) -1
   (D) $\frac{\pi}{4}$
   (E) None of these

4. (5%) Let X have the probability density function
   $$f(x) = e^{-x-1}, \quad -1 < x < \infty.$$
   Which of the following is correct?
   (A) $\Pr(1 \le X) = e^{-1}$
   (B) The moment-generating function of X is $M(t) = e^{-t}$
   (C) $E[X] = -1$
   (D) $Var[X] = 1$
   (E) The distribution function of X is $F(x) = -e^{-(1+x)}, -1 < x < \infty$

5. (5%) Let X and Y have the joint probability density function
   $$f(x,y) = 2, \quad 0 \le x \le y \le 1.$$
   Which of the following is correct?
   (A) $E[X] = 1$
   (B) $E[Y] = 1/2$
   (C) $E[Y^2] = 1/2$
   (D) $\Pr(0 \le X \le Y, 0 \le Y \le \frac{1}{2}) = 1/2$
   (E) None of these

6. (5%) Let the joint probability mass function of X and Y be defined by

$$f(x,y)=\frac{x+y}{32},\quad x=1,2, y=1,2,3,4.$$

Which of the following is wrong?

(A) $\Pr(X>Y)=3/32$
(B) $\Pr(Y=2X)=9/32$
(C) $\Pr(X+Y=3)=3/16$
(D) $\Pr(X\leq 3-Y)=1/4$
(E) X and Y are independent

7. (5%) Let X_1, X_2, X_3 represent the independent failure times in years of three components in parallel. The probability density functions are $f_{X_1}(x_1)=3x_1^2, 0<x_1<1$, $f_{X_2}(x_2)=4x_2^3, 0<x_2<1$, and $f_{X_3}(x_3)=6x_3^5, 0<x_3<1$. Let $Y=\max(X_1,X_2,X_3)$. Which of the following is correct?

(A) $\Pr(y=1/2)=\frac{13}{4096}$
(B) $\Pr(y=1/2)=\frac{1}{8192}$
(C) $\Pr(y\leq 1/3)=13\left(\frac{1}{3}\right)^{12}$
(D) $\Pr(y\leq 1/3)=1-13\left(\frac{1}{3}\right)^{12}$
(E) None of these

8. (5%) Let X be a random variable with moment-generating function $M_X(t), -h<t<h$. Which of the following is correct?

(A) $\Pr(X\geq 1)\leq e^{-t}M_X(t), 0<t<h$
(B) $\Pr(X\geq -2)\leq e^{-2t}M_X(t), 0<t<h$
(C) $\Pr(X\leq 1)\leq 1-e^{-t}M_X(t), -h<t<0$
(D) $M_X(t)+M_X(6t), -h<t<h$, is also a moment-generating function of some random variable
(E) None of these

9. (5%) Let X and Y be independent normal random variables with mean 0 and variance 1. Define a new random variable Z by

$$Z=\begin{cases} X & \text{if } XY>0; \\ -X & \text{if } XY<0. \end{cases}$$

Which of the following is correct?

(A) Z and Y are independent
(B) $E[Z]=1$
(C) $Z=0$
(D) Z has a normal distribution
(E) None of these

10. (5%) Which of the following cannot be cumulative distribution function (CDF)?

(A) $\frac{1}{2}+\frac{1}{\pi}\tan^{-1}(x), x\in(-\infty,\infty)$
(B) $(1-e^{-x})^{-1}, x\in(-\infty,\infty)$
(C) $e^{-e^{-x}}, x\in(-\infty,\infty)$
(D) $1-e^{-x}, x\in(0,\infty)$
(E) None of these

二、問答計算題:

1. (25%) Consider two discrete random variables X and Y with joint pmf:

$P(x,y)$	$X=-1$	$X=0$	$X=1$
$Y=2$	0.15	0.15	0.1
$Y=4$	0.05	0.1	0.15
$Y=6$	0.1	0.15	0.05

(a) (5%) Find the marginal distribution of X
(b) (5%) Find the conditional distribution of X given $Y=4$
(c) (5%) Find the conditional mean $\mathrm{E}[X|Y]$
(d) (5%) Are X and Y independent? Prove it or disprove it.
(e) (5%) Are X and Y uncorrelated? Prove it or disprove it.

2. (25%) Let X be a random variable with PDF

$$f_X(x) = \begin{cases} xe^{-x^2/2}, & \text{if } X > 0; \\ 0 & \text{otherwise.} \end{cases}$$

Suppose that $Y = X^2$. Answer the following questions.
(a) (5%) Find the PDF of Y
(b) (5%) Find the mean value $\mathrm{E}[Y]$
(c) (5%) Find the moment generating function of Y
(d) (10%) Derive the n-th moment $\mathrm{E}[Y^n]$

國立中山大學 109 學年度
碩士暨碩士專班招生考試試題

科目名稱：線性代數【通訊所碩士班甲組】

—作答注意事項—

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In the following, boldface capital and lower-case letters denote matrices and vectors, respectively. For all questions, please provide both answers and justifications.

1. (8%) Let $\mathbf{A} = \begin{bmatrix} a & b & b & \cdots & b \\ b & a & b & \cdots & b \\ b & b & a & \cdots & b \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b & b & b & \cdots & a \end{bmatrix}_{(n+1)\times(n+1)}$ and $a, b \in \mathbb{R}$. Find $\det(\mathbf{A})$.

2. (8%) Please find the eigenvalues and eigenvectors of $(5\mathbf{I}+\mathbf{A})^{400}$ given $\mathbf{A} = \begin{bmatrix} -10 & 15 \\ 25 & -20 \end{bmatrix}$.

3. (8%) Find the singular value decomposition for the matrix $\mathbf{M} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 0 \end{bmatrix}$.

4. (8%) Answer the following questions :
   (a) (4%) Consider a 8×6 matrix $\mathbf{A}$ and 6×11 matrix $\mathbf{B}$. How many possible dimensions of $\ker(\mathbf{AB})$ do we have? What are they? (Note : $\ker(\mathbf{AB})$ is the kernel of matrix $\mathbf{AB}$).

   (b) (4%) Let $\mathbb{R}^{2\times2}$ be the vector space of all 2×2 matrices. Define the linear transformation T from $\mathbb{R}^{2\times2}$ to $\mathbb{R}^{2\times2}$ by $T(\mathbf{K}) = \mathbf{K}\begin{bmatrix} 2 & 5 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 5 \\ 0 & 2 \end{bmatrix}\mathbf{K}$. Find the basis of the image of T with respect to the standard basis $\mathcal{B} = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}$.

5. (8%) Consider the inner product space $C_{0,1}$ which is the set of all functions that have a continuous first order derivative on $0 \le x \le 1$. The inner product of two functions $f(x)$ and $g(x)$ is defined by $< f, g > = \int_0^1 f(x)g(x)dx$.
   (a) (4%) Use the Gram-Schmidt process to find the orthonormal basis $\mathcal{B}$ for the subspace S spanned by the vectors 1 and x.
   (b) (4%) Find the best least squares approximation to the function $\sqrt{x}$ on the interval $[0, 1]$ by a function in S.

6. (8%) Suppose the table below denotes the operation of a group. Fill in the blank entries in the table. (Note : The group G: $\{e, a, b, c, d\}$, and e denotes the identity element.)

	e	a	b	c	d
e	e	__	__	__	__
a	__	b	__	__	e
b	__	c	d	e	__
c	__	d	__	a	b
d	__	__	__	__	__

7. (8%) Let $M_2(\mathbb{R})$ be the vector space of all 2×2 matrices with real coefficients and let P_2 be the vector space of all real polynomials of degree at most 2. Define $T: M_2(\mathbb{R}) \longrightarrow P_2$ by

$$T(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) = (a+b) + 2dx + bx^2.$$

Let $\mathcal{B} = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}$ and $\mathcal{C} = \{1, x, x^2\}$, $\mathcal{D} = \{1+x, x, 1+x^2\}$
Be ordered bases for $M_2(\mathbb{R})$ and P_2, respectively. Determine
(a) (4%) The matrix representation of T relative to ordered bases $\mathcal{B}, \mathcal{C}$.
(b) (4%) The matrix representation of T relative to ordered bases $\mathcal{B}, \mathcal{D}$.

8. (8%) Let $\mathbf{A}_1 = [\,1\,]$, $\mathbf{A}_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\mathbf{A}_3 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $\mathbf{A}_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.

When $n \geq 4$, we have:

$$\mathbf{A}_n = \begin{bmatrix} 1 & 1 & 0 & & 0 & 0 \\ 1 & 1 & 1 & \cdots & 0 & 0 \\ 0 & 1 & 1 & & 0 & 0 \\ & \vdots & & \ddots & \vdots & \\ 0 & 0 & 0 & \cdots & 1 & 1 \\ 0 & 0 & 0 & & 1 & 1 \end{bmatrix}_{n\times n}.$$

Prove that $|\mathbf{A}_{n+2}| = |\mathbf{A}_{n+1}| - |\mathbf{A}_n|$ for $n \geq 1$.

9. (8%) Let $\mathbf{A} = \begin{bmatrix} \cos\left(\frac{\pi}{6}\right) & -\sin\left(\frac{\pi}{6}\right) \\ \sin\left(\frac{\pi}{6}\right) & \cos\left(\frac{\pi}{6}\right) \end{bmatrix}$. Please compute $\mathbf{A}^{100}$.

10. (8%) $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} a_{11} & a_{13} & a_{12} \\ a_{21}+2a_{31} & a_{23}+2a_{33} & a_{22}+2a_{32} \\ a_{31} & a_{33} & a_{32} \end{bmatrix}$,

$$\mathbf{C} = \begin{bmatrix} a_{11} - a_{31}\sin\theta & a_{12} - a_{32}\sin\theta & a_{13} - a_{33}\sin\theta \\ a_{21} - a_{31}\cos\theta & a_{22} - a_{32}\cos\theta & a_{23} - a_{33}\cos\theta \\ a_{31} & a_{32} & a_{33} \end{bmatrix}.$$

(a) (4%) If $|\mathbf{A}| = 10$, please find $|\mathbf{B}|$.

(b) (4%) If $\mathbf{A}^{-1} = \begin{bmatrix} \sin\theta & \cos\theta & -1 \\ \cos\theta & \sin\theta & -\sin 2\theta \\ \sin\theta & \cos\theta & 2 \end{bmatrix}$, find $\mathbf{C}^{-1}$.

11. (10%) Consider the following linear equation:

$$\begin{bmatrix} \mathbf{A} & \mathbf{u} \\ \mathbf{v}^{\mathrm{T}} & d \end{bmatrix}\begin{bmatrix} \mathbf{x} \\ x_{n+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{b} \\ b_{n+1} \end{bmatrix},$$

where $\mathbf{A}$ is a nonsingular $n \times n$ matrix, d is the element in $\mathbb{R}$, $\mathbf{u}, \mathbf{v}, \mathbf{b}, \mathbf{x}$ are $n \times 1$ vectors in $\mathbb{R}^n$, and $\mathbf{0}$ is an $n \times 1$ zero vector.

(a) (5%) Find an $n \times n$ matrix $\mathbf{Y}$ and a vector $\mathbf{z}$ in $\mathbb{R}^n$ (given that $\mathbf{A}^{-1} = \mathbf{H}$, $\mathbf{v}^{\mathrm{T}} = \mathbf{g}$, Please use $\mathbf{H}$ and $\mathbf{g}$ to represent $\mathbf{Y}$and $\mathbf{z}$) so that multiplying both sides of the system by $\begin{bmatrix} \mathbf{Y} & \mathbf{0} \\ \mathbf{z}^{\mathrm{T}} & 1 \end{bmatrix}$ yields an equivalent echelon form, i.e.,

$$\begin{bmatrix} \mathbf{Y} & \mathbf{0} \\ \mathbf{z}^{\mathrm{T}} & 1 \end{bmatrix}\begin{bmatrix} \mathbf{A} & \mathbf{u} \\ \mathbf{v}^{\mathrm{T}} & d \end{bmatrix} = \begin{bmatrix} \mathbf{I}_n & * \\ \mathbf{0}^{\mathrm{T}} & * \end{bmatrix},$$

where $*$ means arbitrary vector or scalar.

(b) (5%) Show that the determinant of $\begin{bmatrix} \mathbf{A} & \mathbf{u} \\ \mathbf{v}^{\mathrm{T}} & d \end{bmatrix}$ has the form

$$\begin{vmatrix} \mathbf{A} & \mathbf{u} \\ \mathbf{v}^{\mathrm{T}} & d \end{vmatrix} = d\big(\det(\mathbf{A})\big) - \mathbf{v}^{\mathrm{T}}\big(\mathrm{adj}(\mathbf{A})\big)\mathbf{u}.$$

12. (10%) Consider the matrix $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

The following steps will form an orthogonal diagonalizing matrix $\mathbf{C}$ for $\mathbf{A}$.

Step 1. Let $g = (a-c)/2$

Step 2. Let $h = \sqrt{g^2 + b^2}$

Step 3. Let $r = \sqrt{b^2 + (g+h)^2}$

Step 4. Let $s = \sqrt{b^2 + (g-h)^2}$

Step 5. Let $\mathbf{C} = \begin{bmatrix} -b/r & -b/s \\ (g+h)/r & (g-h)/s \end{bmatrix}$

(If $b < 0$ and rotation matrix is desired, change the sign of a column vector in $\mathbf{C}$.)

Prove this algorithm by proving the following.

(a) (2%) The eigenvalues of $\mathbf{A}$ are $\lambda = \frac{a+c\pm\sqrt{(a-c)^2+4b^2}}{2}$

(b) (2%) The first row vector of $\mathbf{A} - \lambda\mathbf{I}$ is $\left[g \pm \sqrt{g^2+b^2} \quad b\right]$.

(c) (3%) The columns of the orthogonal matrix $\mathbf{C}$ in **Step 5** can be found using part (b).

(d) (3%) The parenthetical statement following **Step 5** is valid.

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※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　共 4 頁第 1 頁

下面 1-10 題為是非題，總分 20 分。每題答對 2 分，答錯扣 3 分，未作答者以 0 分計。總分低於 0 分者以 0 分計。

1. 微分方程式 $\ddot{y}(t)+3t^2\dot{y}(t)+|t|y(t)=0$ 為非線性方程式。
   (A) 是 (B) 否
2. 微分方程式 $\ddot{x}(t)+(1+x(t)^2)\dot{x}(t)+(1-x(t)^2)x(t)=0$ 有三個平衡點。
   (A) 是 (B) 否
3. 微分方程式 $\ddot{y}(t)+3\dot{y}(t)-y(t)=0$ 的解，不管初值為何，都會收斂到 0。
   (A) 是 (B) 否
4. 拉普拉斯轉換（Laplace transform）為線性轉換。
   (A) 是 (B) 否
5. 令函數 $y(t)$ 的拉普拉斯轉換為 $Y(s)$。則 函數 $\ddot{y}(t)$ 的拉普拉斯轉換為 $s^2Y(s)$。
   (A) 是 (B) 否
6. 函數 $f(t)=\mathrm{e}^{-t}\sin t,\ t\in[0,\infty)$ 的傅立葉轉換（Fourier transform）為 $\frac{1}{(1-\omega^2)(1+j\omega)}$。
   (A) 是 (B) 否
7. 複函數 $f(z)=(z+1)/z$ 在原點之外的所有複平面上皆為解析（analytic）。
   (A) 是 (B) 否
8. 複函數 $f(z)=\sin z$ 之絕對值會隨著 z 的虛部增大而發散。
   (A) 是 (B) 否
9. 定義 Del 操作子為 $\nabla:=\frac{\partial}{\partial x}\mathbf{i}+\frac{\partial}{\partial y}\mathbf{j}+\frac{\partial}{\partial z}\mathbf{k}$。若$\varphi$為具有連續一二偏階導函數的連續純量場函數，則$\nabla\times(\nabla\varphi)\neq 0$，除非$\varphi$是常數函數或線性函數。
   (A) 是 (B) 否
10. 承上題，$\nabla\varphi$ 在 φ 之定義域上的任何封閉路徑積分皆為 0。
    (A) 是 (B) 否

下面 11-15 題為單選題，每題 4 分，答錯或未作答者該題以 0 分計。11-15 題總共 20 分。

考慮微分方程式 $\ddot{z}(t)+b\dot{z}(t)+\sin(z(t))=u(t)$ ，並回答以下第 11 至 15 題。

11. 假設 $u(t)\equiv 0,\ \forall t$。下列哪一組初值 $(\dot{z}(0),z(0))$ 所對應的解不是 $z(t)\equiv 0,\ \forall t$。
    (A) $(0,0)$ (B) $(\pi,0)$ (C) $(0,\pi)$ (D) $(0,-\pi)$
12. 假設 $u(t)\equiv 0,\ \forall t$。將前述方程式就 $(\dot{z}(0),z(0))=(0,0)$ 線性化後之線性方程式，滿足以下哪個敘述?
    (A) 若 $b=0$ ，則任何初值對應的解皆會收斂到 0。
    (B) 若 $b=0$ ，則有些初值對應的解皆會發散。
    (C) 若 $b=1$ ，則任何初值對應的解皆會收斂到 0。
    (D) 若 $b=-1$ ，則有些初值對應的解皆會收斂到 0。

13. 假設 $u(t) \equiv 0,\ \forall t$。將前述方程式就 $(\dot{z}(0), z(0)) = (0, \pi)$ 線性化後之線性方程式，滿足以下哪個敘述?

(A) 若 $b = 0$，則有些初值對應的解皆會收斂到 0。
(B) 若 $b = 0$，則任何初值對應的解皆會收斂到 0。
(C) 若 $b = 1$，則任何初值對應的解皆會收斂到 0。
(D) 若 $b = 1$，則任何初值對應的解皆會發散。

14. 考慮將前述方程式就 $(\dot{z}(0), z(0)) = (0, 0)$ 線性化後之線性方程式。假設 $b = 0$，且該方程式之輸入項（forcing term）為單位步階函數。下列敘述何者為正確?

(A) 該線性方程式的解會收斂到 1。
(B) 如該線性方程式的初值為 $(1,0)$，則方程式的解為 $\sin t$。
(C) 該線性方程式的解會發斂
(D) 該線性方程式的解會不斷震盪。

15. 考慮將前述方程式就 $(\dot{z}(0), z(0)) = (0, 0)$ 線性化後之線性方程式。假設 $b = 2$，且該方程式之輸入項（forcing term）為 $\sin t$。下列敘述何者為正確?

(A) 該線性方程式的解會收斂到 $-\frac{1}{2}\cos t$。
(B) 該線性方程式的解會收斂到 $\sin t$。
(C) 如該線性方程式的初值為 $(0,0)$，則方程式的解為 $\sin t$。
(D) 如該線性方程式的初值為 $(0,1)$，則方程式的解為 $\cos t$。

下面16-23題為複選題，每題5分，總分40分。每錯一個選項扣2分，得分低於零分或未作答者，該題以零分計。

16. 令 $\mathbf{F}(x,y,z) = (y + ay^2 z)\mathbf{i} + (bx - z + 2xyz)\mathbf{j} + (cy + xy^2)\mathbf{k}$。下敘述何者正確?

(A) 有超過一組的 (a,b,c) 值能讓 $\mathbf{F}$ 成為一個保守的向量場。
(B) 只有一組 (a,b,c) 值能讓 $\mathbf{F}$ 成為一個保守的向量場。
(C) $(a,b,c) = (-1,0,1)$ 會讓 $\mathbf{F}$ 成為一個保守的向量場。
(D) $(a,b,c) = (0,1,-1)$ 會讓 $\mathbf{F}$ 成為一個保守的向量場。
(E) $(a,b,c) = (1,1,-1)$ 會讓 $\mathbf{F}$ 成為一個保守的向量場。

17. 令 $\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$。下列關於 $\mathrm{e}^{\mathbf{A}t}$ 之敘述何者正確?

(A) 該矩陣是一個 3×3 的方陣。
(B) 該矩陣在 $t \geq 0$ 時，永為一個可逆矩陣。
(C) 該矩陣 $(3,2)$ 位置那一項為 e^{t}。
(D) 該矩陣 $(1,2)$ 位置那一項為 $t\mathrm{e}^{-t}$。
(E) 當 $t \to \infty$ 時，該矩陣收斂為 0 矩陣。

18. Consider the linear equation $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4] \in \mathbb{R}^{3\times4}$ and $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4$ are column vectors of $\mathbf{A}$. Suppose $\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 + \mathbf{a}_4 = \mathbf{b}$. Which of the following statements are true?
(A) The linear equation has exactly one solution.
(B) The linear equation has infinitely many solutions.
(C) No conclusion can be drawn about the number of solutions to the linear equation.
(D) The vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4$ are linearly dependent.
(E) $\text{rank}([\mathbf{A}, \mathbf{b}]) = \text{rank}(\mathbf{A})$

19. Consider the linear equation $\mathbf{Ax} = \mathbf{b}$ with $\mathbf{A} \in \mathbb{R}^{m\times n}$. Which of the following statements are true?
(A) If $\text{rank}(\mathbf{A}) = m$, then there exists at least one solution.
(B) If $\text{rank}(\mathbf{A}) = n$, then there exists exactly one solution.
(C) If $\text{rank}(\mathbf{A}) = n$, then the column vectors of $\mathbf{A}$ are linearly independent.
(D) If $n > m$, then there exists at least one solution.
(E) If $m > n$, then there exists at most one solution.

20. Consider the linear mapping $L: V \to W$. Let $\mathbf{0}_V$ and $\mathbf{0}_W$ be the zero vectors in V and W, respectively. Which of the following statements are true?
(A) The condition $L(\mathbf{v}_1) = L(\mathbf{v}_2)$ implies $\mathbf{v}_1 = \mathbf{v}_2$.
(B) For any $\mathbf{w} \in W$, there exists $\mathbf{v} \in V$ such that $L(\mathbf{v}) = \mathbf{w}$.
(C) If L is one-to-one, then $L(\mathbf{v}) = \mathbf{0}_W$ implies $\mathbf{v} = \mathbf{0}_V$.
(D) If $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k$ are linearly independent, $L(\mathbf{v}_1), L(\mathbf{v}_2), \cdots, L(\mathbf{v}_k)$ are also linearly independent.
(E) The condition $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_k\mathbf{v}_k = \mathbf{0}_V$ implies $c_1L(\mathbf{v}_1) + c_2L(\mathbf{v}_2) + \cdots + c_kL(\mathbf{v}_k) = \mathbf{0}_W$.

21. Given vectors $\mathbf{x}, \mathbf{y}, \mathbf{z}$ in $\mathbb{R}^n$ and matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ in $\mathbb{R}^{n\times n}$. Which of the following statements are true?
(A) If $\mathbf{x}^T\mathbf{y} = 0$ and $\mathbf{y}^T\mathbf{z} = 0$, then $\mathbf{x}^T\mathbf{z} = 0$.
(B) $\text{rank}(\mathbf{x}^T\mathbf{y}) = \text{rank}(\mathbf{x}\mathbf{y}^T) = 1$
(C) $(\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B}) = \mathbf{A}^2 - \mathbf{B}^2$
(D) If $\mathbf{AC} = \mathbf{BC}$ and $\mathbf{C}$ is not the zero matrix, then $\mathbf{A} = \mathbf{B}$.
(E) If $\mathbf{AB}$ equals the zero matrix, then $\mathbf{BA}$ also equals the zero matrix.

22. Let $\mathbf{A} \in \mathbb{R}^{m\times n}$, $\text{R}(\mathbf{A})$ denote the column space of $\mathbf{A}$, $\text{N}(\mathbf{A})$ denote the null space of $\mathbf{A}$, and $\dim(S)$ denote the dimension of a subspace S. Which of the following statements are true?
(A) For any $\mathbf{x} \in \mathbb{R}^n$, there exists $\mathbf{u} \in \text{R}(\mathbf{A}^T)$ and $\mathbf{v} \in \text{N}(\mathbf{A})$ such that $\mathbf{x} = \mathbf{u} + \mathbf{v}$.
(B) Suppose $\mathbf{u} \in \text{R}(\mathbf{A})$ and $\mathbf{v} \in \text{N}(\mathbf{A}^T)$. Then $\mathbf{u}^T\mathbf{v} = 0$.
(C) $\dim(\text{R}(\mathbf{A})) + \dim(\text{N}(\mathbf{A}^T)) = n$
(D) For any $\mathbf{y} \in \text{R}(\mathbf{A})$, there exists $\mathbf{x} \in \mathbb{R}^m$ such that $\mathbf{y} = \mathbf{A}\mathbf{A}^T\mathbf{x}$.
(E) If $\mathbf{y} \in \text{N}(\mathbf{A}^T) \cap \text{R}(\mathbf{A})$, then $\mathbf{y}$ is the zero vector in $\mathbb{R}^m$.

23. Let $\mathbf{A} \in \mathbb{R}^{n\times n}$ and $\mathbf{x} \in \mathbb{R}^n$. Which of the following statements are true?
(A) If $\mathbf{A}$ is singular, then 0 is an eigenvalue of $\mathbf{A}$.
(B) $\mathbf{A}$ and $\mathbf{A}^T$ share the same eigenvalues and eigenvectors.
(C) If $\mathbf{A}$ is diagonalizable, then $\mathbf{A}$ has n distinct eigenvalues.
(D) Suppose that $\mathbf{A}$ is nonsingular. The condition $\mathbf{Ax} = \lambda\mathbf{x}$ implies $\mathbf{A}^{-1}\mathbf{x} = \lambda^{-1}\mathbf{x}$.
(E) Suppose that all the eigenvalues of $\mathbf{A}$ are real and positive. Then we have $\det(\mathbf{A}) > 0$.

以下第 **24** 題到第 **25** 題需要簡明寫出計算過程，答案正確但沒有計算過程，將酌扣分數或不給分。第 **24** 題到第 **25** 題中 $z = x + jy$ 代表複數，其中 x, y 是實數而 $j = \sqrt{-1}$。

24. (10%)

(a) (5%) Let Γ be the circle $\left|z - \frac{1}{\sqrt{2}}\right| = \frac{2}{\sqrt{2}}$, oriented positively. Evaluate the integral

$$\oint_{\Gamma} \frac{1}{z^4 + 1} dz$$

(b) (5%) Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{1}{x^4 + 1} dx$$

25. (10%)

(a) (5%) Define

$$f(z) = \frac{z}{z^4 + 6z^2 + 1}$$

Let z_k be a pole of $f(z)$. If z_k is inside the unit circle $|z| = 1$, compute the residue of $f(z)$ at z_k.

(b) (5%) Evaluate the integral

$$\int_{0}^{2\pi} \frac{1}{1 + \cos^2 \theta} d\theta$$

國立中山大學109學年度
碩士暨碩士專班招生考試試題

科目名稱：電子學【電波聯合碩士班選考、通訊所碩士班乙組選考、電機系碩士班戊組選考】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學 109 學年度碩士暨碩士專班招生考試試題

科目名稱：電子學【電波聯合碩士班選考、通訊所碩士班乙組選考、電機系碩士班戊組選考】題號：482003

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　共 2 頁第 1 頁

1. (15%) A second-order filter has its poles at $s = -(1/2) \pm j(\sqrt{3}/2)$. The transmission is zero at $\omega = 2$ rad/s and is unity at dc ($\omega = 0$). Find the transfer function. (15%*1)

2. (25%) For the common-emitter amplifier shown in Fig. 1, let $V_{CC} = 9$ V, $R_1 = 27$ kΩ, $R_2 = 15$ kΩ, $R_E = 1.2$ kΩ, and $R_C = 2.2$ kΩ. The transistor has $\beta = 100$ and $V_A = 100$ V. (a) Calculate the dc bias current I_E. If the amplifier operates between a source for which $R_s = 10$ kΩ and a load of 2 kΩ, (b) replace the transistor with its hybrid-π model, and (c) find the values of R_i, (d) the voltage gain v_o/v_s, and (e) the current gain i_o/i_i. (5%*5)

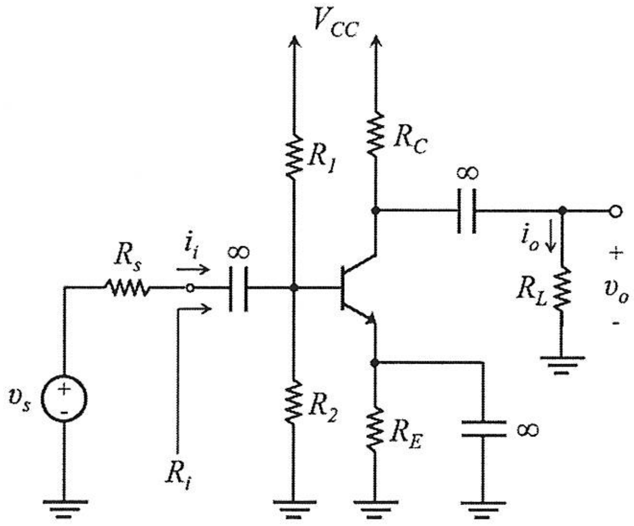

Fig. 1

3. (30%) The current-steering circuit of Fig. 2 is fabricated in a CMOS technology for which $k_n' = 90$ μA/V², $k_p' = 30$ μA/V², $V_{tn} = 0.8$ V, and $V_{tp} = -0.9$ V. If all devices have $L = 2$ μm, design the circuit so that $I_{REF} = 20$ μA, $I_2 = 100$ μA, and $I_5 = 40$ μA. Use the minimum width of 2 μm for as many of the devices as possible. (a) Give the required width for each transistor and the value of R required. (10%) (b) What is the highest voltage possible at the drain of Q_2? (c) What is the lowest voltage possible at the drain of Q_5? If $V_{An} = 8\,L$ and $|V_{Ap}| = 12\,L$, where L is in μm and V_{An} and V_{Ap} are in volts, (d) find the output resistance of the current source Q_2, and (e) the output resistance of the current sink Q_5. (5%*4)

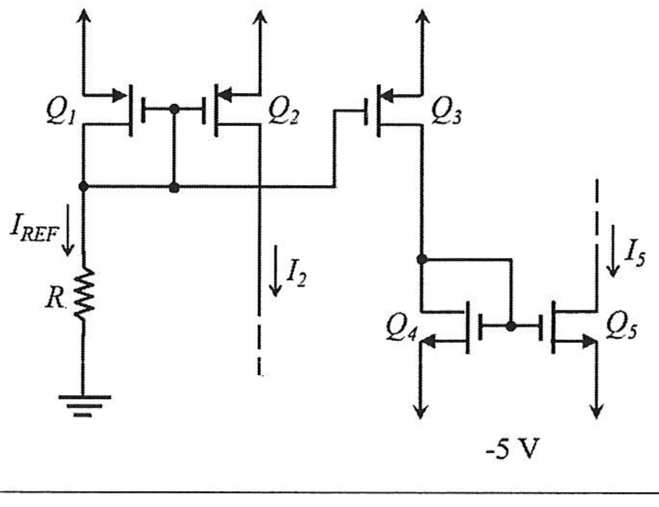

Fig. 2

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4. (30%) For the common-base circuit in Fig. 3, assuming the bias current to be about 1 mA, $\beta = 100$, $C_\mu = 0.8$ pF, $r_e = 25\ \Omega$, and $f_T = 600$ MHz:
   (a) Estimate the midband gain V_o/V_s.
   (b) Use the short-circuit time-constants method to estimate the lower 3-dB frequency, f_L. (*Hint:* In determining the resistance seen by C_1, the effect of the 47-kΩ resistor must be taken into account.)
   (c) Find the high-frequency poles, and estimate the upper 3-dB frequency, f_H. (10%*3)

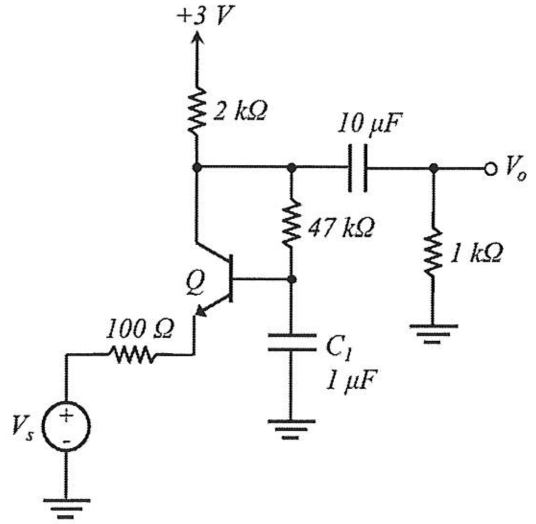

Fig. 3

國立中山大學 109 學年度
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—作答注意事項—

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國立中山大學109學年度碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班電波聯合碩士班選考、通訊所碩士班甲組、乙組選考、電機系碩士班戊組選考】題號：437002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　共3頁第1頁

1. (10%) Explain what carrier and symbol synchronization are in a communication system.

2. (10%) Consider a random variable X with the distribution:

$$p(X=k)=p(1-p)^{k-1},\ k=1,2,3,4,\ldots.$$

Show that $H(X)=-\log_2 p-\dfrac{1-p}{p}\log_2(1-p)$.

Hint: $H(X)=E[-\log_2 p(X)]$.

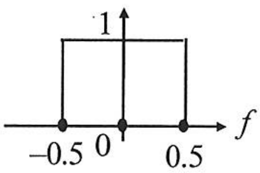
3. (25%) Consider a passband signal given by

$$x(t)=\mathrm{sinc}^2(t)(1+A\cos 4\pi t)\cos\left(w_c t+\frac{\pi}{6}\right),$$

where $w_c \gg 4\pi$.

(a). (10%) Plot the frequency response $|X(w)|$ where $X(w)$ is the Fourier transform of $x(t)$.

Hint: $\mathrm{sinc}(t) \xrightarrow{\mathcal{F}}$



(b). (5%) Find the equivalent baseband signal of $x(t)$.

Hint: $\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$.

(c). (10%) Find the Hilbert transform of $x(t)$.

Hint: Let Hilbert transform of $x(t)$ be $\hat{x}(t)$. $\mathcal{F}\{\hat{x}(t)\}=-j\,\mathrm{sgn}(w)\mathcal{F}\{x(t)\}$.

國立中山大學109學年度碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班電波聯合碩士班選考、通訊所碩士班甲組、乙組選考、電機系碩士班戊組選考】題號：437002

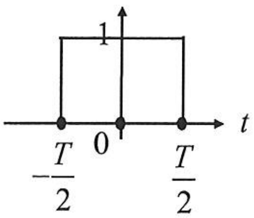
※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）

4. (20%) Consider a sequence $\{a_n\}_{n=-\infty}^{\infty}$ of independent and identically distributed (i.i.d.) random variables. Each a_n takes value of +1 and -1 with equal probability. Let the transmit sequence $b_n = a_n - a_{n-1}$. The sequence is then transmitted by the baseband signal $s(t)$, which is given by

$$s(t) = \sum_{n=-\infty}^{\infty} b_n g(t - nT),$$

where $g(t)$ is shown in Fig. 1.

(a). (10%) Decide the Fourier transform of $g(t)$.

Hint: 

$\xrightarrow{\mathcal{F}} \dfrac{2\sin\frac{T}{2}w}{w}$

(b). (10%) Decide the power spectrum density of $s(t)$.

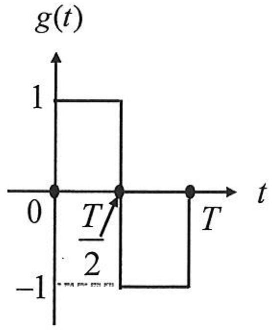


Fig. 1

5. (20%) Consider a real signal $x(t)$ with its Fourier transform $X(jw)$ as shown in Fig. 2. We want to reconstruct the signal $x(t)$ from $x_p(t)$ of the sampling system provided in Fig. 3, where $w_c = \frac{1}{2}(w_1 + w_2)$, and the cutoff frequency of the low-pass filter $H(jw)$ is $\frac{1}{2}(w_2 - w_1)$.

(a). (5%) Plot the spectrum of $x_p(t)$.

(b). (5%) Decide the maximum sampling period T so that $x(t)$ can be perfectly reconstructed from $x_p(t)$.

(c). (10%) Plot a system that can reconstruct $x(t)$ from $x_p(t)$.

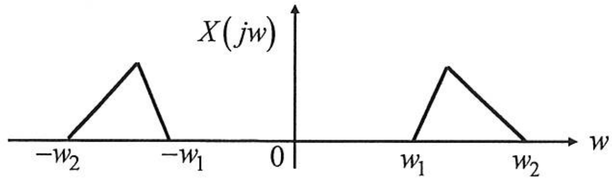


Fig. 2

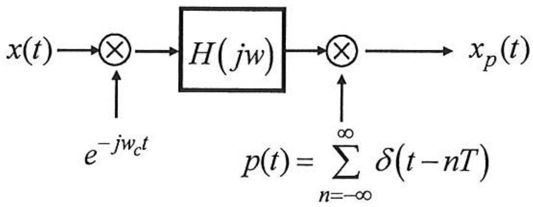


Fig. 3

6. (15%) Consider a communication system as follows

$$Y_i = A_i X_i + N_i,\ i = 1,2,3,4,$$

where $\{X_i\}_{i=1}^4$ are i.i.d. $\mathcal{N}(0,1)$, and $\{N_i\}_{i=1}^4$ are i.i.d. $\mathcal{N}(0,\sigma^2)$. $\mathcal{N}(m,\sigma^2)$ indicates the Gaussian distribution with mean m and variance σ^2. $\{Y_i\}$ are the observed output signals. Let U be the input single binary random variable. If $U=0$, then $A_1 = A_2 = 1$, and $A_3 = A_4 = 0$; $U=1$, then $A_1 = A_2 = 0$, and $A_3 = A_4 = 1$. The input random variable U is independent of $\{N_i\}_{i=1}^4$ and $\{X_i\}_{i=1}^4$.

(a). (5%) Define $\mathbf{Y} = \begin{bmatrix} Y_1 & Y_2 & Y_3 & Y_4 \end{bmatrix}^T$. Determine the joint probability density function $f_{\mathbf{Y}/U}(\mathbf{y} \mid U=0)$.

(b). (5%) Decide the log likelihood ratio

$$LLR(\mathbf{y}) = \ln\left(\frac{f_{\mathbf{Y}/U}(\mathbf{y} \mid U=0)}{f_{\mathbf{Y}/U}(\mathbf{y} \mid U=1)}\right).$$

(c). (5%) Define $E_A = Y_1^2 + Y_2^2$ and $E_B = Y_3^2 + Y_4^2$. Can the difference of $E_A - E_B$ be the sufficient statistics for $LLR(\mathbf{y})$? Provide your justification.