國立中山大學108學年度
碩士暨碩士專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】

—作答注意事項—

考試時間：100分鐘

- **考試開始響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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國立中山大學108學年度碩士暨碩士專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】　　題號：437005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　共3頁第1頁

一、選擇題(單選，計分方式:不倒扣，答對得該題全部分數，答錯及未作答得零分)

1. (5%) Which of the following functions could be the cumulative distribution function (CDF) of some random variable?
(A) $F_X(x) = \frac{1}{2} + \frac{1}{\pi} tan^{-1}(x)$
(B) $F_X(x) = e^{-|x|}$
(C) $F_X(x) = e^{-x^2}$
(D) $F_X(x) = x^2 u(x)$, ($u(x)$ is the unit step function)
(E) None of these

2. (5%) Let X represent a binominal random variable with parameters n and p. Find $E[X^2]$.
(A) np
(B) $n(n-1)p^2$
(C) $n^2p^2 + np(1-p)$
(D) n^2p^2
(E) None of these

3. (5%) A nonnegative integer-valued random variable X has the moment generating function $M(s) = \exp\{2(\exp\{e^s - 1\} - 1)\}$. Find $P(X = 0)$.
(A) 0
(B) e^{-e}
(C) e^{-2}
(D) $e^{2(e^{-1}-1)}$
(E) None of these

4. (5%) Let X be a Gaussian random variable with mean μ_X and variance σ_X^2, and let Y be a Bernoulli random variable with $P(Y = 1) = p$ and $P(Y = -1) = 1 - p$. Assume that X and Y are independent. Let $Z = XY$. Under which of the following conditions is Z a Gaussian random variable?
(A) $p = 1/2$
(B) $p = 1/2$ and $\sigma_X^2 = 1$
(C) $\sigma_X^2 = 1$
(D) $\mu_X = 1$ and $\sigma_X^2 = 1$
(E) $\mu_X = 0$

5. (5%) Two random variables X and Y are independent and uniformly distributed in [0,1]. Let f_X and f_{XY} be the probability density function (PDF) of X and joint PDF of X and Y, respectively. Which of the following statements is correct?
(A) $P(X > 2Y | X > Y) = 1/4$
(B) $f_X(x | X > Y) = 2x,\ 0 \le x \le 1$
(C) $f_{XY}(x, y | X > Y) = 2xy, 0 \le y < x \le 1$
(D) $f_{XY}(x, y | X > Y) = 1, 0 \le y < x \le 1$
(E) None of these

6. (5%) Let X be a random variable with probability distribution function given by

$$f(x) = \begin{cases} k(1-x)x^3, & 0 \le x \le 1 \\ 0, & \text{else} \end{cases}$$

for some constant $k > 0$. Find the variance of X.

(A) $2/3$ (B) $10/21$ (C) $2/63$ (D) $1/20$ (E) None of these

7. (5%) Suppose two random variables X and Y are jointly Gaussian with $\mu_X = 1, \sigma_X^2 = 4, \mu_Y = -1, \sigma_Y^2 = 1, \rho_{X,Y} = 1/3$. Let $Z = X - 3Y$. Which of the following statements is wrong?

(A) Z is Gaussian distributed
(B) $\mathbf{E}[Z] = -2$
(C) $\sigma_Z^2 = 9$
(D) $\text{Cov}(X, Z) = 2$
(E) None of these

8. (5%) Consider a random variables X with probability distribution function

$$f(x) = xe^{-x^2/2}, \quad \text{for } x \ge 0$$

Which of the following statements is wrong?

(A) CDF of X is $F(x) = 1 - e^{-x^2/2}$, for $x \ge 0$
(B) $\mathbf{E}[X] = \sqrt{\pi}$
(C) $\sigma_X^2 = 2 - \pi/2$
(D) Let $Y = X^2$. PDF of Y is $f(y) = \frac{1}{2}e^{-y/2}$, for $y \ge 0$
(E) None of these

9. (5%) A fair and six-sided die is rolled just once. Let X be 1 if the number is 3 and zero otherwise. Let Y be 1 if the number is 2 and zero otherwise. Find the correlation coefficient of X and Y.

(A) $-1/5$
(B) 0
(C) $5/36$
(D) $-1/36$
(E) None of these

10. (5%) Suppose two random variables X and Y are i.i.d. exponential distributed with PDF

$$f_X(u) = f_Y(u) = e^{-u}, \quad \text{for } u \ge 0$$

Let $\text{Z} = \min\{\text{X}, \text{Y}\}$. Which of the following statements is wrong?

(A) The CDF of Z is $F(z) = 1 - e^{-2z}$, for $z \ge 0$
(B) $\mathbf{E}[Z] = 1/2$
(C) $\sigma_Z^2 = 1/4$
(D) Z is also exponentially distributed
(E) None of these

國立中山大學 108 學年度碩士暨碩士專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】 題號：437005
※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）

二、問答計算題:

1. (10%) Let X be a random variable with the following probability mass function:
$$P(X=k)=\frac{1}{k^4}-\frac{1}{(k+1)^4} \quad \text{for } k=1,2,....$$
Find the cumulative distribution function (CDF) of X.

2. (15%) Let X and Y be independent normal random variables with zero mean and common variance σ^2. Let $U=X+Y$ and $V=X^2+Y^2$. Find the joint probability density function (PDF) of U and V. Are U and V independent?.

3. (10%) Let U be uniformly distributed over [0, 1], and let $F(x)$ be a cumulative distribution function of a random variable X. Suppose that the inverse function $F^{-1}(u)$ is well-defined for $u \in [0,1]$. Prove that the random variable $Z=F^{-1}(U)$ has the same CDF with X.

4. (15%) Over the circle $X^2+Y^2 \le r^2$, the random variables X and Y are uniformly distributed as:
$$f(x,y)=\begin{cases}\frac{1}{\pi r^2}, & x^2+y^2 \le r^2 \\ 0, & \text{else}\end{cases}$$
(a) (5%) Find the marginal PDF of X.
(b) (5%) Given $X=r/2$, find the conditional mean of Y: $\mathbf{E}[Y|X=r/2]$
(c) (5%) Are X and Y independent? Please explain why or why not.

試題隨卷繳回

國立中山大學 108 學年度
碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組、乙組選考】

—作答注意事項—

考試時間：100 分鐘

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科目名稱：通訊理論【通訊所碩士班甲組、乙組選考】　　題號：437002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. (30%) Fig. 1(a) indicates a system where the output $y(t)$ is expressed as the input signal $x(t)$ multiplied by $p(t)$. Consider $x(t)$ as a band limited signal with $|X(jw)| = 0$ for $|w| \geq w_M$ and $p(t)$ as a periodic square waveform shown in Fig. 1(b).

(a). (15%) Determine the frequency response $P(jw)$ of $p(t)$.

(b). (5%) Decide the frequency response $Y(jw)$ of the output signal $y(t)$ in terms of $X(jw)$.

(c). (10%) Assume that $T_1 = T/4$. Decide the maximum value of T so that aliasing does not exist among the replicas of $X(jw)$ in $Y(jw)$.

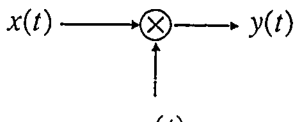

Fig. 1(a).

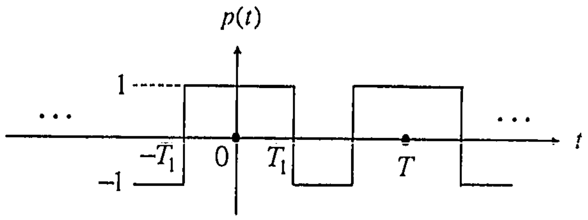

Fig. 1(b).

2. (25%) A baseband-equivalent signals

$$x_b(t) = (x_1 + jx_2)\,\text{sinc}(t)$$

is convoluted by the complex filter

$$h_1(t) = \delta(t) - j\delta(t-1).$$

(a). (5%) Compute $y_b(t) = h_1(t) * x_b(t)$.

(b). (10%) Assume $y_b(t)$ is convolved by the following filter

$$h_2(t) = 2j\,\text{sinc}(t).$$

And the result is

$$r_b(t) = h_2(t) * y_b(t) = h_b(t) * x_b(t).$$

Decide $r_b(t)$. (Hint: $\text{sinc}(t) * \text{sinc}(t-k) = \text{sinc}(t-k)$ with an integer k.)

(c). (10%) Define

$$r(t) = \text{Re}\left\{r_b(t)e^{jw_c t}\right\} = h(t) * x(t),$$

where $x(t) = \text{Re}\left\{x_b(t)e^{jw_c t}\right\}$. Show that

$$h(t) = 4\,\text{sinc}(t-1)\cos(w_c t) - 4\,\text{sinc}(t)\sin(w_c t).$$

3. (25%) Let $\{a_n\}_{n=-\infty}^{\infty}$ be a sequence of independent and identically distributed (i.i.d.) random variables and $a_n = \{0,1\}$ with equal probability. Denote b_n as a new sequence and $b_n = a_{n-2} \oplus a_n$, where $\oplus$ is an EXCLUSIVE-OR operation. The pulse-amplitude modulation (PAM) signal can then be defined as

$$s(t) = \sum_{n=-\infty}^{\infty} b_n g(t - nT),$$

where

$$g(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

and

$$|G(f)|^2 = T^2 \text{sinc}^2(Tf)$$

with $G(f)$ being the Fourier transform of $g(t)$.

(a). (5%) Decide $P(b_n = 0)$ and $P(b_n = 1)$.

(b). (12%) Decide the autocorrelation for the sequence b_n.

(c). (8%) Decide the power spectrum density of the PAM signal.

4. (10%) A passband signal can be transferred to a baseband signal with the phase splitter shown in Fig. 2(a). Here, $\hat{x}(t)$ is the Hilbert transform of $x(t)$, and the Fourier transform of $\hat{x}(t)$ is $-j\,\text{sgn}(w)X(w)$. $\text{sgn}(\bullet)$ is denoted as the sign function. If we replace the Hilbert transform by the inverse Hilbert transform with its frequency response $j\,\text{sgn}(w)$, the phase splitter can be modified as Fig. 2(b). Consider the frequency response of $x(t)$ in Fig. 2(c). Plot the frequency response of $x_{b,1}(t)$ and $x_{b,2}(t)$.

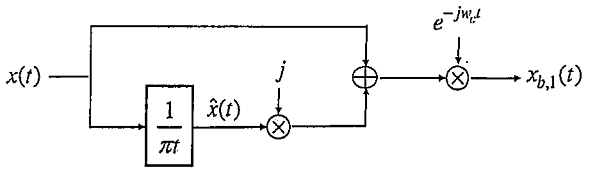

Fig. 2(a).

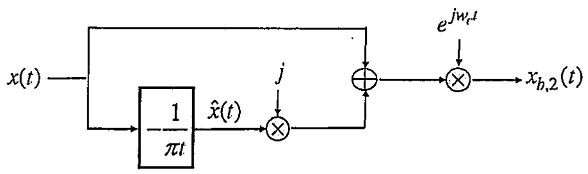

Fig. 2(b).

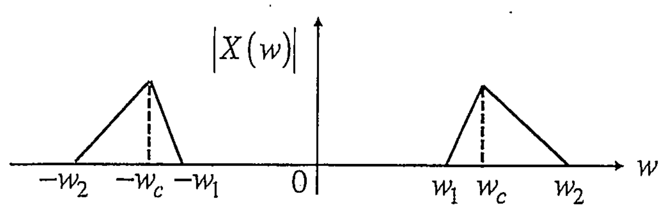

Fig. 2(c).

背面有題

試題請隨卷繳回

國立中山大學 108 學年度碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組、乙組選考】 題號：437002

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5. (10%) A single-input multiple-output (SIMO) system is constructed by an antenna deployed at the transmitter and multiple antennas utilized at the receiver. Consider a simple SIMO model shown in Fig. 3. The transmitter uses equiprobable M signals $\{s_m(t)\}_{m=1}^{M}$ for transmission. The received signals at two antennas can be expressed as

$$y_1(t) = s_m(t) + n_1(t) \quad \text{and} \quad y_2(t) = s_m(t) + n_2(t),$$

where two noises $n_1(t)$ and $n_2(t)$ are independent zero-mean white Gaussian noise with powers σ_1^2 and σ_2^2, respectively. Determine the optimum detector based on the observation of both $y_1(t)$ and $y_2(t)$.

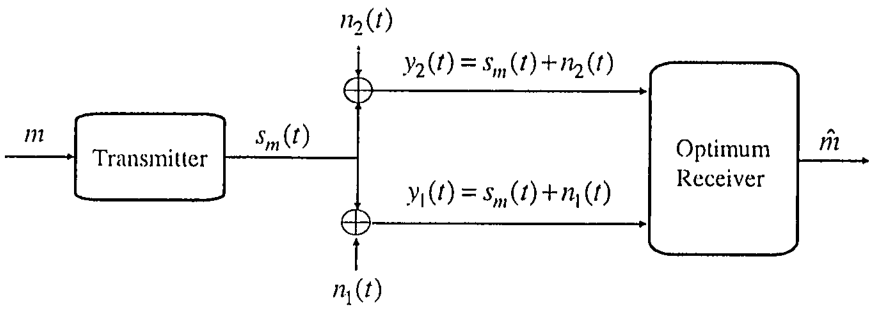

Fig. 3.

國立中山大學108學年度
碩士暨碩士專班招生考試試題

科目名稱：電子學【電波聯合碩士班選考】

—作答注意事項—

考試時間：100分鐘

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1. (30%) Figure 1 shows a differential pair of matched N-channel MOSFET transistors. Given the mathematical model $V_{GS}-V_{TH}=\sqrt{2I_{DS}/K}$, $K=\mu_n C_{ox}W/L=1\ \text{mA/V}^2$, and the bias current $I_S=80\ \mu\text{A}$. **(1) (15%)** Derive the maximum allowable differential input voltage $\Delta V_{\max}$ in Fig. 1(b). Hint: $\Delta V_{\max}$ is the voltage that is about to saturate the amplifier and turn off one MOSFET. **(2)(15%)** Calculate the transconductance g_m of the amplifier when ΔV is small.

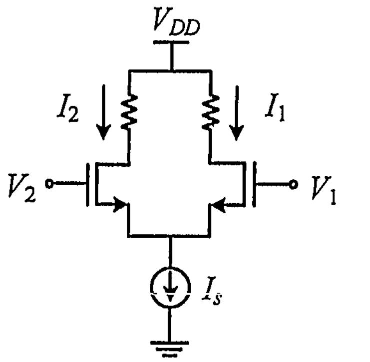

Fig. 1 (a)

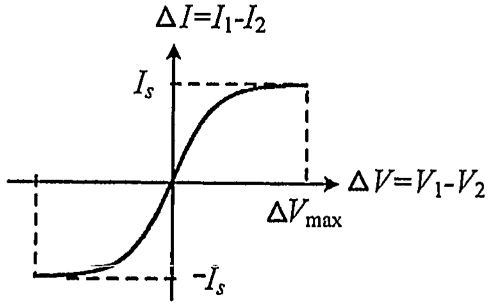

Fig. 1(b)

2. (15%) Figure 2 shows a MOSFET transistor M that makes a constant current source for a 100Ω load. The body terminal and source terminal of MOSFET M are tied together. Assume that the op-amp has an open-loop gain A=10, and M has the device parameters: $K=\mu_n C_{ox}W/L=1\ \text{mA/V}^2$, V_{TH}=0.7 V, λ = 0.1. Suppose the supply voltage V_{DD} has 1% variations on it. Estimate the percentage change in current I_D. Note: $I_D=\frac{1}{2}K(V_{GS}-V_{TH})^2(1+\lambda V_{DS})$ for M in saturation.

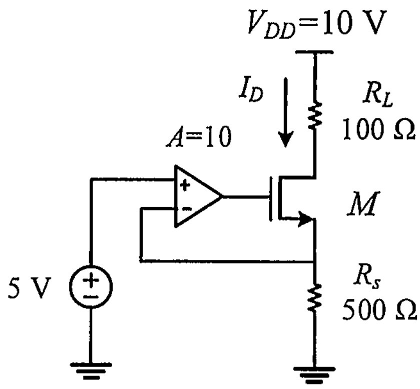

Fig. 2

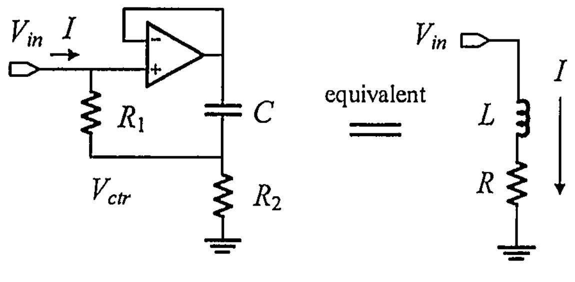

Fig. 3(a)　　Fig. 3(b)

3. (20%) An RC OP-Amp circuit shown in Fig. 3(a) behaves just like a series LR circuit in Fig. 3(b). Derive the expression of L and R in terms of C, R_1, and R_2, so that both circuits have the equivalent impedances looking from their inputs.

4. (35%) A power amplifier is shown in Fig. 4. **(1) (5%)** Explain the purpose of transistor Q_3. **(2) (15%)** C forms a high-pass filter with the voltage divider R_1 and R_2. Determine the value of C so that the cutoff frequency is 20 Hz. **(3) (15%)** Determine the voltage gain of the power amplifier.

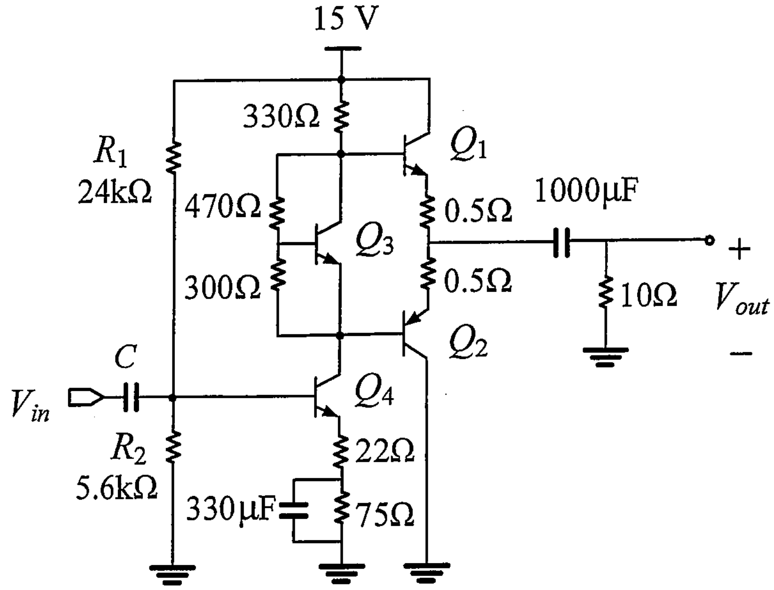

Fig. 4

國立中山大學 108 學年度
碩士暨碩士專班招生考試試題

科目名稱：線性代數【通訊所碩士班甲組】

—作答注意事項—

考試時間：100 分鐘

- **考試開始響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，不得另攜帶紙張，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，其後果由考生自行負擔。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

科目名稱：線性代數【通訊所碩士班甲組】　　　　　　　　　　　　　　　　題號：437006

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

In the following, boldface capital and lower-case letters denote matrices and vectors, respectively. For questions 1~12, please provide both answers and justifications.

1. (8%) Let matrix $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 3 & 0 & 0 \\ 0 & -4 & 5 & 0 \\ 0 & 0 & -6 & 7 \end{bmatrix}$ and $\mathbf{B} = (\mathbf{I} + \mathbf{A})^{-1}(4\mathbf{I} - \mathbf{A})$. Please calculate $(\mathbf{I} + \mathbf{B})^{-1}$, where $\mathbf{I}$ is the identity matrix.

2. (8%) Find the factorization $\mathbf{PA} = \mathbf{LDU}$ for $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 5 & 8 \\ 3 & 4 & 10 \end{bmatrix}$.

3. (8%) Let $\mathbf{A} = \begin{bmatrix} 16 & 7 & -4 \\ 0 & 4 & 6 \\ 0 & 0 & 25 \end{bmatrix}$. Find a matrix $\mathbf{B}$ such that $\mathbf{B}^2 = \mathbf{A}$.

4. (8%) Let $\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Please compute $\mathbf{A}^6$.

5. (8%) Find the best least square (error) sense by linear function $y = ax + b$ to the model:

x	1	3	5	7	9
y	-1	-2	-1	-8	-4

6. (8%) Let $\mathbf{A} = \begin{bmatrix} 1 & 3 & 6 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 4 \end{bmatrix}$. Find the QR-factorization of matrix $\mathbf{A}$.

7. (8%) Consider the set S = {0,1,2,3} with the operations:

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

−	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	3	1
3	0	3	1	2

Is this a field? Why?

8. (8%) Let $\mathbf{A}$ and $\mathbf{B}$ be 3x3 matrices with det($\mathbf{A}$) = 3 and det($\mathbf{B}$) = 7. Please find
   (a) (2%) $\det(\mathbf{AB})$
   (b) (2%) $\det(3\mathbf{A})$
   (c) (2%) $\det(2\mathbf{AB})$
   (d) (2%) $\det(A^{-1}\mathbf{B})$

9. (8%) Let $\mathbf{A} = \begin{bmatrix} 1&1&1&0 \\ 0&1&1&1 \\ 1&1&0&1 \\ 0&0&0&1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0&1&1&1&0&1 \\ 1&1&0&0&1&1 \\ 1&1&1&0&0&0 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 1&1&0&1&0&0 \\ 0&1&1&0&1&0 \\ 1&0&1&0&0&1 \end{bmatrix}$ be matrices with entries from the binary field with addition and multiplication defined in the following:

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

(a) (3%) Find the inverse matrix of **A** in the binary field.
(b) (5%) Prove that the rows of **C** span the null space of **B** in the binary field.

10. (8%) Assume that L is a linear transmission system from $\mathbb{R}^4$ to $\mathbb{R}^3$, where

$$L\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x+y+z \\ y+z+w \\ -y-z-w \end{bmatrix}.$$

Please find:
(a) (2%) the range of L.
(b) (2%) the null space of L.
(c) (2%) Is L one-to-one? Explain why.
(d) (2%) Is L onto? Explain why.

11. (10%) Let **A** and **B** denote two non-singular square matrices. Please prove that matrix **BA** has the same eigenvalues as matrix **AB**.

12. (10%) Let $\mathbf{X} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ and $\mathbf{Y} = \begin{bmatrix} E & F \\ G & H \end{bmatrix}$, where A, B, C, D, E, F, G and H are constants. Define $\mathbf{X} \cdot \mathbf{Y} = \begin{bmatrix} A \cdot E + B \cdot G & A \cdot F + B \cdot H \\ C \cdot E + D \cdot G & C \cdot F + D \cdot H \end{bmatrix}$ as the matrix multiplication using 8 multiplication operators. Is it possible to reduce the number of multiplication operators to 7? Explain why.

試題隨卷繳回