國立中山大學 107 學年度碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組、乙組選考】　　題號：437002
※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　共3頁第1頁

1. (10%) A double-sideband amplitude modulation (AM) transmitted signal can be expressed as

$$u(t)=2\left[1+\pi\sin 2\pi t\right]\cos\left(2\pi f_c t+\phi_c\right),$$

where f_c is the carrier frequency and ϕ_c is the phase. Can a simple envelope detector perfectly detect the message of $\pi\sin 2\pi t$? Explain your answer.

2. (10%) A binary PSK demodulation and detection receiver with a carrier phase error ϕ_e is considered, and the error probability can be expressed as

$$P(\phi_e)=Q\left(\sqrt{\frac{2E_b}{N_0}\cos^2\phi_e}\right),$$

where $Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp\left(\frac{-u^2}{2}\right)du$. Assume that the probability density function (PDF) of ϕ_e is $p(\phi_e)=\frac{1}{\sqrt{2\pi}\sigma_{\phi_e}}e^{\frac{-\phi_e^2}{2\sigma_{\phi_e}^2}}$. Decide the expression for the average error probability in an integral form.

3. (25%) If two equiprobable messages of $s_1(t)$ and $s_2(t)$ are transmitted over an AWGN channel with the noise power spectral density of $N_0/2$, the error probability can be expressed as

$$P_e=Q\left(\sqrt{\frac{d^2}{2N_0}}\right),$$

where $d^2=\int_{-\infty}^{\infty}\left(s_1(t)-s_2(t)\right)^2 dt$. Now, we further consider the case that $s_1(t)=u(t)$ and $s_2(t)=u(t-1)$, where $u(t)$ is shown in Fig. 1. Answer the following questions.

(a). (10%) Plot the diagram of an optimal matched filter receiver for the system. Explicitly label the required parameters.

(b). (5%) Decide the error probability for the system.

(c). (10%) Consider $s_1(t)=u(t)$ and $s_2(t)=\begin{cases}u(t-1) & \text{with probability } 0.5\\ u(t) & \text{with probability } 0.5\end{cases}$. Determine the optimum detection rule.

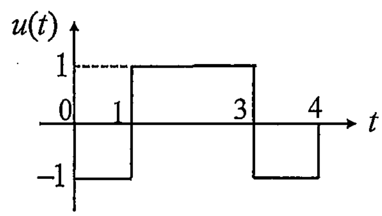

Fig. 1

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4. (20%) A carrier-modulated signal of $x(t) = a(t)\cos\left(w_c t + \theta(t)\right)$ can be transformed into an equivalent baseband signal, as shown in Fig. 2.

If $x(t) = \text{sinc}\left(10^5 t\right)\cos\left(2\pi 10^6 t\right) + \text{sinc}\left(10^5 t\right)\sin\left(2\pi 10^6 t\right)$, answer the following questions.

(a). (10%) Decide the values of w_c, $a(t)$, and $\theta(t)$.

(b). (10%) Determine $x_b(t)$ and $x_A(t)$

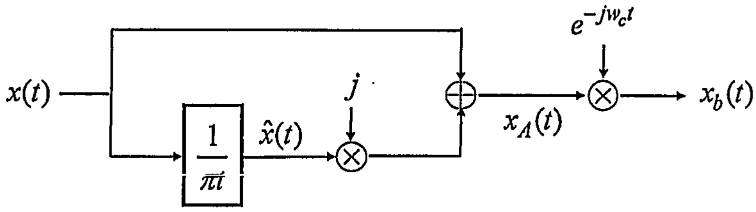

Fig. 2. Transform a passband signal to a baseband signal.

5. (15%) For a binary communication system shown in Fig. 3, the receiver obtains the two corrupted signals, y_1 and y_2, where two noises, n_1 and n_2, are not necessarily Gaussian distributions. The maximum a posteriori probability (MAP) receiver can be used to optimally detect the transmitted signal s from the observed signals, y_1 and y_2, i.e., $\hat{s} = \max_s p\left(s \mid y_1, y_2\right)$. If n_1 and n_2 are independent, can the optimum decision be based only on y_1 (i.e., $\hat{s} = \max_s p\left(s \mid y_1, y_2\right) = \max_s p\left(s \mid y_1\right)$)? Please justify your answer.

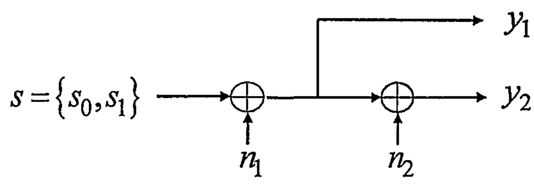

Fig. 3.

6. (20%) Denote $x(t)$ and $y(t)$ as the two bandpass real signals, and $x_L(t)$ and $y_L(t)$ are the corresponding lowpass equivalents with respect to the carrier frequency f_0. Thus, they have the following relations in frequency domain:

$$X(f)=\frac{1}{2}X_L(f-f_0)+\frac{1}{2}X_L^*(-f-f_0)$$

and

$$Y(f)=\frac{1}{2}Y_L(f-f_0)+\frac{1}{2}Y_L^*(-f-f_0),$$

where $X(f)$, $Y(f)$, $X_L(f)$, and $Y_L(f)$, are the frequency responses of $x(t)$, $y(t)$, $x_L(t)$, and $y_L(t)$, respectively.

(a). (15%) Assume $X_L(f-f_0)$ and $Y_L(-f-f_0)$ do not overlap. Show that $\int_{-\infty}^{\infty} x(t)y(t)dt=\frac{1}{2}\mathrm{Re}\left\{\int_{-\infty}^{\infty} x_L(t)y_L^*(t)dt\right\}$, where Re{$X$} represents the real part of the complex number X. Hint: Use Parseval's relation $\int_{-\infty}^{\infty} x(t)y(t)dt=\int_{-\infty}^{\infty} X(f)Y^*(f)df$.

(b). (5%) Prove that the energy in a bandpass signal is just one-half the energy in its lowpass equivalent.

國立中山大學 107 學年度碩士暨碩士專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】　　題號：437005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）(混合題)　　共 3 頁第 1 頁

一、選擇題(單選，計分方式:不倒扣，答對得該題全部分數，答錯及未作答得零分)

1. (5%) Ninety students, including Vivien and Victoria, are to be split into three classes of equal size, and this is to be done at random. What is the probability that Vivien and Victoria end up in the same class?
(A) $\frac{29}{89}$
(B) $\frac{29}{267}$
(C) $\frac{1}{3}$
(D) $\frac{1}{30}$
(E) None of these

2. (5%) Suppose that $M(t)$ is a moment-generating of some random variable. Which of the following is also a moment-generating function of some random variable?
(A) $M(t) + M(5t)$
(B) $3M(t)$
(C) $e^{-t}M(t)$
(D) $\frac{M(t)}{t}$
(E) None of these

3. (5%) Assume that a random X satisfies
$$E[X] = 0, E[X^2] = 1, E[X^3] = 0, E[X^4] = 3,$$
and let
$$Y = 1 + X + X^2.$$
Which of the following is the correlation coefficient $\rho(X, Y)$?
(A) 0
(B) 1
(C) $\frac{1}{3}$
(D) $\frac{1}{\sqrt{3}}$
(E) None of these

4. (5%) A defective coin minting machine produces coins whose probability of heads is a random variable P with probability density function
$$f_P(p) = \begin{cases} pe^p, & p \in [0,1], \\ 0, & \text{otherwise.} \end{cases}$$
A coin produced by this machine is selected and tossed repeatedly, with successive tosses assumed independent. What is the probability that a coin toss results in heads?
(A) $\frac{1}{2}$
(B) p
(C) pe^p
(D) $e-2$
(E) None of these

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5. (5%) Let X and Y be two random variables and $g(Y)$ be a function of Y. Assume that

$$E[Xg(Y)|Y] = g(Y)E[X|Y].$$

Which of the following statements is correct?
(A) X and Y are independent
(B) X and Y are uncorrelated
(C) $g(Y)$ is a constant
(D) $g(Y)$ is a linear function
(E) None of these

6. (5%) Let X be a random variable with probability distribution function given by

$$f(x) = \frac{1}{2}e^{-|x|}$$

Which of the following statements is wrong?
(A) $\mathbf{E}[X] = 0$
(B) $\mathrm{Var}(X) = 1$
(C) $P(|X| \leq u) = 1 - e^{-u}$
(D) The MGF of X is $\mathrm{M_X}(t) = \frac{1}{1-t^2}$, for $|t| < 1$.
(E) None of these

7. (5%) Consider two random variables X and Y with joint probability mass function:

$P(x,y)$	$X=1$	$X=2$	$X=3$
$Y=-1$	0.16	0.06	0.08
$Y=0$	0.16	0.08	0.16
$Y=1$	0.16	0.06	0.08

Which of the following statements is wrong?
(A) $\mathbf{E}[X] = 1.84$
(B) $P(Y=0) = 0.4$
(C) $\mathbf{E}[XY] = 0$
(D) X and Y are uncorrelated
(E) X and Y are independent

8. (5%) Consider two random variables X and Y with joint probability distribution function $f(x,y) = ke^{-3x-2y}$, for $x \geq 0, y \geq 0$.
Which of the following statements is wrong?
(A) $k = 6$
(B) $\mathbf{E}[Y] = 2$
(C) $\mathbf{E}[XY] = 1/6$
(D) X and Y are uncorrelated
(E) X and Y are independent

9. (5%) Two fair and six-sided dies are rolled at the same time. Let A and B be the events:
Event A : "Sum of two dies is four"
Event B : "At least one die shows as one"
What is the conditional probability $P(B|A)$?
(A) 2/36
(B) 11/36
(C) 3/36
(D) 2/11
(E) 2/3

國立中山大學 107 學年度碩士暨碩士專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】 題號：437005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題） 共 3 頁第 3 頁

10. (5%) Let X be a random variable with probability distribution function given by

$$F(x) = \begin{cases} 0, & x < -2 \\ 0.3, & -2 \le x < 0 \\ 0.7, & 0 \le x < 2 \\ 1, & x \ge 2 \end{cases}$$

Which of the following statements is wrong?

(A) $P(X = 0) = 0.4$

(B) $P(X > 1) = 0.3$

(C) $\mathbf{E}[X] = 0$

(D) $\mathbf{Var}(X) = 0.24$

(E) None of these

二、問答計算題:

1. (10%) The lifetimes of two light bulbs are modeled as independent and exponential random variables X and Y, with parameters λ and μ, respectively. The time at which a light bulb first burns out is

$$Z = \min\{X, Y\}.$$

Show that Z is an exponential random variable with parameter $\lambda + \mu$.

2. (15%) Let X and Y have joint probability density function given by $f(x,y) = \lambda^2 e^{-\lambda y}$ for $0 \le x \le y$, where $\lambda > 0$. Find the probability density function of $X + Y$.

3. (15%) Consider two discrete random variables X and . The random variable X has probability mass function

$$P(X = x) = \begin{cases} 0.4, & x = 1, \\ 0.6, & x = -1. \end{cases}$$

The conditional probability mass function of Y given X is given by:

$$P(Y = y|X = 1) = \begin{cases} 0.5, & y = 2 \\ 0.4, & y = 0 \\ 0.1, & y = -2 \end{cases}, P(Y = y|X = -1) = \begin{cases} 0.2, & y = 2 \\ 0.3, & y = 0 \\ 0.5, & y = -2 \end{cases}$$

(a) (5%) Find the marginal probability distribution function of Y

(b) (5%) Find $\mathbf{E}[XY]$. Are X and Y uncorrelated? Please explain your reason.

(c) (5%) Find the probability $P(X = 1|Y = 0)$

4. (10%) Consider a random variable X with probability distribution function:

$$f(x) = xe^{-\frac{x^2}{2}}, \qquad x \ge 0$$

(a) (5%) Find the cumulative distribution function of X.

(b) (5%) Let $Y = \frac{1}{2}X^2$. Find the probability distribution function of Y.

國立中山大學 107 學年度碩士暨碩士專班招生考試試題

科目名稱：線性代數【通訊所碩士班甲組】　　題號：437006

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　共 2 頁第 1 頁

In the following, boldface capital and lower-case letters denote matrices and vectors, respectively.
For questions 1~3, please select the best answer from the choices provided. (單選)
For questions 4~13, please provide both answers and justifications.

1. (5%) Suppose a 4×5 matrix **A** has rank 4. Then the equation **Ax**=**b**
   (a) always has a unique solution.
   (b) always has no solution.
   (c) always has many solutions.
   (d) sometimes but not always has a unique solution.
   (e) sometimes but not always has many solutions.

2. (5%) Suppose a 3×5 matrix **A** has rank 3.
   (a) The orthogonal complement of the range space of **A** is a 3-dimensional space.
   (b) The null space of **A** is a 3-dimensional space.
   (c) The column space of **A** is a 3-dimensional space.
   (d) The kernel of **A** is a 3-dimensional space.
   (e) The orthogonal complement of the kernel of **A** has dimension 2.

3. (5%) Which of the following matrices is a linear combination of $\begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix}$, $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$, and $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$?
   (a) $\begin{bmatrix} 2 & 3 \\ -4 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & 6 \\ -13 & 4 \end{bmatrix}$ (e) $\begin{bmatrix} 3 & -1 \\ 8 & 2 \end{bmatrix}$.

4. (10%) Let **A** and **B** be 3×3 matrices with det(**A**) = 5, det(**B**) = 10, and det(**A**+**B**) = 60. Decide the following values.
   (a) (5%) det(**A**+**A**).
   (b) (5%) $\det(\mathbf{A}^2\mathbf{B}+\mathbf{A}\mathbf{B}^2)$.

5. (10%) Let **A** be an 2x2 real symmetric matrix

$$\mathbf{A} = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}.$$

   (a) (5%) Find $\mathbf{A}^{\frac{1}{2}}$.
   (b) (5%) $F(\mathbf{x}) = \frac{\mathbf{x}^T\mathbf{A}\mathbf{x}}{\mathbf{x}^T\mathbf{x}}$, find the maximum and minimum values of $F(\mathbf{x})$ over the set of nonzero vectors in R^2.

6. (10%) Let dim(Z) denote the dimension of the vector space Z and rank(**C**) denote the rank of the matrix **C**. Show that:
   (a) (5%) If X and Y are subspaces of a vector space V, then $\dim(X+Y)=\dim(X)+\dim(Y)-\dim(X \cap Y)$.
   (b) (5%) rank(**A**+**B**) ≤ rank(**A**)+ rank(**B**) where **A** and **B** are $m \times n$ matrices. (Hint: use the result in (a))

7. (5%) Given the following matrix:

$$\begin{bmatrix} 2 & 1-i \\ 1+i & 1 \end{bmatrix}.$$

Determine whether it is Hermitian, unitary, singular and positive definite.
Please explain your reasons to each answer.

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8. (5%) Let $\mathbf{u}_1 = (1,1,1)^T$, $\mathbf{u}_2 = (1,2,2)^T$, $\mathbf{u}_3 = (2,3,4)^T$.
(a) (2%) Find the transition matrix corresponding to the change of basis from $[\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3]$ to $[\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$.
(b) (3%) Find the coordinates of $(2,3,2)^T$ with respect to $[\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$.

9. (10%) Suppose that **A** is a 5×3 real matrix of rank 3.Let $\mathbf{W} = \mathbf{A}^T\mathbf{A}$ and $\mathbf{S} = \mathbf{A}\mathbf{A}^T$.
(a) (3%) Find the ranks of **W** and **S**.
(b) (3%) Explain why $\lambda = 0$ is an eigenvalue of **S**.
(c) (4%) What is the (algebraic) multiplicity of the eigenvalue $\lambda = 0$ of **S** ?

10. (5%) Find the Jordan canonical form of

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

11. (10%) Let **M** be the vector space of all (3×3) real-valued matrices over the real field. Let **T**: **M**→**M** be a linear transformation given by

$$\mathbf{T}(\mathbf{X}) = \mathbf{A}\mathbf{X}, \text{ where } \mathbf{A} = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}.$$

(a) (5%) Find a basis for the kernel of **T**.
(b) (5%) For each eigenvalue of **T**, find a basis for the corresponding eigenspace.

12. (10%) If $\mathbf{K} = \begin{bmatrix} 1 & 2 & 2^2 & 2^3 & 2^4 & 2^5 \\ 2 & 1 & 2 & 2^2 & 2^3 & 2^4 \\ 2^2 & 2 & 1 & 2 & 2^2 & 2^3 \\ 2^3 & 2^2 & 2 & 1 & 2 & 2^2 \\ 2^4 & 2^3 & 2^2 & 2 & 1 & 2 \\ 2^5 & 2^4 & 2^3 & 2^2 & 2 & 1 \end{bmatrix}$, find det(**K**).

13. (10%) On $P_2(R)$, consider the inner product given by $< p, q >= \int_0^1 p(x)q(x)dx$.
(a) (5%) Show that the basis $(1, x, x^2)$ is NOT orthonormal.
(b) (5%) Apply the Gram-Schmidt procedure to $(1, x, x^2)$ to produce an orthonormal basis of $P_2(R)$.
Note that $P_2(R)$ is the set of all polynomials of degree 2 with real valued coefficients.

國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：電子學【電波領域聯合】　　題號：482003

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）　　共1頁第1頁

1. **(30%)** Figure 1 shows an amplifier made of a single MOSFET that is biased with I_D = 0.5 mA. Assume that all capacitors C_1, C_2 and C_3 are large enough to act like shorted in the frequency band of interest, and the parasitic capacitances of the MOSFET Q and the series gate resistance are negligible. The transistor Q has the device parameters: W/L = 80, $\mu_n C_{ox}$ = 50 μA/V^2, V_{TH} =0.7 V, λ = 0.1. **(a)(20%)** Draw the ac equivalent circuit. Determine the frequency (in rad/s) at which the amplifier achieves the peak gain, and determine this maximum gain. **(b)** **(10%)** Find the bandwidth of the amplifier (in rad/s). Note: $I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$ for Q in saturation.

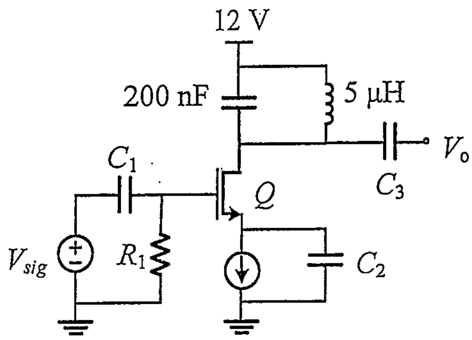


Fig. 1

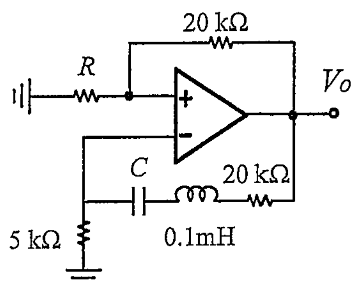


Fig. 2

2. **(20%)** Use the Barkhausen criterion to determine the values of R and C so that the Wien-bridge circuit in Fig. 2 oscillates at 100 kHz.

3. **(30%)** **(a)** **(20%)** Determine the values of R and C in Fig. 3 so that the average power dissipation on resistor R is maximized. **(b)** **(10%)** Calculate this maximum power.

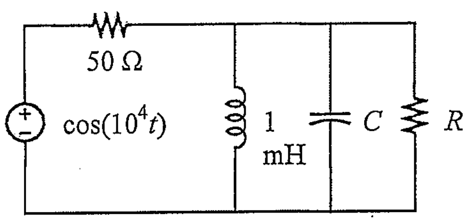


Fig. 3

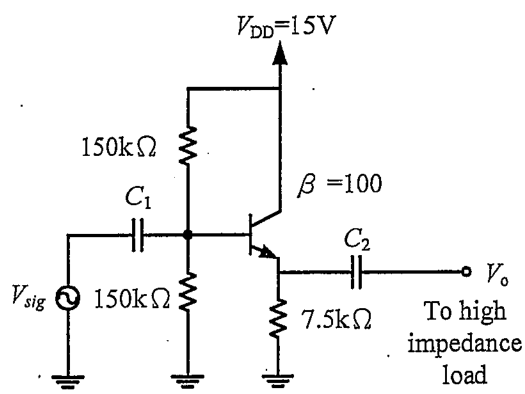


Fig. 4

4. **(20%)** An emitter follower in Fig. 4 is used to drive a very high impedance. C_1 forms a high-pass filter with the divider resistances and the resistance looking into the base. Choose the value of C_1 so that the resulting cutoff frequency is 1 kHz.

試題隨卷繳回

1. (10%) (a) Explain Gradient, Divergence, and Curl. (b) Curl of a Gradient field, $\nabla \times \nabla V = 0$, Divergence of a Curl, $\nabla \cdot \nabla \times \mathbf{A} = 0$. V is a scalar field, A is a vector field. 說明其物理意義(why?)，或舉例。

2. (10%) For a coaxial transmission line, Fig. 1, the capacitance per unit length is $c' = \frac{2\pi \cdot \varepsilon_0}{\ln\frac{b}{a}} \left[\frac{\text{F}}{\text{m}}\right]$, and the inductance per unit length is $\ell' = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi}\ln\frac{b}{a} \left[\frac{\text{H}}{\text{m}}\right]$. At high frequencies, the internal inductance drops off. Find the characteristic impedance of the coaxial line, $Z_c = \sqrt{\frac{\ell'}{c'}}$, at high frequencies. Please also write down the unit of Z_c, *i.e.*, what is the square root of (H/F)? What is the speed the wave travels in the coaxial cable? You can find it by calculating velocity $= \frac{1}{\sqrt{\ell' \cdot c'}}$.

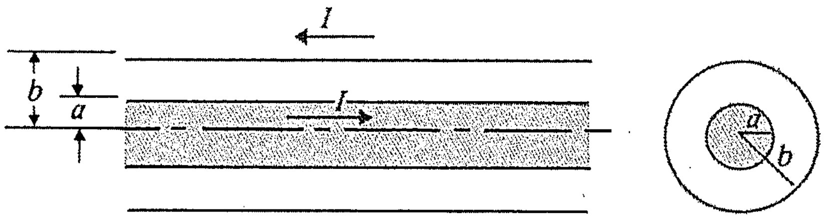


Fig. 1. Coaxial cable

3. (10%) In the following configurations, Fig. 2, assuming both grounds are perfect conductors, current directions are as indicated (the solid arrows); the image current for both cases are shown. Using $a_n \times H = J$, H is the magnetic field intensity on the ground, a_n is the normal vector of the top surface of the grounds, determine the direction of the currents on the top surface of the grounds.

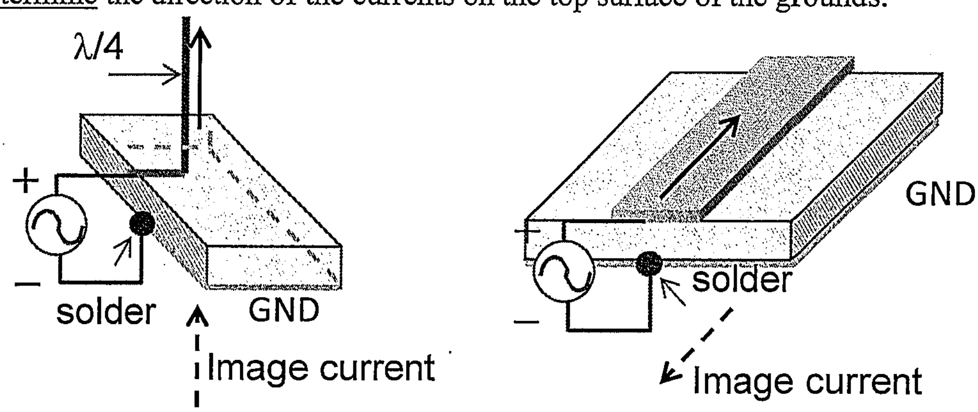


Fig. 2. Current relative to the ground.

Considering both the original current and its induced current on the ground, which one, left or right, is likely to be an effective antenna structure, why?

4. (5%) Using the Method of Image, write down the potential distribution, $V(x, y, z)$, for an observing point $P(x, y, z)$ in the space, Fig. 3. The dielectric constant of the space is ε_0. Q is a positive point

charge of Q *Coul.*

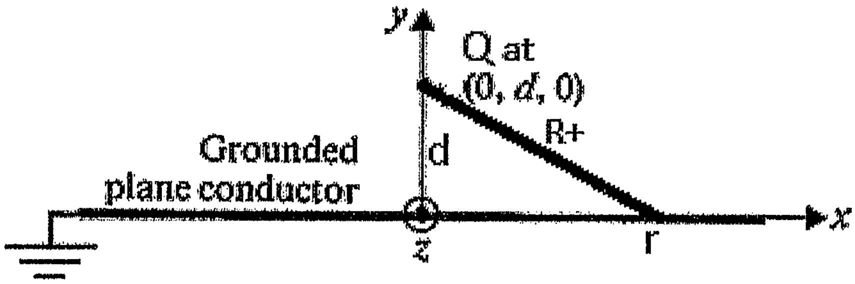
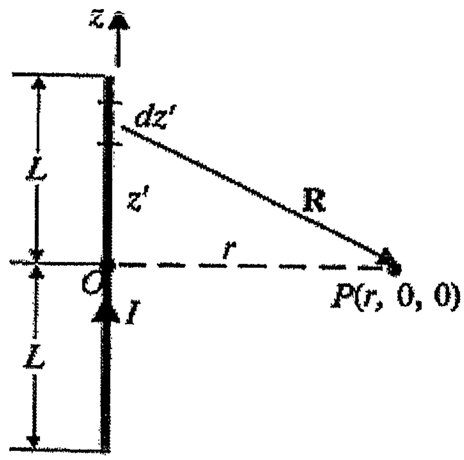


Fig. 3. A charge source Q above a ground.

5. (10%) 下圖 Fig. 4 之 magnetic flux density B can be found as,



$$\mathrm{B} = \mathrm{a}_\phi \frac{\mu_0 I L}{2\pi r\sqrt{L^2+r^2}}$$

Fig. 4. An observing point P near a current source I.

簡化上列之 B as a function of r, μ_0, and a_ϕ when $L>>r$. In cylindrical coordinate system,

$\nabla\cdot\mathrm{B} = \frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{1}{r}\frac{\partial B_\phi}{\partial \phi} + \frac{\partial B_z}{\partial z}$. Show that $\nabla\cdot\mathrm{B} = 0$.

6. (5%) What are the permittivity ε and permeability μ of Copper, a very good conductor? Provide your reasoning.

7. (10%) Analyze the performance of a right-hand circularly polarized wave received respectively by linearly or circularly polarized antennae.

8. (15%) As shown in Fig. 5, a waveguide filled with a material whose $\varepsilon_r = 2.25$ has dimensions $a = 2$ cm and $b = 1.4$ cm. If the guide is to transmit 13.5 GHz signals, what possible modes can be used for the transmission? Please respectively calculate the cutoff frequencies of the possible modes.

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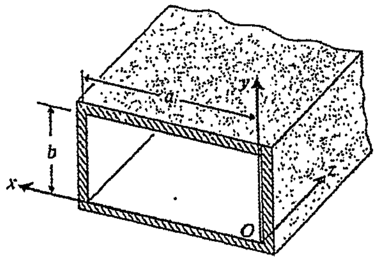

Fig. 5

9. (15%) According to Fig. 6, write the input impedance of the transmission line in differential special cases.
   (a) (6%) Open-circuit termination, and also plot the reactance-line length diagram
   (b) (6%) Short-circuit termination, and also plot the reactance-line length diagram
   (c) (3%) Quarter-wave section

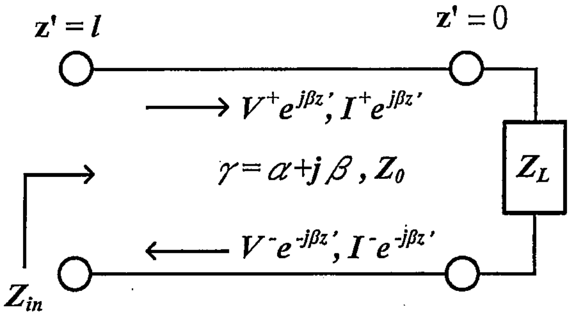

Fig. 6

10. (10%) As shown in Fig. 7, calculate the average power transmitted into the infinite 150 Ω line. The λ/2 line is lossless and the infinitely long line is slightly lossy. (Hint: The input impedance of an infinitely long line is equal to its characteristic impedance so long as the line is not lossless)

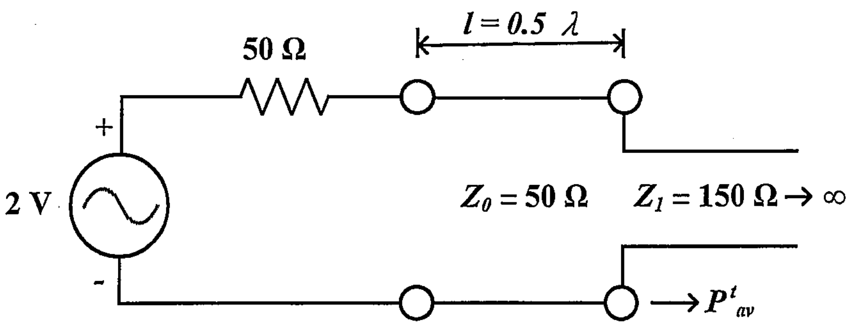

Fig. 7