國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組】　　題號：437002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　共3頁第1頁

(50 %) Multiple Choice. Please mark the answers on your computer scoring answer sheet.

1. (　) (5%) True or false. The signal $x(t)=\text{sinc}(t)$ is a power-type signal.

   A. True.　　B. False.

2. (　) (5%) True or false. If $Y=3X+2$, then $H(Y|X)=0$. Here, $H(Y|X)$ is the conditional entropy of random variable Y given the random variable X.

   A. True.　　B. False.

3. (　) (5%) True or false. A time-domain signal has a frequency domain representation that can be obtained using Fourier transform.

   A. True.　　B. False.

4. (　) (5%) True or false. Frequency modulation is a nonlinear modulation while phase modulation is linear.

   A. True.　　B. False.

5. (　) (5%) True or false. The matched filter can maximize the signal-to-noise ratio (SNR) of the sampled signals even in COLOR noise environment.

   A. True.　　B. False.

6. (　) (5%) True or false. Fig. 1 can be a regular autocorrelation function.

   A. True.　　B. False.

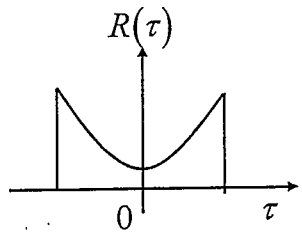

Fig 1.

7. (　) (5%) True or false. Consider a linear-time invariant system. If the input signal in time-domain $x(t)=e^{3t}$ and system impulse response $h(t)=5\delta(t)$, then the output signal does not exist since the Fourier transform of $x(t)$ does not converge.

   A. True.　　B. False.

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8. (　) (5%) True or false. A conventional AM signal $u(t)$ contains a large carrier component in addition to the AM modulated signal, i.e., $u(t)=A_c\left(1+m(t)\right)\cos\left(2\pi f_c t+\phi_c\right)$. The message $m(t)$ can be completely recovered by a envelop detector as $m(t)$ is constrained to $|m(t)|\geq 1$.

A. True.　　B. False.

9. (　) (5%) True or false. A white noise process with power spectrum $N_0/2$ passes the filter with frequency response in Fig. 2. Then, the power of the filter output is $N_0/3$.

A. True.　　B. False.

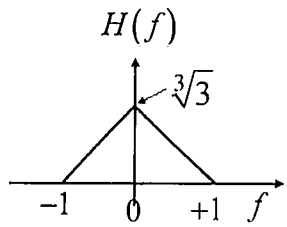

Fig. 2

10. (　) (5%) True or false. For a real signal $x(t)$, its Fourier transform is also real.

A. True.　　B. False.

11. (15%) For a binary PAM system, the received signal is expressed as $r=\pm\sqrt{E_b}+N$, where N is a zero-mean Gaussian random variable with variance σ_n^2. Assume the two signals are transmitted with unequal probability $P\left(a_m=\sqrt{E_b}\right)=1/3$ and $P\left(a_m=-\sqrt{E_b}\right)=2/3$.

(a). (10%) Decide the optimum threshold at the detector.

(b). (5%) Compute the average probability of error in terms of Q-function.

Hint: $Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{u^2}{2}}du$.

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12. (35%) The system in Fig. 3(a) shows that a low pass signals $x(t)$ with a bandwidth of w is sampled at the Nyquist rate. The sampled signal is expressed as $x_p(t)=\sum_{n=-\infty}^{\infty}(-1)^n x(nT_s)\delta(t-nT_s)$, where T_s is the sampling period. Assume the Fourier transform of $x(t)$ be $X(f)$ given in Fig. 3(b). Answer the following questions.

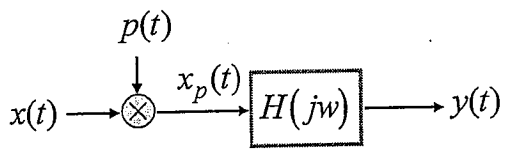
(a). (10%) Find the Fourier transform of $x_p(t)$.

(b). (5%) Can we reconstruct $x(t)$ from $x_p(t)$ with a linear time-invariant system? Please justify your answer.

(c). (5%) Can we reconstruct $x(t)$ from $x_p(t)$ with a time-varying system? Please justify your answer.

(d). (10%) Assume $h(t)$ be a bandpass filter with frequency response shown in Fig. 3(c). Plot the frequency response of $y(t)$.

(e). (5%) How can you reconstruct $x(t)$ from $y(t)$?



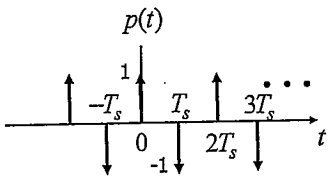


Fig. 3(a)

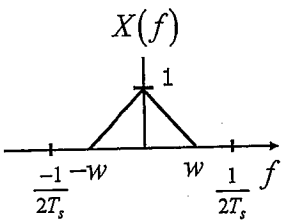


Fig. 3(b)

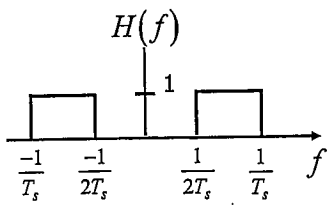


Fig. 3(c)

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科目名稱：機率【通訊所碩士班甲組】　　題號：437005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　

選擇題(單選，計分方式:不倒扣，答對得該題全部分數，答錯及未作答得零分)

1. (5%) Let $S=\{1,2,3,4\}$ be a sample space of an experiment. If G is the smallest field that contains the sets $\{1\}$ and $\{2,3\}$, which of the following sets is also contained in G?

   (A) $\{3\}$

   (B) $\{4\}$

   (C) $\{1,3\}$

   (D) $\{2,4\}$

   (E) $\{1,3,4\}$

2. (5%) Let X be a random variable with probability mass function given by

$$p_X(x)=\begin{cases} x^2/a, & \text{if } x=-3,-2,-1,0,1,2,3, \\ 0, & \text{otherwise.} \end{cases}$$

   What is the variance of X?

   (A) 28

   (B) 14

   (C) 9

   (D) 7

   (E) 1

3. (5%) Suppose that X and Y are random variables with the same variance σ^2. What is the covariance of $X-Y$ and $X+Y$?

   (A) 0

   (B) 1

   (C) σ^2

   (D) $2\sigma^2$

   (E) σ^4

4. (5%) Let X be a random variable. Let $M_X(s)$ be the moment generating function associated with X. Which of the following expressions cannot be $M_X(s)$?

   (A) $M_X(s)=\dfrac{e^s}{3-2e^s}$

   (B) $M_X(s)=\dfrac{1}{4}e^{-s}+\dfrac{1}{2}+\dfrac{1}{8}e^{4s}+\dfrac{1}{8}e^{5s}$

   (C) $M_X(s)=\dfrac{1}{3}\cdot\dfrac{3}{2-s}+\dfrac{2}{3}\cdot\dfrac{2}{3-s}$

   (D) $M_X(s)=\dfrac{e^{3s}}{1-2s}$

   (E) $M_X(s)=\dfrac{2}{2-s}e^{3(e^s-1)}$

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5. (5%) A six-sided die is rolled three times independently. What is the probability that we obtain a sum of 12?
   (A) 1/6
   (B) 1/18
   (C) 1/36
   (D) 1/216
   (E) 25/216.

6. (5%) Two coins are tossed simultaneously. If one of them turned head, what is the probability that the other one also turn head?
   (A) 0.1
   (B) 0.25
   (C) 0.5
   (D) 0.75
   (E) None of these

7. (5%) If the pdf of a continuous random variable is given as
$$f(x) = \begin{cases} x/2, & 0 \le x \le 2, \\ 0, & \text{otherwise}, \end{cases}$$
   what is the value of $P(X = 1)$?
   (A) 0
   (B) 0.25
   (C) 0.5
   (D) 1
   (E) None of these

8. (5%) If a continuous random variable X has the pdf
$$f(x) = \frac{1}{\pi} \cdot \frac{1}{1+x^2}, \quad -\infty < x < \infty,$$
   what is its mean?
   (A) 0
   (B) 1
   (C) π
   (D) $ln\ (1 + \pi)$
   (E) None of these

9. (5%) If a continuous random variable X has the pdf
$$f(x) = \frac{1}{2} e^{-\frac{x}{2}}, \qquad x \ge 0.$$
   Which of the following statements is wrong?
   (A) $P(0 \le x \le 2) = e^{-1}$
   (B) $\mathbf{E}[X] = 2$
   (C) $\mathbf{Var}[X] = 8$
   (D) The CDF of X is
$$F(x) = 1 - e^{-\frac{x}{2}}, \qquad x \ge 0.$$
   (E) None of these

國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】　　題號：437005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　

10. (5%) Consider two i.i.d. random variables X_1 and X_2 with Poisson distribution:

$$P(X_i = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \qquad k = 0,1,2,3, \ldots\ldots$$

Which of the following statements is wrong?

(A) The mean of X_i is λ

(B) The variance of X_i is λ

(C) The moment generating function of X_i is $e^{\lambda(e^t-1)}$

(D) $X_1 + X_2$ is Poisson distributed

(E) None of these

問答計算題:

1. (10%) Consider two discrete random variables X and Y with joint pmf:

$P(x,y)$	$X=-1$	$X=0$	$X=1$
$Y=2$	0.15	0.15	0.1
$Y=4$	0.05	0.1	0.15
$Y=6$	0.1	0.15	0.05

(a) (5%) Are X and Y independent? Prove it or disprove it.

(b) (5%) Are X and Y uncorrelated? Prove it or disprove it.

2. (15%) Consider a random variable X with pmf:

$$P(X=x) = \begin{cases} 0.4, & x = \pm 1, \\ 0.2, & x = 0. \end{cases}$$

Given $X = x$, the conditional distribution of a random variable Y is Gaussian with $N(x,1)$.

(a) (5%) Find the marginal distribution of Y.

(b) (5%) Find the conditional probability $P(X=1|Y=1)$.

(c) (5%) Find the conditional mean $\mathbf{E}[X|Y=1]$

3. (10%) An exponential random variable has a PDF of the form

$$f_Z(z) = \begin{cases} \lambda e^{-\lambda z}, & \text{if } z \geq 0, \\ 0, & \text{otherwise,} \end{cases}$$

where λ is a positive parameter. Suppose that X and Y are independent exponential random variables with common parameter λ. Please find $E[\max(2X,Y)]$.

4. (15%) Let X be a random variable that takes nonnegative integer values, and is associated with a moment generating function of the form

$$M_X(s) = \frac{2}{9} \cdot \frac{3+4e^{2s}+2e^{3s}}{3-e^s}.$$

(a) (5%) Find $E[X]$.

(b) (10%) Find $E[X \mid X \neq 0]$.

國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：線性代數【通訊所碩士班甲組】　　題號：437006

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）　　

For each of the following questions, please select the best answer from the choices provided. (單選)
You do NOT need to provide any justification.

1. (5%) Which of the following statement is **False**?
(A) If an augmented matrix $[\mathbf{A}\ \mathbf{b}]$ is transformed into $[\mathbf{C}\ \mathbf{d}]$ by elementary row operations, then the equations $\mathbf{Ax} = \mathbf{b}$ and $\mathbf{Cx} = \mathbf{d}$ have exactly the same solution sets.
(B) If a system $\mathbf{Ax} = \mathbf{b}$ has more than one solution, then so does the system $\mathbf{Ax} = \mathbf{0}$.
(C) If matrices $\mathbf{A}$ and $\mathbf{B}$ are row equivalent, they have the same reduced echelon form.
(D) If $\mathbf{A}$ is an $m \times n$ matrix and the equation $\mathbf{Ax} = \mathbf{b}$ is consist for every $\mathbf{b}$ in $\mathbb{R}^m$, then $\mathbf{A}$ has m pivot column.
(E) If $\mathbf{A}$ is an $m \times n$ matrix and the equation $\mathbf{Ax} = \mathbf{b}$ is consistent for some $\mathbf{b}$, then the columns of $\mathbf{A}$ span $\mathbb{R}^m$.

2. (5%) Which of the following statement is **False**?
(A) If $\mathbf{A}$ and $\mathbf{B}$ are row equivalent $m \times n$ matrices and if the columns of $\mathbf{A}$ span $\mathbb{R}^m$, then so do the columns of $\mathbf{B}$.
(B) In some cases, it is possible for four vectors to span $\mathbb{R}^5$.
(C) If $\mathbf{u}$ and $\mathbf{v}$ are in $\mathbb{R}^m$, then $-\mathbf{u}$ is in $\mathrm{Span}\{\mathbf{u}, \mathbf{v}\}$.
(D) If $\mathbf{A}$ is a 6×5 matrix, the linear transformation $\mathbf{x} \mapsto \mathbf{Ax}$ cannot map $\mathbb{R}^5$ onto $\mathbb{R}^6$.
(E) A linear transform is a function.

3. (5%) Which of the following statement is **False**?
(A) If $\mathbf{A}$ and $\mathbf{B}$ are $m \times n$, then both $\mathbf{AB}^T$ and $\mathbf{A}^T\mathbf{B}$ are defined.
(B) Left-multiplying a matrix $\mathbf{B}$ by a diagonal matrix $\mathbf{A}$, with nonzero entries on the diagonal, scales the rows of $\mathbf{B}$.
(C) If $\mathbf{BC} = \mathbf{BD}$, then $\mathbf{C} = \mathbf{D}$.
(D) If $\mathbf{AB} = \mathbf{BA}$ and if $\mathbf{A}$ is invertible, then $\mathbf{A}^{-1}\mathbf{B} = \mathbf{BA}^{-1}$.
(E) An elementary $n \times n$ matrix has either n or $n + 1$ nonzero entries.

4. (5%) Which of the following statement is **False**?
(A) If $\mathbf{B}$ is formed by adding to one row of $\mathbf{A}$ a linear combination of other rows, then $\det(\mathbf{A}) = \det(\mathbf{B})$.
(B) $\det(\mathbf{A}^T\mathbf{A}) \geq 0$.
(C) If $\mathbf{A}^3 = \mathbf{0}$, then $\det(\mathbf{A}) = 0$.
(D) $\det(-\mathbf{A}) = -\det(\mathbf{A})$.
(E) If $\mathbf{A}$ is invertible, then $\det(\mathbf{A})\det(\mathbf{A}^{-1}) = 1$.

5. (5%) Which of the following statement is **False**?
(A) If $\mathbf{B}$ is obtained from a matrix $\mathbf{A}$ by several elementary row operations, then $\mathsf{rank}(\mathbf{B}) = \mathsf{rank}(\mathbf{A})$.
(B) Row operations on a matrix $\mathbf{A}$ can change the linear dependence relations among the rows of $\mathbf{A}$.
(C) A change-of-coordinates matrix is always invertible.
(D) If $\mathbf{A}$ is $m \times n$ and linear transformation $\mathbf{x} \mapsto \mathbf{Ax}$ is onto, then $\mathsf{rank}\mathbf{A} = m$.
(E) If $\mathbf{A}$ is $m \times n$ and $\mathsf{rank}\mathbf{A} = m$, then the linear transform $\mathbf{x} \mapsto \mathbf{Ax}$ is one-to-one.

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6. (5%) Which of the following statement is **False**?
   (A) If $\mathbf{A}$ is invertible and 1 is an eigenvalue of $\mathbf{A}$, then 1 is also an eigenvalue of $\mathbf{A}^{-1}$.
   (B) If $\mathbf{A}$ contains a row or column of zeros, then 0 is an eigenvalue of $\mathbf{A}$.
   (C) Each eigenvector of $\mathbf{A}$ is also an eigenvector of $\mathbf{A}^2$.
   (D) Each eigenvalue of $\mathbf{A}$ is also an eigenvalue of $\mathbf{A}^2$.
   (E) Eigenvectors much be nonzero vectors.

7. (5%) Which of the following statement is **False**?
   (A) There exists a 2×2 matrix that has no eigenvectors in $\mathbb{R}^2$.
   (B) If $\mathbf{A}$ is diagonalizable, then the column of $\mathbf{A}$ are linearly independent.
   (C) A nonzero vector cannot correspond to two different eigenvectors of $\mathbf{A}$.
   (D) If $\mathbf{A}$ and $\mathbf{B}$ are invertible $n \times n$ matrices, then $\mathbf{AB}$ is similar to $\mathbf{BA}$.
   (E) If $\mathbf{A}$ is an $n \times n$ diagonalizable matrix, then each vector in $\mathbb{R}^n$ can be written as a linear combination of eigenvectors of $\mathbf{A}$.

8. (5%) Which of the following statement is **False**?
   (A) Two eigenvectors corresponding to the same eigenvalue are always linearly dependent.
   (B) Similar matrices always have exactly the same eigenvalues.
   (C) The matrices $\mathbf{A}$ and $\mathbf{A}^T$ have the same eigenvalues, counting multiplicities.
   (D) Each eigenvector of an invertible matrix $\mathbf{A}$ is also an eigenvector of $\mathbf{A}^{-1}$.
   (E) If $\mathbf{A}$ is similar to a diagonalizable matrix $\mathbf{B}$, then $\mathbf{A}$ is also diagonalizable.

9. (5%) Which of the following statement is **False**?
   (A) If $\mathbf{A}$ is orthogonally diagonalizable, then $\mathbf{A}$ is symmetric.
   (B) If $\mathbf{A}$ is orthogonal matrix, then $\|\mathbf{Ax}\| = \|\mathbf{x}\|$ for all $\mathbf{x}$ in $\mathbb{R}^n$.
   (C) By a suitable change of variable, any quadratic form $\mathbf{x}^T\mathbf{Ax}$ can be changed into one with no cross-product term.
   (D) The largest value of a quadratic form $\mathbf{x}^T\mathbf{Ax}$, for $\|\mathbf{x}\| = 1$, is the largest entry on the diagonal of $\mathbf{A}$.
   (E) If $\mathbf{P}$ is an $n \times n$ orthogonal matrix, then the change of variable $\mathbf{x} = \mathbf{Pu}$ transforms $\mathbf{x}^T\mathbf{Ax}$ into a quadratic form whose matrix is $\mathbf{P}^{-1}\mathbf{AP}$.

10. (5%) Which of the following statement is **False**?
    (A) The set of all vectors in $\mathbb{R}^n$ orthogonal to one fixed vector is a subspace of $\mathbb{R}^n$.
    (B) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal set and if c_1, c_2, and c_3 are scalars, then $\{c_1\mathbf{v}_1, c_2\mathbf{v}_2, c_3\mathbf{v}_3\}$ is an orthogonal set.
    (C) If a square matrix has orthonormal columns, then it also has orthonormal rows.
    (D) If a vector $\mathbf{y}$ coincides with its orthogonal projection onto a subspace $\mathbf{W}$, then $\mathbf{y}$ is in $\mathbf{W}$.
    (E) If a matrix $\mathbf{U}$ has orthonormal columns, then $\mathbf{UU}^T = \mathbf{I}$.

11. (10%) The dimension of the subspace

$$H = \left\{ \begin{bmatrix} a-3b+6c \\ 5a+4d \\ b-2c-d \\ 5d \end{bmatrix} : a,b,c,d \in \mathbb{R} \right\}$$

is

(A) 1.
(B) 2.
(C) 3.
(D) 4.
(E) 5.

12. (10%) Let

$\mathbf{A} = \begin{bmatrix} .4 & -.3 \\ .4 & 1.2 \end{bmatrix}$. As $k \to \infty$, we obtain $\mathbf{A}^k$

(A) $\begin{bmatrix} -.5 & -1.75 \\ 1.0 & 1.50 \end{bmatrix}$.

(B) $\begin{bmatrix} -.75 & -.5 \\ 1.0 & 1.50 \end{bmatrix}$.

(C) $\begin{bmatrix} -.5 & 1.50 \\ 1.0 & -.75 \end{bmatrix}$.

(D) $\begin{bmatrix} -1.5 & -.75 \\ 1.0 & 2.50 \end{bmatrix}$.

(E) $\begin{bmatrix} -.5 & -.75 \\ 1.0 & 1.50 \end{bmatrix}$.

13. (10%) Let $\mathbf{J}$ be the $n \times n$ matrix of all 1's, and consider $\mathbf{A} = (a-b)\mathbf{I} + b\mathbf{J}$; that is

$$\mathbf{A} = \begin{bmatrix} a & b & b & \dots & b \\ b & a & b & \dots & b \\ b & b & a & \dots & b \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b & b & b & \dots & a \end{bmatrix}.$$

Then the eigenvalues of $\mathbf{A}$ are

(A) $a+b$, and $a+(n-1)b$.
(B) $a-nb$, and $a+nb$.
(C) $a-b$, and $a+(n-1)b$.
(D) $a-2b$, and $a+nb$.
(E) $a+b$, and $a-(n-1)b$.

14. (10%) The determinant of

$$\mathbf{A} = \begin{bmatrix} 3a & -7 & 8 & 9 & -6 \\ 0 & 2 & -5 & 7 & 7 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 2 & 4 & -1 \\ 0 & 0 & 0 & -2 & 0 \end{bmatrix}$$

is

(A) -11a.
(B) -12a.
(C) -13a.
(D) -14a.
(E) -15a.

15. (10%) Let $\mathbf{A}$ and $\mathbf{B}$ be 4×4 matrices, with $\det \mathbf{A} = -1$ and $\det \mathbf{B} = 2$. Then, $\det \mathbf{B}^{-1}\mathbf{AB} + \det \mathbf{A}^T\mathbf{A} + \det 2\mathbf{A}$ =

(A) -12.
(B) -14.
(C) -16.
(D) -18.
(E) -20.