國立中山大學105學年度碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組】　　　　題號：437007

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）(混合題)　共3頁第1頁

(50 %) Multiple Choice. Please mark the answers on your computer scoring answer sheet.

1. (　　) (5%) True or false. Viterbi algorithm provides a low-complexity maximum-likelihood decoding for convolutional codes. The complexity of the algorithm increases linearly with the constraint length of the convolutional codes.

   A. True.　　B. False.

2. (　　) (5%) True or false. Consider a transmission system given by Fig. 1. Herein, $x(t) = \mathrm{sinc}(2000t)$ and $\mathrm{sinc}(y) = \dfrac{\sin(\pi y)}{\pi y}$. The ideal lowpass and highpass filters are cascaded with cutoff frequencies at 900 and 800 Hz, respectively. Then $y(t)$ is a baseband signal.

   A. True.　　B. False.

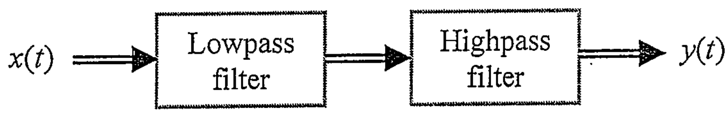

Fig. 1.

3. (　　) (5%) In Figs. 2(a) and 2(b), which one is the possible signal waveform of frequency modulation if the input is a square-wave signal.

   A. Fig 2(a).　　B. Fig 2(b).

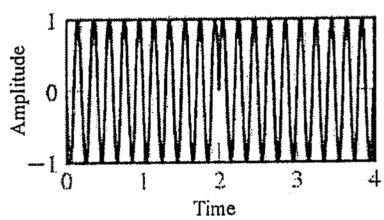

Fig. 2(a)

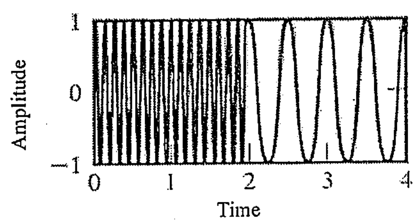

Fig. 2(b)

4. (　　) (5%) True or false. The condition of the zero inter-symbol interference can be satisfied in a digital communication system if we define the pulse shape function as $h(t) = \mathrm{sinc}\left(\dfrac{\alpha t}{T}\right)$, where $0 < \alpha < 1$ and T is the sampling interval.

   A. True.　　B. False.

5. (　　) (5%) True or false. The condition of the zero inter-symbol interference is satisfied in a digital communication system if we adopt the pulse shape function $h(t) = e^{-|t|}\mathrm{sinc}\left(\dfrac{t}{T}\right)$, where T is the sampling interval.

   A. True.　　B. False.

試題隨卷繳回　背面有題

國立中山大學105學年度碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組】　　題號：437007

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　共3頁第2頁

6. (　　) (5%) True or false. Consider pulse-amplitude modulation (PAM) of ANALOG signals in a transmission system. Assume that the message bandwidth is W = 5k Hz, the sampling rate is f_s = 12k Hz, and the pulse is rectangular with pulse width $T < 1/f_s$. Then, we can exactly recover the message signal $m(t)$ from the PAM signal $s(t)$ by first sampling $s(t)$ to retrieve the sampled signals, and then applying an ideal lowpass filter with cut-off frequency 6k Hz.

   A. True.　　B. False.

7. (　　) (5%) True or false. Consider the constellation map in Fig. 3. The corresponding bit rate is $4/T$ bits/sec., where T is the symbol interval.

   A. True.　　B. False.

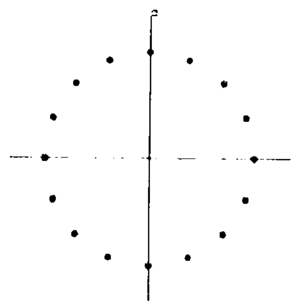

Fig. 3

8. (　　) (5%) Figs. 4(a) and 4(b) show two different 8-point QAM signal constellations. Assume the minimum distance between adjacent points is 2A. Which constellation is more power efficient?

   A. Fig. 4(a).　　B. Fig. 4(b).

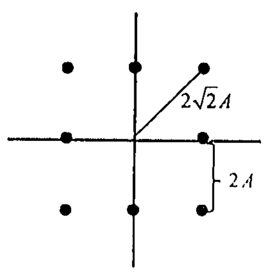

Fig. 4(a)

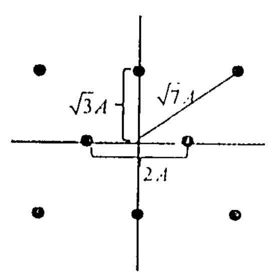

Fig. 4(b)

9. (　　) (5%) True or false. If a periodic physical signal has finite power, the corresponding coefficients of the Fourier series expansion x_n tend to zero as $n \to \infty$.

   A. True.　　B. False.

10. (　　) (5%) True or false. If the input and the impulse response of a linear time-invariant system are energy-type, then the output is also energy-type.

   A. True.　　B. False.

背面有題

國立中山大學 105 學年度碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組】 題號：437007

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）(混合題) 共 3 頁第 3 頁

11. (10%) A simple repetition code of block length 5 can only contain two code words one (1,1,1,1,1) and the other (0,0,0,0,0). Decide the generation matrix and the parity check matrix of this code in the systematic form.

12. (20%) A simple communication system is constructed by cascading n-1 regenerative repeaters and a terminal receiver. Herein, a binary signal is used for transmission. Assume that the probability of detection error at each receiver is p and that error among the repeaters are statistically independent.

(a). (5%) Derive the probability of i out of n repeaters to produce an error.

(b). (15%) Show that the binary error probability at the terminal receiver is given by

$$P_n = \frac{1}{2}\left(1-(1-2p)^n\right)$$

(Hint: A bit error at the output of the terminal receiver occurs only when an odd number of repeaters produces an error.)

13. (20%) Fig. 5 illustrates a system diagram of a detector for amplitude modulation (AM). The received signal $s(t) = A_c\left(1+k_a m(t)\right)\cos\left(2\pi f_c t\right)$ is an AM signal. Assume that $1+k_a m(t) \geq 0$ for all t, $m(t)$ is the message signal, and ϕ specifies the phase error of the local oscillator at the receiver.

(a). (15%) Show that the system can exactly recover the message signal by appropriately choosing a lowpass filter. Specifically, you should express the $u_1(t)$, $u_2(t)$, $x_1(t)$, $x_2(t)$, $y_1(t)$, $y_2(t)$, $z_1(t)$, and $z_2(t)$ of Fig. 5 in detail. Then, use $z_2(t)$ to recover the message signal.

(b). (5%) Is this a coherent detector or a noncoherent detector? Please give the brief justification.

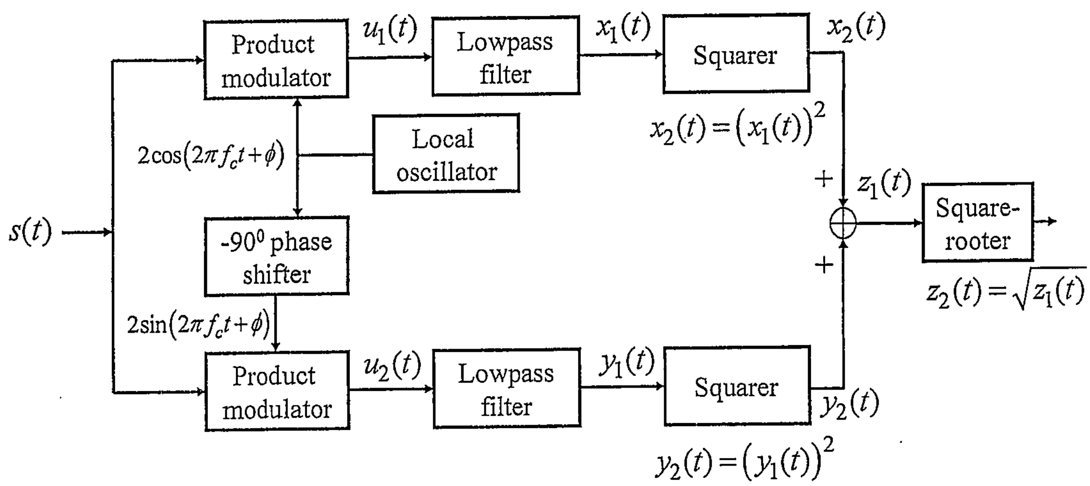

Fig. 5.

試題隨卷繳回

國立中山大學105學年度碩士暨碩士專班招生考試試題

科目名稱：線性代數【通訊所碩士班甲組】　　　　　　　　題號：437004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）　　　　共4頁第1頁

For each of the following questions, please select <u>the best answer</u> from the choices provided. (單選) You do NOT need to provide any justification.

1. (5%) Which of the following statement is **true**?
(A) Every matrix is row equivalent to a unique matrix in echelon form.
(B) Any system of n linear equations in n variables has at most n solutions.
(C) If a system of linear equations has two different solutions, it must have infinitely many solutions.
(D) If a system of linear equations has no free variables, then it has a unique solution.
(E) If $\mathbf{A}$ is an $m \times n$ matrix and the equation $\mathbf{Ax} = \mathbf{b}$ is consistent for some $\mathbf{b}$, then the columns of $\mathbf{A}$ span $\mathbb{R}^m$.

2. (5%) Which of the following statement is **true**?
(A) If an augmented matrix $[\mathbf{A}\ \mathbf{b}]$ can be transformed by elementary row operations into reduced echelon form, then the equation $\mathbf{Ax} = \mathbf{b}$ is consistent.
(B) If matrices $\mathbf{A}$ and $\mathbf{B}$ are row equivalent, they have the same reduced echelon form.
(C) The equation $\mathbf{Ax} = \mathbf{0}$ has the trivial solution if any only if there are no free variables.
(D) If an $m \times n$ matrix $\mathbf{A}$ has a pivot positions in every row, then the equation $\mathbf{Ax} = \mathbf{b}$ has a unique solution for each $\mathbf{b}$ in $\mathbb{R}^m$.
(E) If none of the vectors in the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ in $\mathbb{R}^3$ is a multiple of one of the other vectors, then S is linearly independent.

3. (5%) Which of the following statement is **false**?
(A) If an augmented matrix $[\mathbf{A}\ \mathbf{b}]$ is transformed into $[\mathbf{C}\ \mathbf{d}]$ by elementary row operations, then the equations $\mathbf{Ax} = \mathbf{b}$ and $\mathbf{Cx} = \mathbf{d}$ has exactly the same solution sets.
(B) If a system $\mathbf{Ax} = \mathbf{b}$ has more than one solution, then so does the system $\mathbf{Ax} = \mathbf{0}$.
(C) If an $n \times n$ matrix $\mathbf{A}$ has n pivot positions, then the reduced echelon form of $\mathbf{A}$ is the $n \times n$ identity matrix.
(D) If 3×3 matrices $\mathbf{A}$ and $\mathbf{B}$ each have three pivot positions, then $\mathbf{A}$ can be transformed into $\mathbf{B}$ by elementary row operations.
(E) In some cases, it is possible for four vectors to span $\mathbb{R}^5$.

4. (5%) Determine a and b such that the solution set of the system
$x_1 + 3x_2 = b,$
$4x_1 + ax_2 = 8$
is empty
(A) $(a, b) = (12, 2)$.
(B) $(a, b) = (12, 3)$.
(C) $(a, b) = (4, 2)$.
(D) $(a, b) = (3, 2)$.
(E) $(a, b) = (3, 12)$.

5. (5%) Which of the following statement is **true**?
(A) If $\mathbf{A}$ and $\mathbf{B}$ are $m \times n$, then both $\mathbf{AB}^T$ and $\mathbf{A}^T\mathbf{B}$ are defined.
(B) If $\mathbf{AB} = \mathbf{C}$ and $\mathbf{C}$ has two columns, then $\mathbf{A}$ has two column.
(C) If $\mathbf{BC} = \mathbf{BD}$, then $\mathbf{C} = \mathbf{D}$.
(D) If $\mathbf{AC} = \mathbf{0}$, then either $\mathbf{A} = \mathbf{0}$ or $\mathbf{C} = \mathbf{0}$.
(E) If $\mathbf{A}$ and $\mathbf{B}$ are $n \times n$ matrix, then $(\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B}) = \mathbf{A}^2 - \mathbf{B}^2$.

6. (5%) Which of the following statement is **false**?
(A) Left-multiplying a matrix $\mathbf{B}$ by a diagonal matrix $\mathbf{A}$, with nonzero entries on the diagonal, scales the rows of $\mathbf{B}$.
(B) An elementary $n \times n$ matrix has either n or $n+1$ nonzero entries.
(C) The transpose of an elementary matrix is an elementary matrix.
(D) The elementary matrix must be square.
(E) Every square matrix is a product of elementary matrices.

7. (5%) Which of the following statement is **true**?
(A) If $\mathbf{A}$ is a 2×2 matrix with a zero determinant, then one column of $\mathbf{A}$ is a multiple of the other.
(B) If $\mathbf{A}$ is a 3×3 matrix, then $\det(5\mathbf{A}) = 5\det(\mathbf{A})$.
(C) If $\mathbf{A}$ and $\mathbf{B}$ are $n \times n$ matrices, with $\det(\mathbf{A}) = 2$ and $\det(\mathbf{B}) = 3$, then $\det(\mathbf{A}+\mathbf{B}) = 5$.
(D) If $\mathbf{A}$ is $n \times n$ and $\det(\mathbf{A}) = 2$, then $\det(\mathbf{A}^3) = 6$.
(E) If $\mathbf{B}$ is produced by interchanging two rows of $\mathbf{A}$, then $\det(\mathbf{A}) = \det(\mathbf{B})$.

8. (5%) Which of the following statement is **false**?
(A) If two rows of a 3×3 matrix $\mathbf{A}$ are the same, then $\det(\mathbf{A}) = 0$.
(B) If $\mathbf{B}$ is produced by multiplying row 3 of $\mathbf{A}$ by 5, then $\det(\mathbf{B}) = 5 \cdot \det(\mathbf{A})$.
(C) If $\mathbf{B}$ is formed by adding to one row of $\mathbf{A}$ a linear combination of the other rows, then $\det(\mathbf{A}) = \det(\mathbf{B})$.
(D) $\det(\mathbf{A}^T) = -\det(\mathbf{A})$.
(E) $\det(-\mathbf{A}) = -\det(\mathbf{A})$

9. (5%) Which of the following statement is **true**?
(A) If $\mathbf{A}$ is row equivalent to the identity matrix $\mathbf{I}$, then $\mathbf{A}$ is diagonalizable.
(B) If $\mathbf{A}$ contains a row or column of zeros, then 0 is an eigenvalue of $\mathbf{A}$.
(C) Each eigenvalue of $\mathbf{A}$ is also an eigenvalue of $\mathbf{A}^2$.
(D) Eigenvalues must be nonzero scalars.
(E) Two eigenvectors corresponding to the same eigenvalue are always linearly dependent.

10. (5%) Which of the following statement is **false**?
(A) If $\mathbf{A}$ is invertible and 1 is an eigenvalue of $\mathbf{A}$, then 1 is also an eigenvalue of $\mathbf{A}^{-1}$.
(B) Each eigenvector of $\mathbf{A}$ is also eigenvector of $\mathbf{A}^2$.
(C) Each eigenvector of an invertible matrix $\mathbf{A}$ is also eigenvector of $\mathbf{A}^{-1}$.
(D) Similar matrices always have exactly the same eigenvectors.
(E) The matrices $\mathbf{A}$ and $\mathbf{A}^T$ have the same eigenvalues, counting multiplicities.

背面有題

11. (10%) Let $\mathbf{A} = \begin{bmatrix} 16 & -35 \\ 6 & -13 \end{bmatrix}$. $\mathbf{A}^8$ is

(A) $\begin{bmatrix} 8925 & -3826 \\ 1530 & -3569 \end{bmatrix}$.

(B) $\begin{bmatrix} 3826 & -8925 \\ 1530 & -3569 \end{bmatrix}$.

(C) $\begin{bmatrix} 3826 & -8925 \\ 3569 & -1530 \end{bmatrix}$.

(D) $\begin{bmatrix} -3569 & -8925 \\ 1530 & 3826 \end{bmatrix}$.

(E) $\begin{bmatrix} 3826 & 1530 \\ -8925 & -3569 \end{bmatrix}$.

12. (10%) Let $\mathbf{J}$ be the $n \times n$ matrix of all 1's, and consider $\mathbf{A} = (a-b)\mathbf{I} + b\mathbf{J}$; that is

$$\mathbf{A} = \begin{bmatrix} a & b & b & \dots & b \\ b & a & b & \dots & b \\ b & b & a & \dots & b \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b & b & b & \dots & a \end{bmatrix}.$$

Then $\det(\mathbf{A})$ is

(A) $(a-b)^n[a+(n-1)b]$.
(B) $(a-b)^{n-1}[a+nb]$.
(C) $(a+b)^{n-1}[a+(n-1)b]$.
(D) $(a-b)^{n-1}[a+(n-1)b]$.
(E) $(a-b)^{n+1}[a+(n+1)b]$.

13. (10%) Suppose $\mathbf{x}$ is an eigenvector of $\mathbf{A}$ corresponding to an eigenvalue λ. What are the corresponding eigenvalue and the eigenvector of $5\mathbf{I} - \mathbf{A}$?
(A) eigenvalue $= 5 - \lambda$, eigenvector $= 51 - \mathbf{x}$
(B) eigenvalue $= \lambda$, eigenvector $= 5\mathbf{x}$
(C) eigenvalue $= 5 - \lambda$, eigenvector $= \mathbf{x}$
(D) eigenvalue $= -\lambda$, eigenvector $= 51 - \mathbf{x}$
(E) eigenvalue $= 5 + \lambda$, eigenvector $= \mathbf{x}$

14. (10%) The determinant of

$$\mathbf{A} = \begin{bmatrix} 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \\ -5a & -8 & 0 & 3 \\ 0 & 5 & 0 & -6 \end{bmatrix}$$

is
(A) 0. (B) $20a$. (C) $10a$. (D) $-10a$. (E) $-20a$.

15. (10%) If

(A) $\mathbf{A} = \begin{bmatrix} a_{11}+v_1 & a_{12} & u_1 \\ a_{21}+v_2 & a_{22} & u_2 \\ a_{31}+v_3 & a_{32} & u_3 \end{bmatrix}$, (B) $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12}+v_1 & u_1 \\ a_{21} & a_{22}+v_2 & u_2 \\ a_{31} & a_{32}+v_3 & u_3 \end{bmatrix}$,

(C) $\mathbf{A} = \begin{bmatrix} a_{11}+v_1 & a_{12} & u_1 \\ a_{21} & a_{22}+v_2 & u_2 \\ a_{31} & a_{32} & u_3+v_3 \end{bmatrix}$, (D) $\mathbf{A} = \begin{bmatrix} a_{11}+v_1 & a_{12}+v_2 & u_1+v_3 \\ a_{21} & a_{22} & u_2 \\ a_{31} & a_{32} & u_3 \end{bmatrix}$,

(E) $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & u_1+v_1 \\ a_{21} & a_{22} & u_2+v_2 \\ a_{31} & a_{32} & u_3+v_3 \end{bmatrix}$,

then $\det(\mathbf{A}) = \det(\mathbf{B}) + \det(\mathbf{C})$, where

$\mathbf{B} = \begin{bmatrix} a_{11} & a_{12} & u_1 \\ a_{21} & a_{22} & u_2 \\ a_{31} & a_{32} & u_3 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} a_{11} & a_{12} & v_1 \\ a_{21} & a_{22} & v_2 \\ a_{31} & a_{32} & v_3 \end{bmatrix}$.

背面有題

國立中山大學105學年度碩士暨碩士專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】　　　　題號：437006

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　共1頁第1頁

1. (Totally, 25 pts) Let T be the random variable of the lifetime (in years) of an LED lamp. The lifetime T satisfies

$$\Pr[T > t] = \frac{100}{t^2}, \text{ for } t > 10.$$

 (a) (8 pts) Please find $\Pr[T \le 3]$.

 (b) (7 pts) Please find the expected value of T.

 (c) (10 pts) Find the conditional probability of failure in the next year for a working LED lamp that is now t years old for each $t > 10$.

2. (Totally, 10 pts) Let X be uniformly distributed on the interval $[-1,1]$. In addition, let $Y = X^2$ and $Z = X^4$.

 (a) (5 pts) Are X and Y uncorrelated? Why?

 (b) (5 pts) Are Z and Y uncorrelated? Why?

3. (Totally, 25 pts) We transmit a bit of information which is "0" with probability $1-p$ and "1" with probability p. Because of channel noise, each transmitted bit is received correctly at the output of the channel with probability $1-\varepsilon$. That is

$$\Pr(\text{"1" is received} \mid \text{"1" is transmitted}) = \Pr(\text{"0" is received} \mid \text{"0" is transmitted}) = 1-\varepsilon.$$

 (a) (5 pts) Suppose we receive a "1" at the output. Please find the conditional probability that the transmitted bit is a "1".

 (b) (10 pts) A bit of information is chosen to transmit. Assume that we transmit the information bit n times over the channel. Also, assume that the uses of the channel are independent. Let q_n be the probability that the transmitted information bit is a "1" given that we have received "1" n times at the output. Please find q_n. Please also find q_n when $n \to \infty$.

 (c) (10 pts) Assume that we transmit the symbol "1" a total of n times over the channel. At the output of the channel, we receive the symbol "1" three times in the n received bits, and that we receive a "1" at the n-th transmission. Given these observations, what is the probability that j-th received bit is a "1"?

4. (Totally, 25 pts) Assume that a pair of random variables X and Y has a joint probability density function given by

$$f_{XY}(x,y) = \begin{cases} \alpha, & \text{if } x \in [0,1] \text{ and } y \in [x, x+1]; \\ 0, & \text{otherwise.} \end{cases}$$

 (a) (10 pts) Find α such that $f_{XY}(x,y)$ is a joint probability density function.

 (b) (15 pts) Are X and Y independent? Please explain your answer.

5. (Totally, 15 pts) Assume that X has a moment generating function given by

$$M_X(t) = \frac{1}{6}e^{-2t} + \frac{1}{3}e^{-t} + \frac{1}{8}e^{t} + \frac{3}{8}e^{2t}.$$

Please find $\Pr(|X| \le 1)$.

試題隨卷繳回