國立中山大學103學年度碩士暨碩士專班招生考試試題

科目名稱：線性代數【通訊所碩士班甲組】 題號：437002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘） 共2頁第1頁

1. (10%) Suppose a 4 by 5 matrix A has rank 4. Then the equation $Ax = b$
   (a) always has a unique solution.
   (b) always has no solution.
   (c) always has many solutions.
   (d) sometimes but not always has a unique solution.
   (e) sometimes but not always has many solutions.

Please select the best answer and you do NOT need to provide any justification.

2. (10%) Suppose a 3 by 5 matrix A has rank 3.
   (a) The orthogonal complement of the range space of A is a 3-dimensional space.
   (b) The nullspace of A is a 3-dimensional space.
   (c) The column space of A is a 3-dimensional space.
   (d) The kernel of A is a 3-dimensional space.
   (e) The orthogonal complement of the kernel of A has dimension 2.

Please select the best answer and you do NOT need to provide any justification.

3. (10%) Suppose that A is the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \\ 2 & 4 \end{bmatrix}.$$

Which of the following is the column space of A?
   (a) $[8, \ 28, \ 14]^T$.
   (b) $[28, \ 8, \ 14]^T$.
   (c) $[14, \ 28, \ 8]^T$.
   (d) $[28, \ 14, \ 8]^T$.

Please select the best answer and you do NOT need to provide any justification.

4. (10%) Which of the following matrix A can project every vector b in $\mathbb{R}^3$ onto the line in the direction of $a = [2, 1, 3]$?

(a) $A = \frac{1}{14}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$. (b) $A = \frac{1}{14}\begin{bmatrix} 4 & 2 & 6 \\ 2 & 1 & 3 \\ 6 & 3 & 9 \end{bmatrix}$.

(c) $A = \frac{1}{14}\begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 6 & 3 & 9 \end{bmatrix}$. (d) $A = \frac{1}{14}\begin{bmatrix} 3 & 1 & 2 \\ 6 & 2 & 4 \\ 9 & 3 & 6 \end{bmatrix}$.

Please select the best answer and you do NOT need to provide any justification.

5. (10%) Suppose that A is the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}.$$

Which of the following is all the eigenvalues of A?
   (a) $\lambda = (1, 1, 1)$.
   (b) $\lambda = (1, 1, 0)$.
   (c) $\lambda = (1, 0, 0)$.
   (d) $\lambda = (0, 0, 0)$.

Please select the best answer and you do NOT need to provide any justification.

背面有題

6. (10%) Suppose the n by n matrix A that has 0 on its main diagonal and all other entries are -1; i.e., $\mathrm{A} = \mathbf{I} - \mathbf{1}\mathbf{1}^T$ with $\mathbf{1} = [1, 1, \ldots, 1]^T$. The determinant of A is
(a) $n^2 - 1$.
(b) $n^2 + 1$.
(c) $n - 1$.
(d) $n + 1$.
Please select the best answer and you do NOT need to provide any justification.

7. (10%) Suppose that A is the matrix

$$\mathrm{A} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Which of the following is all the eigenvectors of A?
(a) $[1,\ -1]^T$, $[1,\ 1]^T$.
(b) $[1,\ -i]^T$, $[1,\ i]^T$.
(c) $[1,\ -i]^T$, $[1,\ 1]^T$.
(d) $[1,\ -i]^T$, $[i,\ 1]^T$.
Please select the best answer and you do NOT need to provide any justification.

8. (10%) If A is any m by n matrix with $m > n$, then
(a) AA^T is always a positive semidefinite matrix.
(b) AA^T is always invertible.
(c) $\mathrm{A}^T\mathrm{A}$ is always a positive definite matrix.
(d) A is a symmetric matrix.
Please select the best answer and you do NOT need to provide any justification.

9. (10%) Let $\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3, \mathrm{v}_4$ be four non-zero vectors. If $\mathrm{v}_i^T \mathrm{v}_j = 0$ for all $i \neq j$, then
(a) they must be linearly dependent.
(b) they must be linearly independent.
(c) they must be either linearly independent or linearly dependent.
(d) none of the above hold.
Please select the best answer and you do NOT need to provide any justification.

10. (10%) Suppose that A is the matrix

$$\mathrm{A} = \begin{bmatrix} 2-k & 1 \\ 3 & 4-k \end{bmatrix}.$$

For which value of constant k does matrix A fail to be invertible?
(a) $k = 4$.
(b) $k = 3$.
(c) $k = 2$.
(d) $k = 1$.
Please select the best answer and you do NOT need to provide any justification.

國立中山大學 103 學年度碩士暨碩士專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】 題號：437004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘） 共1頁第1頁

1. (Totally, 15 pts) A coin is chosen to have a random bias. That is, the coin has probability Q of coming up heads, where Q is a random variable uniformly distributed over the interval [0,1]. Then, Chris and Judy each flip the coin once. Let X be the random variable which is 1 if Chris flips a head and 0 if he flips a tail. Similarly, let Y be the random variable which is 1 if Judy flips a head and 0 if she flips a tail.
   - **(a)** (7 pts) What is the probability that both Chris and Judy flip heads?
   - **(b)** (8 pts) Are the random variables X and Y independent? Please explain your answer.

2. (Totally, 15 pts) Let X_1, X_2 and X_3 be independent random variables, each of which is normally distributed with mean μ and variance σ^2. Define $Y = X_1 + X_2$ and $Z = X_3 + X_2$.
   - **(a)** (5 pts) Find the covariance between Y and Z.
   - **(b)** (10 pts) Find $E[E[Z|Y]]$.

3. (Totally, 15 pts) Let X be a Bernoulli random variable with parameter α, that is,
$$P(X=x)=\begin{cases}1-\alpha & \text{if } x=0\\ \alpha & \text{if } x=1\\ 0 & \text{otherwise.}\end{cases}$$
Please find $E[X^k]$ for all $k > 0$.

4. (Totally, 20 pts) Let Z be a continuous random variable with density function $f_Z(z)$. We say that Z is symmetric about α if for all z,
$$P(Z \geq \alpha + z) = P(Z \leq \alpha - z).$$
   - (a) (10 pts) Prove that Z is symmetric about α if and only if for all z, we have
$$f_Z(\alpha - z) = f_Z(\alpha + z).$$
   - (b) (10 pts) Let Y be a continuous random variable with probability density function given by
$$f_Y(y) = \frac{1}{\pi[1+(y-1)^2]}, \qquad y \in \boldsymbol{R}.$$
Find the point about which Y is symmetric.

5. (Totally, 20 pts) Suppose X and Y are two jointly distributed random variables with joint probability density function given by
$$f_{XY}(x,y)=\begin{cases}12xy(c-x) & 0<x<1, 0<y<1\\ 0 & \text{otherwise,}\end{cases}$$
where c is a constant.
   - **(a)** (10 pts) Find c.
   - **(b)** (10 pts) Find the probability $P\left(Y<\frac{1}{2}|X>\frac{1}{2}\right)$.

6. (Totally, 15 pts) For $\beta > 0$, let
$$F_{XY}(x,y)=\begin{cases}1-\beta e^{-\beta(2x+y)} & \text{if } x>0, y>0;\\ 0 & \text{otherwise,}\end{cases}$$
Determine if F_{XY} can be the joint probability distribution function of two random variables X and Y.

國立中山大學103學年度碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組】　　　　題號：437005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）　　　　

1. (30 %) True or False. You do NOT need to provide any justification.

(a) (　　) Every time domain signal can be expressed in terms of overlasting sinusoids and exponentials.

(b) (　　) The Fourier spectrum of a signal indicates only the relative amplitude of the sinusoids/exponentials that are required to synthesize that signal.

(c) (　　) If $x(t)$ is periodic, the spectrum is also periodic.

(d) (　　) In double sideband suppressed-carrier (DSB-SC) signal, the envelope of its bandpass signal is proportional to the amplitude of the message signal.

(e) (　　) A fast time-varying signal suffers more slope-overload distortion than the slow one.

(f) (　　) If $Y = g(X)$ with g being a deterministic function, then $H(Y \mid X) = 0$.

2. (20%) A single-sideband (SSB) AM signal is transmitted via modulating an 1000 Hz carrier by the signal $m(t) = \cos(200\pi t) + 2\sin(200\pi t)$. The amplitude of the carrier is $A_c = 20$.

(a) (5%) Decide the Hilbert transform of $m(t)$.

(b) (10%) Decide the time domain expression for the lower sideband of the SSB AM signal.

(c) (5%) Decide the magnitude spectrum of the lower sideband SSB signal.

3. (15%) Show that

$$\operatorname{sinc}(t) * \operatorname{sinc}(t) * \operatorname{sinc}(t) = \operatorname{sinc}(t),$$

where $*$ represents the convolution operation and $\operatorname{sinc}(t) = \dfrac{\sin \pi t}{\pi t}$.

4. (15%) Using the properties of the Fourier transform, compute $\int_{-\infty}^{\infty} \operatorname{sinc}^3(t)\,dt$.

背面有題

國立中山大學 103 學年度碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組】　　題號：437005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）　　

5. (20%) A Z channel is given as Fig. 1(a). Decide the input probability distribution that achieves capacity. Also, determine the capacity of the infinitely cascaded Z channel, as shown in Fig. 1(b).

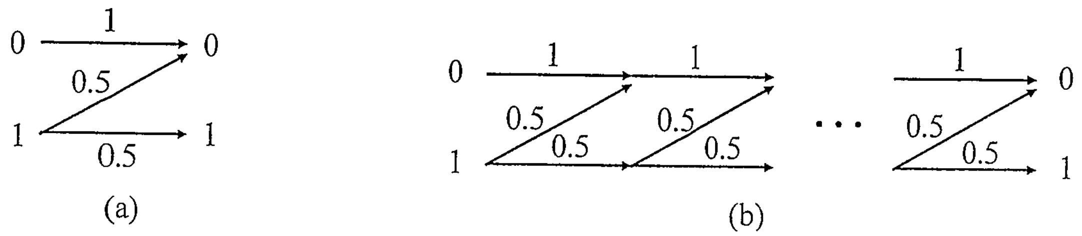

Fig. 1 (a) Z channel. (b) Cascaded Z channel.