國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：電磁學【電機系碩士班戊組、通訊所碩士班乙組、電波聯合碩士班】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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國立中山大學 113 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：電磁學【電機系碩士班戊組、通訊所碩士班乙組、電波聯合碩士班】題號：482004
※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題） 共1頁第1頁

1. (25%) The radius of the inner conducting sphere and the inner radius of the outer spherical conductor are R_i and R_o, respectively. The voltage between these two concentric spherical conductors is V. The space between the conductors is filled with a dielectric medium with the permittivity ε. Determine the stored electrostatic energy.

2. (25%) The magnetic flux density vector is $\mathbf{B} = (5kx+4)\boldsymbol{a}_x - (3ky+10y)\boldsymbol{a}_y + (8kz)\boldsymbol{a}_z$ in free space. Please determine the value of the constant k.

3. (10%) Write the frequency-domain Maxwell's equations with time-varying source. Define phase velocity and group velocity.

4. (15%) The magnetic field intensity of a linearly polarized uniform plane wave propagating in the +y-direction in seawater [$\varepsilon_r = 80$, $\mu_r = 1$, $\sigma = 4$ (S/m)] is

$$\vec{H} = \hat{a}_x 0.1 \sin\left[10^{10}\pi t - \frac{\pi}{3}\right] \quad \text{(A/m)}$$

at $y = 0$.

a) Determine the attenuation constant, the phase constant, the intrinsic impedance, the phase velocity, the wavelength, and the skin depth. (5%)

b) Find the location at which the amplitude of $\vec{H}$ is 0.01 (A/m). (5%)

c) Write the expressions for $\vec{E}(y, t)$ and $\vec{H}(y, t)$ at $y = 0.5$ (m) as function of t. (5%)

5. (10%) For the case of oblique incidence of a uniform plane wave with perpendicular polarization on a perfectly conducting plane boundary as shown in Fig. 1, write (a) the instantaneous expressions

$$\vec{E}_1(x,z;t) \text{ and } \vec{H}_1(x,z;t)$$

For the total field in medium 1, using a cosine reference, (5%) and (b) the time-average Poynting vector. (5%)

6. (5%) A standard air-filled S-band rectangular waveguide has dimensions $a = 7.21$ (cm) and $b = 3.40$ (cm). What mode types can be used to transmit electromagnetic waves having the 5-cm wavelengths?

7. (10%) Find the input impedance of the lossless transmission line shown in Fig. 2.

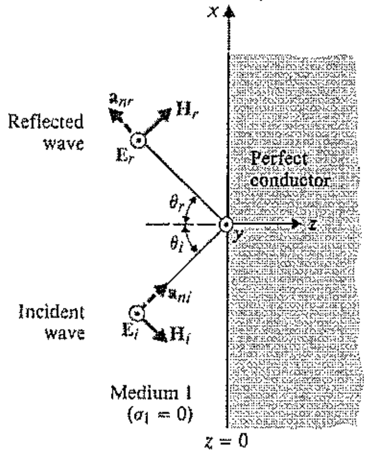

Fig. 1

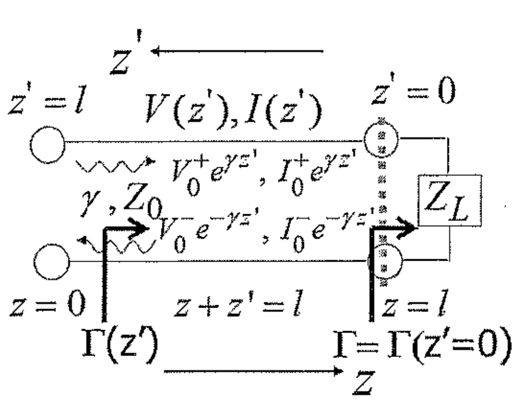

Fig. 2

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

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國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

科目名稱：電子學【電機系碩士班戊組選考、通訊所碩士班乙組選考、電波聯合碩士班選考】題號：482003

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　共2頁第1頁

1. (15%) A third-order low-pass filter has transmission zeros at $\omega = 2$ rad/s and at $\omega = \infty$. Its natural modes are at $s = -1$ and $s = -0.5 \pm j0.8$. The dc gain is unity. Find the transfer function $T(s)$. (15%*1)

2. (30%) For the common-base circuit in Fig. 1, assuming the bias current to be about 1 mA, $\beta = 100$, $C_\mu = 0.5$ pF, $r_e = 25\ \Omega$, and $f_T = 1000$ MHz:
   (a) Estimate the midband gain V_o/V_s.
   (b) Use the short-circuit time-constants method to estimate the lower 3-dB frequency, f_L. (*Hint:* In determining the resistance seen by C_1, the effect of the 47-kΩ resistor must be taken into account.)
   (c) Find the high-frequency poles, and estimate the upper 3-dB frequency, f_H. (10%*3)

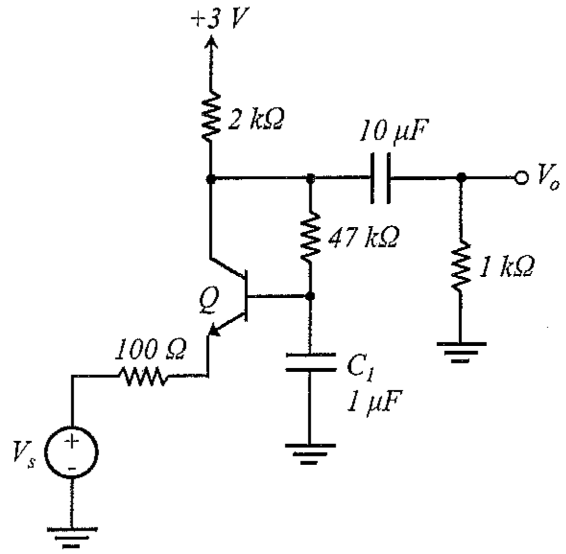

Fig. 1

3. (20%) For the emitter-follower circuit shown in Fig. 2 the BJT used is specified with a β value of 100, find :
   (a) I_E, V_E, and V_B. (10%)
   (b) the input resistance R_i. (5%)
   (c) the voltage gain v_o/v_s. (5%)

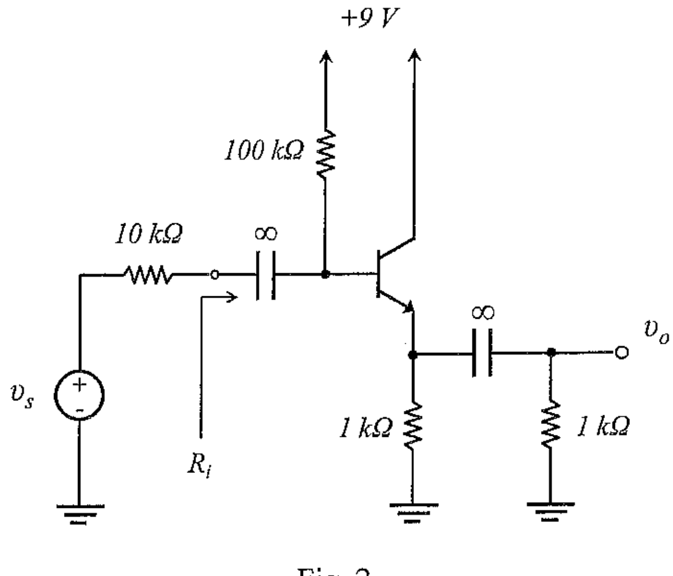

Fig. 2

試題請隨卷繳回，請留意背面是否有題

國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

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4. (35%) Consider the common-emitter amplifier of Fig. 3 under the following conditions: $R_s = 5$ kΩ, $R_1 = 33$ kΩ, $R_2 = 22$ kΩ, $R_E = 3.9$ kΩ, $R_C = 4.7$ kΩ, $R_L = 5.6$ kΩ, $V_{CC} = 5$ V. The dc emitter current can be shown to be $I_E \approx 0.33$ mA, at which $\beta_0 = 120$, $r_o = 300$ kΩ, and $r_x = 50$ Ω.

(a) Find the input resistance, R_{in}. *(Hint: $R_{in} = R_1 // R_2 // (r_x + r_\pi)$)* (10%)

(b) Find the midband gain, A_M. (10%)

(c) For $C_{C1} = C_{C2} = 5$ μF and $C_E = 20$ μF, estimate the low-frequency 3-dB frequency. Also find the frequency of the zero introduced by C_E. (15%)

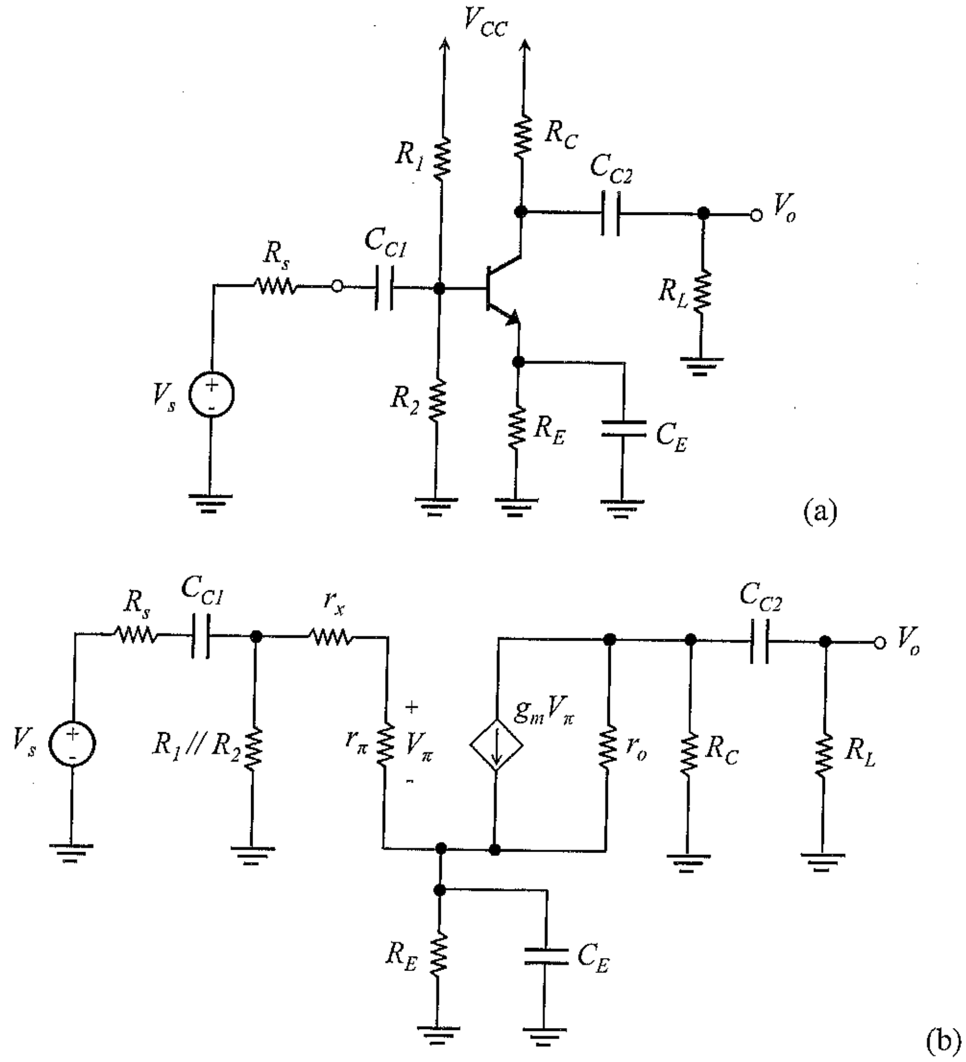

Fig. 3. (a) Common-emitter amplifier stage; (b) Equivalent circuit for the amplifier of Fig. 3(a) in the low-frequency band.

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：通訊理論【電機系碩士班戊組選考、通訊所碩士班甲組、乙組選考、電波聯合碩士班選考】

—作答注意事項—

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國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

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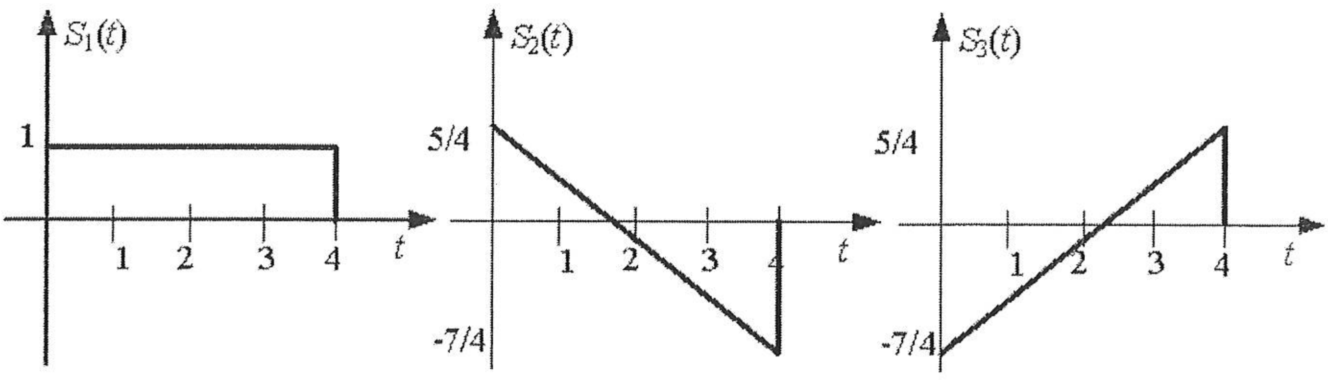
※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　共3頁第1頁

1. (20%) We are given the complex baseband signal $x_z(t) = \text{sinc}(t-1) + j2\text{sinc}(t)$.
   (A) (5%) Calculate the real and imaginary parts of the Fourier transform $X_z(f)$. To express the transforms, please use the function $\text{rect}(t)$, which is defined as a rectangle of unit height and spanning the interval [-1/2,1/2].
   (B) (5%) Plot the real and imaginary parts of $X_z(f)$.
   (C) (10%) Plot the real and imaginary parts of the Fourier transform of the bandpass signal obtained by upconverting $x_z(t)$ to the carrier frequency of 10 Hz.

2. (20%) Let $x(t) = m(t) + \cos(\omega_c t)$. Let W be the bandwidth of $m(t)$. Assume that the average value of $m(t)$ is zero and that the maximum value of $|m(t)|$ is M. Also assume that the square-law device is defined by $y(t) = 4x(t) + 2x^2(t)$.
   (A) (5%) Write the equation for $y(t)$.
   (B) (10%) Describe the filter with input signal $y(t)$ that produces an AM signal for $g(t)$, where $g(t)$ represents the output of the filter.
   (C) (5%) Specify the requirement of M to ensure no distortion when using envelope demodulation.

3. (10%) A transmitter uses a carrier frequency of 1000 Hz, with the unmodulated carrier represented as $A_c \cos(2\pi f_c t)$. Determine both the phase and frequency deviation for each of the following transmitter outputs:
   (A) (5%) $x_c(t) = \cos[2\pi(1000)t + 40\sin(5t^2)]$
   (B) (5%) $x_c(t) = \cos[2\pi(600)t]$

4. (10%) Consider the following three signals:



   (A) (6%) Use Gram-Schmidt procedure to find the set of basis functions from the three signals and determine the dimensionality of the set.

(B) (2%) Find the signal-space representation of the three signals based on the basis functions obtained in (a). (Represent the signals in terms of vectors)

(C) (2%) Determine the minimum distance between any pair of waveforms.

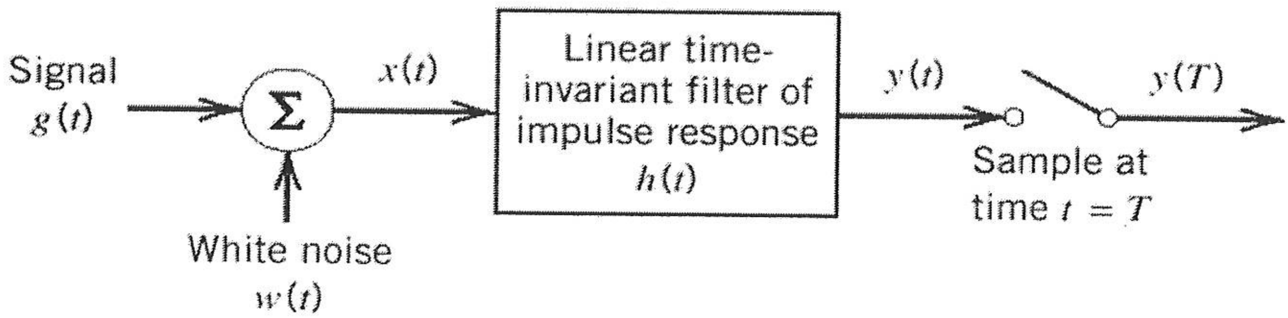
5. (20%) Let $x(t)$ denote a real valued WSS random process with an autocorrelation function $R_x(\tau)$ and $y(t) = x(t)\cos(2\pi f_c t + \theta)$, $\theta \sim U(0,2\pi)$.

(A) (5%) Find $R_y(\tau)$.

(B) (5%) $y(t)$ is passed through a low-pass filter with a frequency-domain response H(f) $=\begin{cases} 1, & |f| < f_c \\ 0, & |f| > f_c \end{cases}$ and $\tilde{y}(t)$ denotes the filter output. Find $E[y(t)^2]$.

(C) (10%) If we let $\theta = \frac{\pi}{4}$ and $r(t) = s(t)\cos(2\pi f_c t) + y(t)$, please show how to demodulate $s(t)$ based on $r(t)$ in detail.

6. (10%) Let $x(t) = g(t) + w(t),\ 0 \le t \le T$, be the received noisy signal, where $g(t)$ denotes the transmitted pulse that represents a binary symbol 0 or 1 and $w(t)$ denotes an additive white noise process with zero mean and power spectral density (PSD) $\frac{N_0}{2}$. Since the filter is linear, the result output can be express as $y(t) = g_o(t) + n(t)$, where $g_o(t)$ denotes the response to $g(t)$ and $n(t)$ denotes the response to $w(t)$. We know that the peak pulse signal to noise ratio of the match filter is $\eta = \frac{|g_0(T)|^2}{E[n^2(t)]}$. Please show that $\eta \le \frac{2}{N_0}\int_{-\infty}^{\infty}|G(f)|^2\,df$.



7. (10%) Consider two discrete random variables X and Y with the joint distribution:

$P(x,y)$	$X=-1$	$X=0$	$X=1$
$Y=2$	0.1	0.15	0.15
$Y=4$	0.05	0.2	0.15
$Y=6$	0.05	0.05	0.1

(A) (2%) Find the entropy $H(X,Y)$.

(B) (2%) Find the entropy $H(X)$.

(C) (4%) Find the entropy $H(X|Y)$.

(D) (2%) Find the mutual information $I(X;Y)$.

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學甲【電機系碩士班甲組、戊組選考、己組、庚組、通訊所碩士班乙組選考、電波聯合碩士班選考】

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※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）　　共4頁第1頁

下面 1-8 題為單選題，總分 40 分。每題答對 5 分，答錯或未作答者以 0 分計。總分低於 0 分者以 0 分計算。

1. Use the Fourier series analysis equation to calculate the coefficients a_k (when $k \neq 0$) for the continuous-time periodic signal

   $x(t) = \begin{cases} 1.5, & 0 \leq t < 1 \\ -1.5, & 1 \leq t < 2 \end{cases}$ with fundamental frequency $\omega_0 = \pi$.

   (A) $\frac{3}{2k\pi} e^{-jk\pi/2} sin(\frac{k\pi}{2})$　(B) $\frac{3}{2k\pi} e^{-jk\pi} sin(k\pi)$

   (C) $\frac{3}{k\pi} e^{-jk\pi} sin(k\pi)$　(D) $\frac{3}{k\pi} e^{-jk\pi/2} sin(\frac{k\pi}{2})$

2. Let $x_1(t)$ be a continuous-time periodic signal with fundamental frequency ω_1 and Fourier coefficients a_k. Given that $x_2(t) = x_1(1-t) + x_1(t-1)$. Find a relationship between the Fourier series coefficients b_k of $x_2(t)$ and the coefficients a_k.

   (A) $b_k = e^{-jk\omega_1}(a_k + a_{-k})$　(B) $b_k = e^{-jk\omega_1}(a_k - a_{-k})$

   (C) $b_k = jk\omega_1(a_k + a_{-k})$　(D) $b_k = jk\omega_1(a_k - a_{-k})$

3. What is the Fourier transform of $e^{-2|t-1|}$?

   (A) $2e^{-j\omega}/(4+\omega^2)$ (B) $4e^{-j\omega}/(4+\omega^2)$ (C) $4e^{-j\omega}/(2+j\omega)$ (D) $2e^{-j\omega}/(2+j\omega)$

4. Use the Fourier transform synthesis equation to determine the inverse Fourier transforms of $X(j\omega) = 2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)$.

   (A) $1 + \pi cos(4\pi t)$　(B) $1 + sin(4\pi t)$　(C) $1 + cos(4\pi t)$　(D) $1 + \pi sin(4\pi t)$

5. Use the Fourier transform synthesis equation to determine the inverse Fourier transforms of

   $$X(j\omega) = \begin{cases} 2, & 0 \leq \omega \leq 2 \\ -2, & -2 \leq \omega < 0 \\ 0, & |\omega| > 2 \end{cases}$$

   (A) $(4jcos^2 t)/\pi t$　(B) $(4sin^2 t)/\pi t$　(C) $(4jsin^2 t)/\pi t$　(D) $(4cos^2 t)/\pi t$

6. What is the Laplace transform of $e^{-5t}u(t-1)$?

   (A) $e^{(5+s)}/(s+5)$　(B) $e^{-(5+s)}/(s+5)$ (C) $e^{-(5+s)}/(s-5)$　(D) $e^{(5+s)}/(s-5)$

7. Determine the inverse Laplace transform of

$$X(s) = \frac{2(s+2)}{s^2+7s+12}, \ \mathcal{Re}\{s\} > -3.$$

(A) $4e^{-4t}u(-t) - 2e^{-3t}u(-t)$　(B) $4e^{4t}u(t) - 2e^{3t}u(t)$

(C) $4e^{4t}u(-t) - 2e^{3t}u(-t)$　(D) $4e^{-4t}u(t) - 2e^{-3t}u(t)$

8. One corner of a rectangular parallelepiped is at $(1,1,1)$, and three incident sides extend from this point to $(-2,1,6)$, $(3,5,7)$ and $(0,1,6)$. Please identify the volume of this solid.
(A) 20　(B) 40　(C) 18　(D) 42

下面 9-20 題為複選題，每題 5 分，總分 60 分，每題有五個選項，其中至少有一個是正確答案，答錯 1 個選項者，得 3 分，答錯 2 個選項者，得 1 分，答錯多於 2 個選項或未作答者，該題以 0 分計算。

9. Consider the wave equation $a^2u_{xx} = u_{tt}$ for a string tied to the x-axis at $x = 0$ and at $x = \pi$. When the string starts to vibrate, the motion takes place in the xu-plane. Let $u(x,t)$ denote the vertical displacement from the x-axis for $t > 0$ and a^2 be a real constant. The initial displacement is $f(x)$, $0 < x < \pi$ and the string is released from rest. Given the product solution $u(x,t) = X(x)T(t)$, which of the following is/are correct?
(A) The wave equation of this string is elliptic.
(B) All the boundary conditions belong to Dirichlet conditions.
(C) The initial condition yields $T'(0) = 0$.
(D) This is referred to as a boundary-value problem.
(E) One of the boundary conditions yields $X(0) = \pi$.

10. Assume the product solution to the wave equation in Question 9 is found as $u(x,t) = A \cdot \cos(B \cdot at) \cdot \sin(Cx)$. If the initial displacement $f(x)$ is a sinusoidal wave with the amplitude of $1/100$ and the frequency of $3/2\pi$, which of the following is/are correct?
(A) There exists a trivial solution.
(B) $B =$ arbitratry integer.
(C) $C = 3$.
(D) There is no motion at $x = 2\pi/3$ and all other x points vibrate vertically over time.
(E) The vertical displacement is reversed $(-f(x))$ when $t = k\pi/a, k = 1,2,3...$

11. Consider the differential equation $P'(t) = aP(t) - bP(t)^2$ with the initial condition of $P(0) = p_0$, where a and b are positive real constants. Which of the following is INCORRECT?
(A) This is a linear differential equation.
(B) This is a Bernoulli's equation.
(C) None of the critical points are attractors.
(D) When $t \to \infty$, $P(t) = a/b$.
(E) The solution interval is $(-\infty, \infty)$.

12. Given $X' = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} X$, which of the following is correct?
(A) There are two distinct eigenvalues.
(B) The stationary point is a repeller.
(C) All solutions will converge onto the origin.
(D) One of the eigenvalues is $\lambda_2 = -1$.
(E) One of the solution vectors is $K_1 = \binom{1}{1} e^{-4t}$.

13. Define the matrix $\mathbf{A}$ as $\mathbf{pq}^T$, where $\mathbf{p}$ and $\mathbf{q}$ are nonzero vectors in $\mathbb{R}^{n\times 1}$. Which of the following statements are true?
(A) $\mathrm{rank}(\mathbf{A})$ may be 2.
(B) $\mathbf{q}$ is an eigenvector of $\mathbf{A}$.
(C) $\mathbf{q}^T\mathbf{p}$ is an eigenvalue of $\mathbf{A}$.
(D) The linear equation $\mathbf{Ax} = \mathbf{0}$ has infinitely many solutions.
(E) The linear equation $\mathbf{Ax} = \mathbf{b}$ with $\mathbf{b} = \mathbf{q}$ has infinitely many solutions.

14. Define the matrix $\mathbf{M}$ as $\mathbf{I} - 2\mathbf{uu}^T$, where $\mathbf{u}$ is a unit vector in $\mathbb{R}^{n\times 1}$, meaning $\mathbf{u}^T\mathbf{u} = 1$. Which of the following statements are true?
(A) $\mathbf{M} = \mathbf{M}^T$.
(B) $\mathbf{M}^T\mathbf{M} = \mathbf{I}$.
(C) $\mathbf{M} = \mathbf{M}^{-1}$.
(D) $\det(\mathbf{M}) = 1$.
(E) $\mathbf{M}$ has eigenvalues that are complex with non-zero real components.

15. Let $N(\mathbf{A})$ denote the null space of $\mathbf{A}$, and $R(\mathbf{A})$ denote the range space of $\mathbf{A}$. Suppose $\mathbf{u} \in R(\mathbf{A})$, $\mathbf{v} \in N(\mathbf{A}^T)$, and both $\mathbf{u}$ and $\mathbf{v}$ are non-zero. Define $\|\mathbf{x}\| = \sqrt{\langle \mathbf{x}, \mathbf{x}\rangle}$. Suppose

$$\mathbf{x}_1 = \mathbf{u} + 2\mathbf{v},\ \mathbf{x}_2 = 2\mathbf{u} - \mathbf{v},\ \|\mathbf{x}_1\|^2 = 29,\ \|\mathbf{x}_2\|^2 = 41$$

Which of the following statements are true?
(A) $\|\mathbf{u}\| = \sqrt{5}$.
(B) $\|\mathbf{u}\| = 3$.
(C) $\|\mathbf{v}\| = 7$.
(D) $\|\mathbf{v}\| = \sqrt{5}$.
(E) $\|\mathbf{u}\| + \|\mathbf{v}\| = 8$.

16. Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be an orthogonal basis for an inner product space. Suppose $\|\mathbf{u}_1\| = 2$, $\|\mathbf{u}_2\| = 3$, $\|\mathbf{u}_3\| = 5$. If $\mathbf{x} = c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3\mathbf{u}_3$ is a vector with the properties $\langle \mathbf{x},\ \mathbf{u}_1\rangle = 12$, $\|\mathbf{x}\|^2 = 145$, and the orthogonal projection of $\mathbf{x}$ onto $\mathbf{u}_3$ is $-2\mathbf{u}_3$, then which of the following statements are true?
(A) $c_1 = 12$, (B) $c_2 = 2$, (C) $c_3 = -2$, (D) $\langle \mathbf{x},\ \mathbf{u}_2\rangle = 18$, (E) $\langle \mathbf{x},\ \mathbf{u}_3\rangle = 50$

17. Let $L: V \to W$ be a linear transformation, and $E = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ and $F = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ be ordered bases for V and W, respectively. Suppose $\mathbf{A}$ is the matrix representing L relative to E and F, and

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 7 \\ 3 & 4 & 8 \\ 5 & 6 & 9 \end{bmatrix}.$$

If $L(\mathbf{v}_1 + 2\mathbf{v}_2) = c_1\mathbf{w}_1 + c_2\mathbf{w}_2 + c_3\mathbf{w}_3$, then which of the following statements are true?

(A) $c_1 = 7$, (B) $c_2 = 11$, (C) $c_3 = 17$, (D) $c_1 + c_2 + c_3 = 34$, (E) $c_1 + c_2 + c_3 = 40$

18. Let

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \\ -1 & 3 & 2 \\ 2 & 0 & 3 \end{bmatrix}.$$

Which of the following vectors are in the column space of $\mathbf{A}\mathbf{A}^T$?

(A) $[0, 2, 3, 7]^T$
(B) $[3, 0, 2, 4]^T$
(C) $[3, 1, 0, 4]^T$
(D) $[5, 4, -1, 2]^T$
(E) $[-1, 2, 4, 5]^T$

19. Define $\mathrm{i} = \sqrt{-1}$. Suppose $\int_C f(z)dz = a + bi$, where $f(z) = (z + 2)/z$ and C is the semicircle $z = 2e^{i\theta} (0 \le \theta \le \pi)$. Which of the following statements are true?

(A) $a = -4$, (B) $a = 2$, (C) $b = -2\pi$, (D) $b = 0$, (E) $b = 2\pi$

20. Define $\mathrm{i} = \sqrt{-1}$. Suppose $\int_C f(z)dz = a + bi$, where $f(z) = 1/(z^2 + 4)$ and C is the positively oriented circle $|z - i| = 2$. Which of the following statements are true?

(A) $a = \pi/8$, (B) $a = \pi/4$, (C) $a = \pi/2$, (D) $b = \pi/8$, (E) $b = \pi/4$