國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學甲【電機系碩士班甲組、己組、庚組、電波聯合選考：電機系碩士班戊組、通訊所碩士班乙組】

—作答注意事項—

考試時間：100分鐘

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※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）

下面 1-15 題為單選題，總分 45 分。每題答對 3 分，答錯扣 4 分，未作答者以 0 分計。總分低於 0 分者以 0 分計算。

1. Consider the autonomous differential equation $y' = (2/\pi)y - \sin y$. Which of the following is INCORRECT?
   (A) There are three critical points.
   (B) One of critical point is semi-stable.
   (C) Two of critical points are unstable.
   (D) One of the critical points is 0.

2. If $y = e^{3x}\cos x$ is the solution to $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + ky = 0$, what is the value of k?
   (A) 3 (B) -2 (C) 10 (D) 8

3. The differential equation $e^x\frac{dy}{dx} + 3y = x^2y$ is linear and separable.
   (A) True (B) False

4. The improved Euler's method is what type of Runge-Kutta method?
   (A) First order (B) Second order (C) Third order (D) Fourth order

5. Consider $y(x)$ is the solution to the initial-value problem $x^2y'' - 2xy' + 2y = 0$ where $x > 0$, $y(1) = 4$, and $y'(1) = 9$, use Euler's method to compute $y(1.2)$. Given $h = 0.1$, which of the following is correct?
   (A) The general solution is $y = C_1x - C_2x^2$, where $C_1 + C_2 = 6$.
   (B) The general solution is $y = C_1x + C_2x^2$, where $C_1 + C_2 = 6$.
   (C) $y(1.2) = 5.9$.
   (D) $y(1.2) = 6$.

6. Given the three vectors (1, 0, 3, 1), (0, 1, -6, -1) and (0, 2, 1, 0) in R^4, they are linearly dependent.
   (A) True (B) False

7. Provided the system below, the rank is
$$X_1 - X_3 + 2X_4 + X_5 + 6X_6 = -3$$
$$X_2 + 2X_3 + 3X_4 + 2X_5 + 4X_6 = 1$$
$$X_1 - 4X_2 + 3X_3 + X_4 + 2X_6 = 0$$
   (A) 1 (B) 2 (C) 3 (D) 4

8. Which one of the following is correct regarding Fourier series?
   (A) $e^{-|x|}$ is odd function.
   (B) f' must be continuous on the interval $[a, b]$ to ensure that the Fourier series of f on $[a, b]$ converges to f.
   (C) $f(x) = |x|$ is continuous on $[-\pi, \pi]$.
   (D) The Fourier series of $f(x) = x^2 + 1$, where $0 < x < 3$, converges to 0 at $x = 0$.

9. Expand $f(x) = 2x^2 - 1, -1 < x < 1$ in a Fourier series and yield $f(x) = A + \sum \frac{B}{n^2\pi^2} C$. Which of the following is correct?
(A) $A = -2/3$　(B) $B = 4$　(C) $C = (-1)^n \cos n\pi x$　(D) None of the above

10. If $y_1(x) = x$ is one of the solutions of the following differential equation, what is the other linear independent solution $y_2(x)$?
$$y'' - \frac{2x}{1+x^2}y' + \frac{2}{1+x^2}y = 0$$
(A) $y_2(x) = 2x^2 + 1$　(B) $y_2(x) = \frac{x^2-1}{x}$　(C) $y_2(x) = \frac{1}{x} - 1$　(D) $y_2(x) = x^2 - 1$

11. Use the Laplace transform to solve the following initial-value problem. If the solution is $y = A + Be^{-t} + Ce^{3t} + De^{4t}$, which of the following is true?
$$y'' - 4y' = 6e^{3t} - 3e^{-t}, y(0) = 1,\ y'(0) = -1$$
(A) $A + B + C + D = 1$.
(B) $B = -2$
(C) $A + B + D = 2$
(D) All of the above

12. The Laplace transform of a function f is denoted by $\mathcal{L}\{f\}$. If $\mathcal{L}\{f(t)\} = F(s)$ and $\mathcal{L}\{g(t)\} = G(s)$, then $\mathcal{L}^{-1}\{F(s)G(s)\} = f(t)g(t)$.
(A) True　(B) False

13. If $\mathcal{L}\{f(t)\}$ represents the Laplace transform of a function $f(t)$. Let $f(t) = \begin{cases} 3 & , if\ 0 \le t \le 2 \\ 5 - t & , if\ t > 2 \end{cases}$, then $\mathcal{L}\{f(t)\}$ is
(A) $\frac{3}{s^2} + \frac{e^{2s}}{s^2}$　(B) $\frac{3}{s} + \frac{e^{-2s}}{s^2}$　(C) $\frac{3}{s} - \frac{e^{-2s}}{s^2}$　(D) $\frac{3}{s^2} - \frac{e^{-2s}}{s^2}$

14. Provided the differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which of the following is true?
(A) first order, linear, non-homogeneous
(B) second order, nonlinear
(C) second order, linear, non-homogeneous
(D) second order, linear, homogeneous

15. The Fourier transform of a function f is denoted by $\Im\{f\}$. Suppose $\Im\{f(t)\} = F(\omega), \Im\{g(t)\} = g(\omega)$, which of the following is INCORRECT?
(A) $\int_{-\infty}^{\infty} f(\tau)g(t-\tau)\,d\tau = \Im^{-1}\{F(\omega)G(\omega)\}$
(B) $\int_{-\infty}^{\infty} f(t-\tau)g(\tau)\,d\tau = \Im^{-1}\{F(\omega)G(\omega)\}$
(C) $\Im\{f(t-\tau)\} = F(\omega)e^{-i\omega\tau}$
(D) None of the above

下面 16-21 題為複選題，每題 5 分，總分 30 分，每題有五個選項，其中至少有一個是正確答案，答錯 1 個選項者，得 3 分，答錯 2 個選項者，得 1 分，答錯多於 2 個選項或未作答者，該題以零分計算。

16. Let $\mathbf{A}$ and $\mathbf{B}$ be matrices in $\mathbb{R}^{n\times n}$. Which of the following statements are true?
(A) $\det(-\mathbf{A}) = -\det(\mathbf{A})$.
(B) If $\mathbf{A}\mathbf{A}^T = \mathbf{I}$, then $\det(\mathbf{A}) = 1$.
(C) If $\mathbf{A}\mathbf{A}^T = \mathbf{I}$, then $\text{trace}(\mathbf{A}) = n$.
(D) If two rows of $\mathbf{A}$ are equal, then $\det(\mathbf{A}) = 0$.
(E) If $\det(\mathbf{A}) = \det(\mathbf{B})$, then $\mathbf{A}$ and $\mathbf{B}$ have the same rank.

17. Let $\mathbf{A} \in \mathbb{R}^{3\times 3}$ and its eigenvalues are λ_1, λ_1, and λ_2, where λ_1 and λ_2 are distinct eigenvalues. Suppose the dimension of $N(\mathbf{A} - \lambda_1\mathbf{I})$ is 1, where $N(\mathbf{A})$ denotes the null space of $\mathbf{A}$. Which of the following statements are true?
(A) λ_1 must be a real number (not a complex number).
(B) λ_2 must be a real number (not a complex number).
(C) The dimension of $N(\mathbf{A} - \lambda_2\mathbf{I})$ equals 1.
(D) $\mathbf{A}$ is diagonalizable.
(E) $\mathbf{A}$ has two linearly independent eigenvectors corresponding to λ_1.

18. Let $\mathbf{A} \in \mathbb{R}^{m\times n}$. Consider the linear equation $\mathbf{A}\mathbf{x} = \mathbf{b}$ or the homogeneous linear equation $\mathbf{A}\mathbf{x} = \mathbf{0}$. Which of the following statements are true?
(A) If $\text{rank}(\mathbf{A}) = m$, then $\mathbf{A}\mathbf{x} = \mathbf{b}$ has at least one solution for any $\mathbf{b} \in \mathbb{R}^m$.
(B) If $\text{rank}(\mathbf{A}) = m$, then $\mathbf{A}\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$.
(C) If $\text{rank}(\mathbf{A}) = n$, then $\mathbf{A}\mathbf{x} = \mathbf{b}$ has at most one solution for any $\mathbf{b} \in \mathbb{R}^m$.
(D) If $\text{rank}(\mathbf{A}) = n$ and $m > n$, then $\mathbf{A}\mathbf{x} = \mathbf{0}$ has infinitely many solutions.
(E) If $\text{rank}(\mathbf{A}) = m$ and $n > m$, then $\mathbf{A}\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

19. Let $\mathbf{A}$ and $\mathbf{B}$ be square matrices. Suppose that $\mathbf{A}$ is similar to $\mathbf{B}$, that is, $\mathbf{B} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ for some nonsingular matrix $\mathbf{P}$. Which of the following statements are true?
(A) If $\mathbf{x}$ is an eigenvector of $\mathbf{B}$, then $\mathbf{x}$ is also an eigenvector of $\mathbf{A}$.
(B) If $\mathbf{y}$ is in the column space of $\mathbf{B}$, then $\mathbf{y}$ is also in the column space of $\mathbf{A}$.
(C) $\text{trace}(\mathbf{A}) = \text{trace}(\mathbf{B})$.
(D) $\mathbf{A} - \mathbf{I}$ is similar to $\mathbf{B} - \mathbf{I}$.
(E) $\mathbf{A}^5$ is similar to $\mathbf{B}^5$.

20. Let $\mathbf{A} \in \mathbb{R}^{m\times n}$, $R(\mathbf{A})$ denotes the column space of $\mathbf{A}$, $N(\mathbf{A})$ denotes the null space of $\mathbf{A}$, and $\dim(S)$ denotes the dimension of a subspace S. Which of the following statements are true?
(A) If $\mathbf{y} \in R(\mathbf{A})$, then $\mathbf{y} \in R(\mathbf{A}\mathbf{A}^T)$.
(B) If $\mathbf{x} \in N(\mathbf{A})$, then $\mathbf{x} \in N(\mathbf{A}\mathbf{A}^T)$.
(C) $\text{rank}(\mathbf{A}) + \dim\big(N(\mathbf{A})\big) = \text{rank}(\mathbf{A}^T) + \dim(N(\mathbf{A}^T))$.
(D) It is possible for a matrix $\mathbf{A}$ to have $[2, 1, -1]^T$ in $N(\mathbf{A})$ and $[1, -2, 3]^T$ in $R(\mathbf{A}^T)$.
(E) Let $\mathbf{y} \in \mathbb{R}^m$. If $\mathbf{y} = \mathbf{u}_1 + \mathbf{v}_1 = \mathbf{u}_2 + \mathbf{v}_2$, where $\mathbf{u}_1, \mathbf{u}_2 \in R(\mathbf{A})$ and $\mathbf{v}_1, \mathbf{v}_2 \in N(\mathbf{A}^T)$, then $\mathbf{u}_1 = \mathbf{u}_2$ and $\mathbf{v}_1 = \mathbf{v}_2$.

21. Let

$$\mathbf{A}=\begin{bmatrix}4 & 1 & 3 & 2\\ 1 & 4 & 3 & 3\\ -1 & 11 & 6 & 7\end{bmatrix}.$$

Which of the following vectors are in the column space of **A**?

(A) $[3,1,2]^T$
(B) $[1,0,-1]^T$
(C) $[0,1,3]^T$
(D) $[2,1,1]^T$
(E) $[4,2,-1]^T$

以下第22題到第23題需要詳明推導計算過程。如推導計算過程錯誤，將酌扣分數或不給分。

22. (10分) 求出以下複平面上之路徑積分值，z為複數。

$\int_C \frac{z^5}{1-z^3}dz$，其中$C$為沿著$\{z:|z|=2\}$正向旋轉一周之封閉路徑。

23. (15分) 利用餘值（residues）求取以下瑕積分，其中參數$a>0$。

$$\int_0^\infty \frac{\cos ax}{x^2+1}dx$$

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1. (30%) For the common-base circuit in Fig. 1, assuming the bias current to be about 1 mA, β = 100, C_μ = 0.8 pF, r_e = 25 Ω, and f_T = 800 MHz:
   (a) Estimate the midband gain V_o/V_s. (10%)
   (b) Use the short-circuit time-constants method to estimate the lower 3-dB frequency, f_L. (10%) (*Hint:* In determining the resistance seen by C_1, the effect of the 47-kΩ resistor must be taken into account.)
   (c) Find the high-frequency poles, and estimate the upper 3-dB frequency, f_H. (10%)

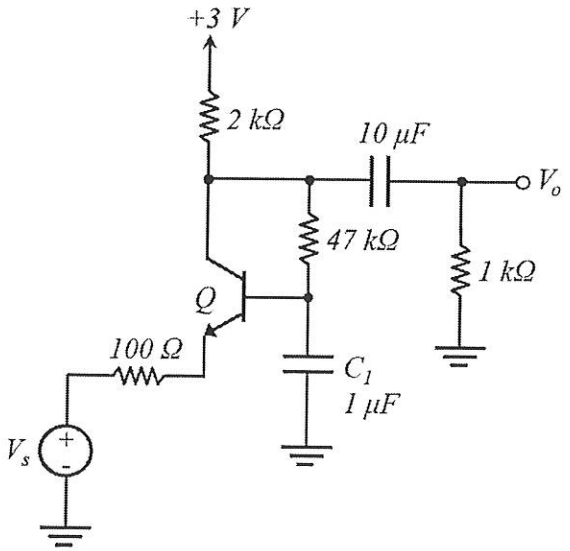

Fig. 1

2. (25%) The current-steering circuit of Fig. 2 is fabricated in a CMOS technology for which k_n' = 90 μA/V², k_P' = 30 μA/V², V_{tn} = 0.8 V, and V_{tp} = -0.9 V. If all devices have L = 2 μm, design the circuit so that I_{REF} = 20 μA, I_2 = 100 μA, and I_5 = 40 μA. Use the minimum width of 2 μm for as many of the devices as possible. (a) Give the required width for each transistor and the value of R required. (10%) (b) What is the highest voltage possible at the drain of Q_2? (5%) (c) What is the lowest voltage possible at the drain of Q_5? If $|V_{Ap}|$ = 16 L, where L is in μm and V_{Ap} is in volts, (5%) (d) find the output resistance of the current source Q_2. (5%)

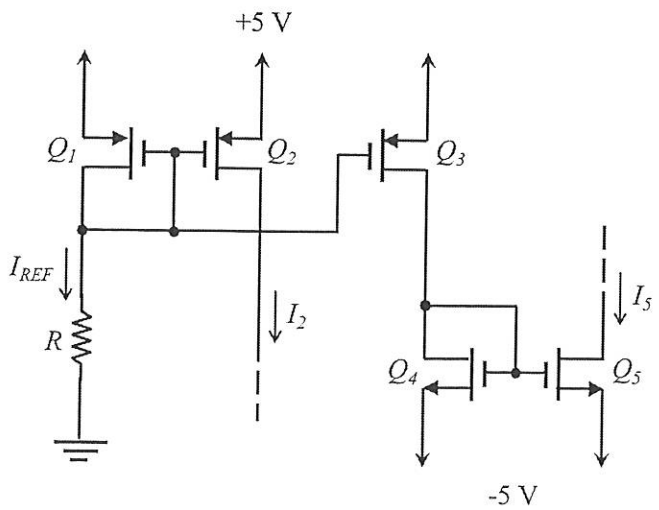

Fig. 2

試題請隨卷繳回，請留意背面是否有題

3. (15%) A second-order filter has its poles at $s = -(1/8) \pm j(\sqrt{63}/8)$. The transmission is zero at $\omega = 5$ rad/s and is unity at dc ($\omega = 0$). Find the transfer function.

4. (30%) For the emitter-follower circuit shown in Fig. 3 the BJT used is specified to have β values in the range of 20 to 200. For the two extreme values of $\beta = 20$ and $\beta = 200$, find :
   (a) I_E, V_E, and V_B. (10%)
   (b) the input resistance R_i. (10%)
   (c) the voltage gain v_o/v_s. (10%)

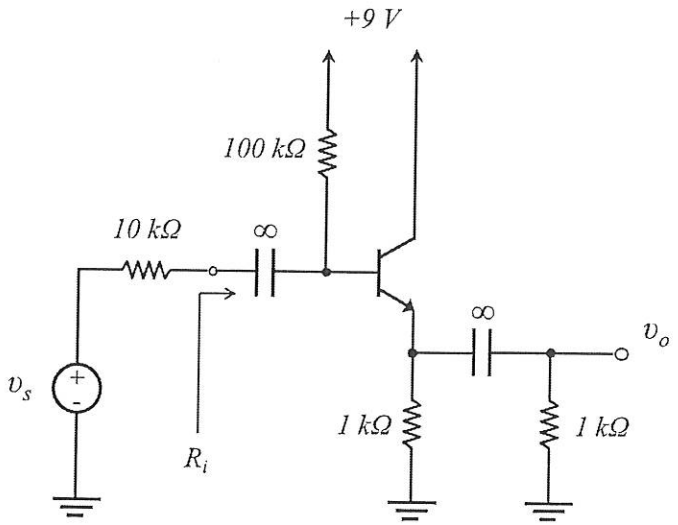

Fig. 3

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1. (3%) (a) At any point (x_0, y_0, z_0) in the domain of a scalar function $V(x, y, z)$, we take a path a_ℓ along $V = c_i$, where c_i is a constant, or take a path a_n along ∇V. Tell me about the main characteristic (主要特徵) of these two paths a_ℓ and a_n, and also what is $a_\ell \cdot a_n$, the dot product of a_ℓ and a_n?
   (3%) (b) $() = \nabla V \cdot (a_\ell)\, d\ell$, where V is a scalar function, $d\ell$為任意方向 a_ℓ 之小路徑。() 裏應填什麼？
   (3%) (c) 利用 Divergence theorem for ∇·E 寫下 E 和 Q (真空中有一 charge Q)的關係。
   (3%) (d) 利用 Stokes' theorem for ∇×B 寫下 B 和 I (真空中有一 current I) 的關係。
   (3%) (e) 在運算 Divergence ∇·A 或 Curl ∇ × A時 (A 為一向量場) ，我們選擇的體積或面積在大小和形狀各有何限制？

2. (5%) Using the *Method of Image*, write down the potential distribution, $V(x, y, z)$, for a point P(x, y, z) in the space, Fig. 1, the dielectric constant of the space is ε_0. Q is a positive point charge of Q 庫侖 *Coul.*

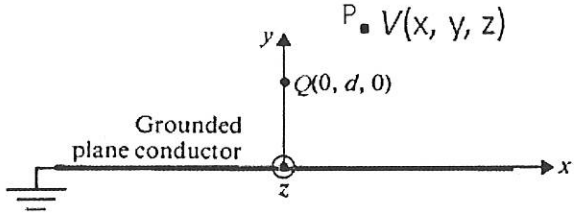

Fig. 1. A point charge Q distance d above the Ground.

3. (3%) (a) 在 dielectric constant 為 ε_r (=1+X_e) dielectric 之內部電場為 E (V/m)，請問 Polarization vector P 為何？
   (3%) (b) 在 relative permeability 為 μ_r (=1+X_m)的一 ferromagnetic material 外面線圈通電流，在其內部產生磁場 H (A/m)，請問 Magnetization vector M 為何？
   (4%) 銅的導電性很好，(c) 它的 permittivityε 和 permeability μ 各為何？請簡單提供你的理由。

4. For a coaxial transmission line shown in Fig. 2, the capacitance per unit length is $c = \frac{2\pi\varepsilon_0}{ln\frac{b}{a}} \left[\frac{\mathrm{F}}{\mathrm{m}}\right]$, and the inductance per unit length is $\ell = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi} \ln\frac{b}{a} \left[\frac{\mathrm{H}}{\mathrm{m}}\right]$. At high frequencies, the internal inductance drops off (that is, approaching 0, and you should know which term is the internal inductance).
   (2%) (a) Find the characteristic impedance of the coaxial line $Z_c=(l/c)^{0.5}$ at high frequencies 請務必寫 Z_c 之單位。
   (2%) (b) 請問在地 (Ground, 即半徑 b 粗體部分) 之外的 magnetic flux density B 值為何？

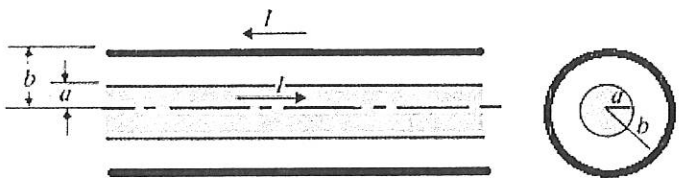

Fig. 2. Coaxial cable side and cross-sectional views, 粗體線代表地 Ground

The capacitance of a line charge of radius a over a ground 0, as shown in Left of Fig. 3 $C = \frac{2\pi\varepsilon_0}{ln\frac{2h}{a}} \left[\frac{\mathrm{F}}{\mathrm{m}}\right]$.
(3%) (c) Find the external inductance L for such a transmission system in air using a quasi-TEM property $L \cdot C = \mu_0 \cdot \varepsilon_0$.
(3%) (d) Obtain the internal inductance from the inductance formula l, also the external inductance found in c), write down the per unit length internal & external inductance for the conductor system shown in the Right of Fig. 3.

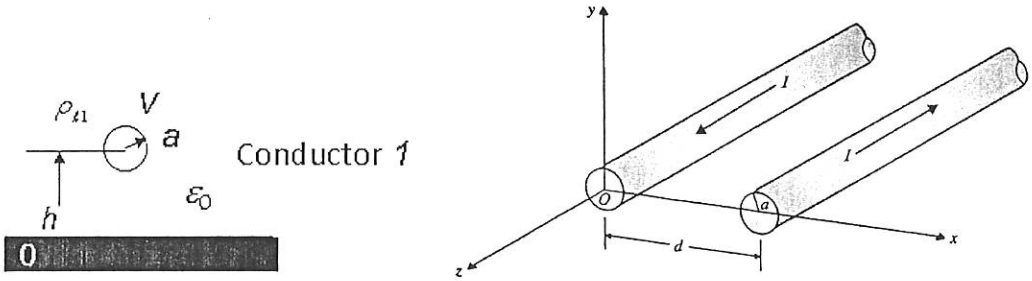

Fig. 3. A single conductor above a Ground (Left); A two-conductor system with currents flow in opposite direction (Right); the two conductors both with radius a and d distance apart.

5. (5%) (a) A position vector $\mathrm{R} = a_x(x-x') + a_y(y-y') + a_z(z-z')$, $R = [(x-x')^2+(y-y')^2+(z-z')^2]^{1/2}$, $a_R = \mathrm{R}/R$, where $P(x, y, z)$ is an observation point, and $P'(x', y', z')$ is a source point. Show that $\nabla'(\frac{1}{R}) = (a_R \frac{1}{R^2})$, $\nabla' f$ is the gradient operator with respect to the source coordinates, that is, $\nabla' f = a_x \frac{\partial f}{\partial x'} + a_y \frac{\partial f}{\partial y'} + a_z \frac{\partial f}{\partial z'}$.
(5%) (b) As shown in Fig. 4, determine B at $P(0, 0, z^*)$?

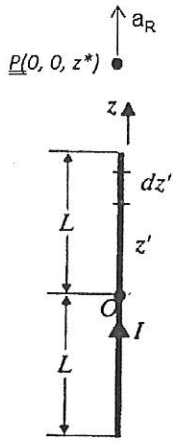

Fig. 4.

6. (10%) (a) Derive the electromagnetic wave equation in free space.
(5%) (b) Explain the traveling-wave factor.

7. A uniform plane wave ($\mathbf{E}_i$, $\mathbf{H}_i$) of an angular frequency ω is incident from air (medium 1) on a very large, perfectly conducting wall (medium 2) at an angle of incidence θ_i with perpendicular polarization. Find
(10%) (a) **E** and **H** in medium 1.
(5%) (b) **E** and **H** in t medium 2.
(5%) (c) the current induced on the wall surface, and
(5%) (d) the time-average Poynting vector in medium 1.

8. (10%) As shown in Fig. 5 with $Z_1 = \sqrt{Z_0 Z_L}$ and $l = \lambda/4$, please explain how the circuit works to achieve impedance matching between Z_0 at Z_L, and obtain the bandwidth with the maximum Γ of Γ_m.

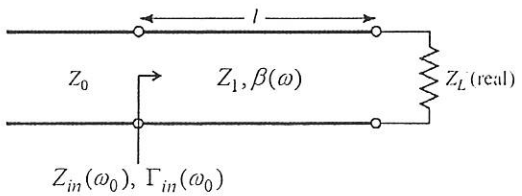

Fig. 5.

國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組、電波聯合選考：電機系碩士班戊組、通訊所碩士班乙組】

－作答注意事項－

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組、電波聯合選考：電機系碩士班戊組、通訊所碩士班乙組】題號：437002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）

1. (10%) Please explain the following concepts as detail as possible:
   (a) (2%) Describe the conditions for a random process to be wide-sense stationary (WSS).
   (b) (2%) What is an Ergodic process?
   (c) (2%) Describe the Wiener-Khinchin Theorem.
   (d) (2%) What is an Additive White Gaussian Noise?
   (e) (2%) What is a Gaussian Process?

2. (12%) Find the Fourier transform of $x(t) = sinc^3(t)$. (Hint: The answer is a piecewise function which consists of five intervals.)

3. (15%) The characteristic function of a random variable X is defined as the statistical average
$$E\left(e^{jvX}\right) \equiv \psi(jvX) = \int_{-\infty}^{\infty} e^{jvx} p(x)dx.$$
   (a) (10%) Find the characteristic function of a Gaussian random variable.
   (b) (5%) Show that the variable Y, which is defined as the sum of N independent and identically distributed (i.i.d.) Gaussian random variables X_i, $i = 1,2, \ldots, N$, is a Gaussian random variable.

4. (10%) An AM signal $s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$ is considered in the following systems:
   (a) (5%) If $s(t)$ is used as the input to a square-law detector which has a transfer characteristic defined as $v_O(t) = a_1 v_I(t) + a_2 v_I^2(t)$, where a_1 and a_2 are constants, $v_I(t)$ denotes the input, and $v_O(t)$ denotes the output. Find the conditions for which the message signal $m(t)$ can be recovered from $v_O(t)$.
   (b) (5%) Let $r(t)$ denote the recovered signal in (a). Suppose that we use an ideal sampling with a sampling interval of T_s to sample $r(t)$ and obtain the sampled signal $r_\delta(t)$, please find the Fourier transform of $r_\delta(t)$.

5. (15%) Please answer the following questions.
   (a) (5%) For a quaternary communication system, the possible transmitted signals are
$$s_k(t) = A\cos\left(\frac{20\pi}{T}t - \frac{(k-1)}{2}\pi\right), 0 \le t \le T, k = 1,\cdots,4.$$
   Assume $T = 40ms, A = 100mV, P\left(s_k(t)\right) = \frac{1}{4}, \forall k$, and the noise PSD $S_n(f) = 20\ \mu$W/Hz. Please calculate the error probability P_e.
$$\left(Hint:\ P_e = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) - Q^2\left(\sqrt{\frac{E_s}{N_0}}\right)\right)$$
   (b) (5%) If T changes to $1ms$, in order to maintain the same P_e obtained in (a), please calculate the required amplitude value A.
   (c) (5%) Please show the orthonormal basis functions for the signal constellation $s_k(t)$.

6. (20%) The definition of entropy is the expected value of the self information:

$$H(X) \triangleq E[I(x_k)] = \sum_{k=0}^{K-1} p_k \log_2\left(\frac{1}{p_k}\right) = -\sum_{x \in X} P(x)\log_2\big(P(x)\big).$$

Let variables X, Y have the joint probability

$$P(X,Y) = \begin{bmatrix} P(x_1,y_1) & P(x_1,y_2) \\ P(x_2,y_1) & P(x_2,y_2) \end{bmatrix} = \begin{bmatrix} 0.54 & 0.06 \\ 0.06 & 0.34 \end{bmatrix}.$$

Please find the following quantities:

(a) (4%) $P(Y|X)$ and $P(X|Y)$

(b) (4%) $H(X)$ and $H(Y)$

(c) (6%) Calculate $H(X|Y)$ and describe the physical meaning of $H(X|Y)$.

(d) (6%) Calculate $I(X;Y)$ and describe the physical meaning of $I(X;Y)$.

7. (18%) Consider the encoder for a binary (3,1,2) convolutional code shown in Fig. 1. There are one input message u, two registers and three outputs v_1, v_2 and v_3.

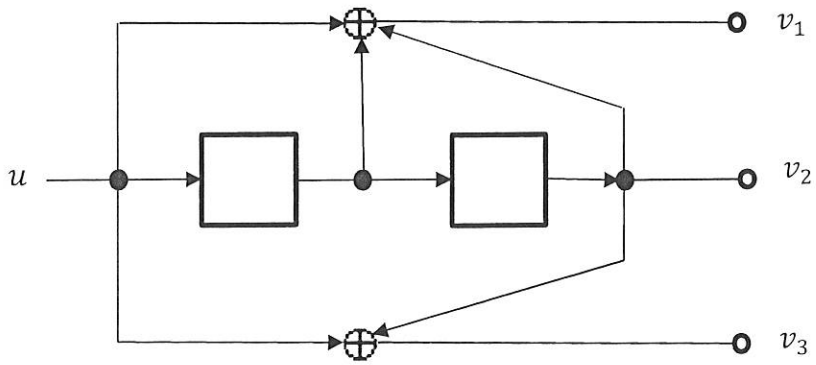

Figure 1

(a) (3%) Find the codeword $\mathbf{v}$ corresponding to the information sequence $\mathbf{u} = (1\ 1\ 1\ 0\ 1\ 0\ 0)$.

(b) (5%) Draw the state diagram of this encoder.

(c) (10%) Please use Viterbi algorithm to decode the received sequence (110 010 111 100 101 001), assuming that a binary symmetric channel with a crossover probability $p < 1/2$ is considered.