Quantum Mechanics

1. Consider a particle of mass μ in a potential $V(x)=\begin{cases}\frac{1}{2}\mu\omega^2x^2, & x>0\\ \infty, & x<0\end{cases}$

   (a) What are the boundary conditions on the wave functions? Find the eigenvalues and eigenfunctions. (10%)

   (b) Two noninteracting boson in this potential, one is in the ground state $|0\rangle$ and the other in the first excited state $|1\rangle$. Show the normalized wave function. (5%)

   (c) Two noninteracting fermions in this potential, one is in the ground state $|0\rangle$ and the other in the first excited state $|1\rangle$. Show the normalized wave function. (5%)

   Hint: The Hermite polynomials $H_m(y)$ satisfy the differential equation

$$\frac{d^2H_m(y)}{dy^2}-2y\frac{dH_m(y)}{dy}+2mH_m(y)=0$$

$$H_0(y)=1,\quad H_1(y)=2y,\quad H_2(y)=4y^2-2,\quad H_3(y)=8y^2-12y$$

2. Find the energy levels of a particle in a spherical box of radius a in the l=0 sector. (20%)

3. A particle is moving along the x-axis. Find the probability for transmission of the particle through a delta-function potential barrier at the origin. (20%)

4. Suppose we have a particle with spin up along the x-direction at time t=0.

   (a) Calculate the eigenvector measurement of S_x. (7%)

   (b) Then, a fixed magnetic field $\vec{B}=B_0\hat{z}$ is applied to the state. Find the expectation value of S_x for such localized electron later. (7%)

   (c) At a later time T, the magnetic field is very rapidly rotated to y-direction, *i.e.*, $\vec{B}=B_0\hat{y}$. After another time interval T, a measurement of S_x is carried out. What is the probability that the value $\hbar/2$ will be found? (6%)

5. Describe the differences between the Schrödinger's and Heisenberg's views of Quantum Mechanics. (10%)

6. Write down the Hamiltonian in Quantum Mechanics for a charged particle moving in electromagnetic field. (10%)

1. A spherical cavity of radius a is inside a grounded conductor. A point charge q inside the cavity is located a distance b from the center as shown in Fig. 1. Use the method of image to calculate the maximum and minimum surface charge density of the cavity..(20%)

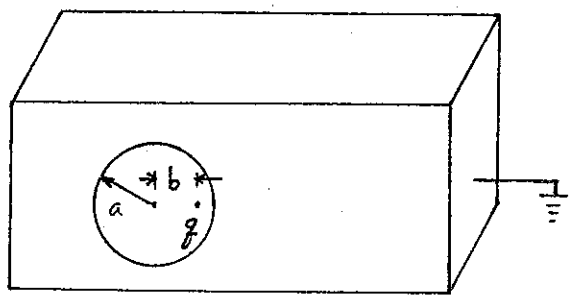

Fig. 1

2. Two concentric spherical conductor shells with inner and outer radius of R_1 and R_2 respectively. The inner shell is grounded and the outer shell keeps in a potential of $\Phi(R_2,\theta)=V_o\cos\theta$. Determine the electric potential of an arbitrary point between these two shells. (20%)

3. A sphere of radius R carries a charge density ρ(r) = kr (where k is a constant). Find the energy of the configuration.(20%)

4. A cylindrical material of radius b and length L is uniformly magnetized alone its axis (in the direction of z). Its magnetization is $\mathbf{M} = M\hat{\mathbf{k}}$. Determine the magnetic field at a point in the axis. (See Fig. 2) (20%)

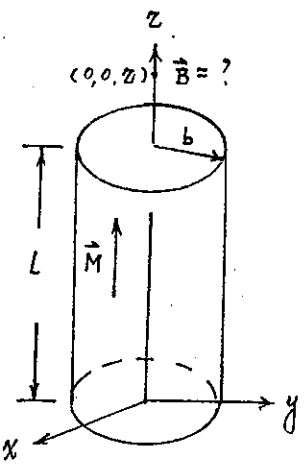

Fig. 2

5. Two long solenoids, with number of turns per unit length of N_a' and N_b' respectively. One of which extends a distance l within the other as in Fig. 3. Neglect end effects, calculate the force exerted on each other. (20%)

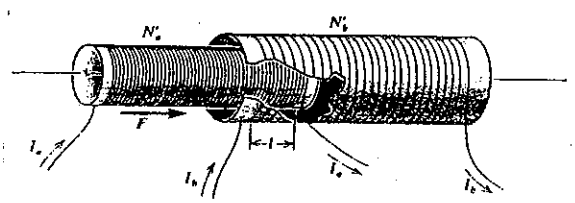

Fig. 3