Some might-be-useful informations:

1. for simple harmonic oscillator:

$$\psi_{\mathcal{E}}(x) \equiv \psi_{(n+1/2)\hbar\omega}(x) \equiv \psi_{n}(x)$$

$$= \left(\frac{m\omega}{\pi \hbar 2^{2n}(n!)^{2}}\right)^{1/4} \exp\left(-\frac{m\omega x^{2}}{2\hbar}\right) H_{n}\left[\left(\frac{m\omega}{\hbar}\right)^{1/2} x\right]$$

$$\int_{-\infty}^{\infty} H_{n}(y) H_{n'}(y) e^{-y^{2}} dy = \delta_{nn'}(\pi^{1/2} 2^{n} n!)$$

$$a = \left(\frac{m\omega}{2\hbar}\right)^{1/2} X + i \left(\frac{1}{2m\omega \hbar}\right)^{1/2} P$$

$$a^{\dagger} = \left(\frac{m\omega}{2\hbar}\right)^{1/2} X - i \left(\frac{1}{2m\omega \hbar}\right)^{1/2} P$$

2. for hydrogen atom:

$$\psi_{1,0,0} = \left(\frac{1}{\pi a_0^3}\right)^{1/2} e^{-r/a_0}$$

$$\psi_{2,0,0} = \left(\frac{1}{32\pi a_0^3}\right)^{1/2} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$$

$$\psi_{2,1,0} = \left(\frac{1}{32\pi a_0^3}\right)^{1/2} \frac{r}{a_0} e^{-r/2a_0} \cos \theta$$

$$\psi_{2,1,\pm 1} = \mp \left(\frac{1}{64\pi a_0^3}\right)^{1/2} \frac{r}{a_0} e^{-r/2a_0} \sin \theta e^{\pm i\phi}$$

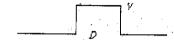
## 國立中山大學九十一學年度博士班招生考試試題

科目:量子力學【物理系】

共工頁第2頁

Each problem weights 20 points.

1. Find the transmission coefficient for a rectangular potential barrier (Fig. 1.) if the



energy exceeds the height V of the barrier.

Fiz. 1

- Using perturbation theory to find the eigenstate's correction up to first order and the eigen energy's correction up to second order for a one –dimensional simple harmonic oscillator under a small constant force f.
- 3. A particle of spin 1 and a particle of spin 2 are at rest in a configuration such that the total spin is 3, and its z-component is 1 (we take  $\pi = 1$ ). If you measured the z-component of the angular momentum of the spin-2 particle, what values might you got on what probabilities.
- 4. A hydrogen atom in its ground state is placed in a homogeneous electric field that has the time dependence:  $\vec{E} = 0$ , t < 0

$$\vec{E} = \vec{A} \exp(-t/b), \quad t > 0.$$

Find the first-order probability of the atom being at 2p states after a long time.

5. Prove a Hermitian operator has only real eigen values and its eigenstates are orthogonal in a non-degenerate case.

## 國立中山大學九十一學年度博士班招生考試試題

科目:電動力學【物理系】

共/頁第/頁

- 1. A planar charge density is given by  $\sigma(z)=\rho_0z\exp(-\eta|z|)$ , obtain the electrostatic potential,  $\Phi(z)$ , with the boundary condition  $\Phi(z)=0$  when  $z\to\infty$ .  $\rho_0$  and  $\eta$  are positive constants. 20%
- 2. If the electronic charge density of an atom with an atomic number of Z is given by  $\rho_e(r,\theta,\phi) = (\rho_0 + \rho_1 \sin\theta \cos\phi) \exp\left(-\frac{r}{\alpha_0}\right)$ , where  $\rho_0 = \frac{Ze}{8\pi\alpha_0^3}$  and  $\rho_1$  is a small constant. Obtain the electrostatic potential,  $\Phi(r,\theta,\phi)$ , in terms of spherical

harmonics. [Hint: 
$$\frac{1}{\left|\vec{r} - \vec{r}'\right|} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{1}{2l+1} \frac{r'_{<}}{r_{>}^{l+1}} Y_{lm}^{*}(\theta', \phi') Y_{lm}(\theta, \phi); \quad Y_{00}(\theta, \phi) = \frac{1}{\sqrt{4\pi}},$$

$$Y_{11}(\theta,\phi) = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\phi}, \quad Y_{10}(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\cos\theta, \quad \text{and} \quad Y_{1-1}(\theta,\phi) = \sqrt{\frac{3}{8\pi}}\sin\theta e^{-i\phi}$$

- 3. A point charge q is placed a distance d away from the surface of a semi-infinite dielectric material with a dielectric constant of  $\varepsilon$ . Using the image charge method to find the electrostatic potential at all points in space. 20%
- 4. Using the symmetry and the Ampère's law to find the magnetic induction, B(r), given rise by a current density  $J(\rho)=J_0kexp(-\eta\rho)$  in cylindrical coordinates  $(\rho,\phi,z)$ , where k is a unit vector in the z direction and  $J_0$  and  $\eta$  are positive constants. 20%
- 5. Derive the boundary conditions of the magnetic induction, **B**, and the magnetic field, **H**, at the interface of two mediums with magnetic permeability of  $\mu_1$  and  $\mu_2$ , respectively. 5%
- 6. Using Maxwell equations to derive the wave equations for the scalar potential,  $\Phi(\mathbf{r})$ , and the vector potential,  $\mathbf{A}(\mathbf{r})$ , in free space. 15%