Quantum Mechanics

1. Use the commutation relation between the momentum p and the position **x** to obtain the equations describing the time dependence of the expectation values of $\langle x \rangle$ and $\langle p \rangle$ given the Hamiltonian $\mathbf{H}=\mathbf{p}^2/(2m)+(1/2)k(\alpha \mathbf{x}^2+\beta \mathbf{x}+\gamma)$.

2. Obtain the standard angular momentum commutation relations, i.e. $[\mathbf{L}_x,\mathbf{L}_y]$, $[\mathbf{L}_y,\mathbf{L}_z]$ and $[\mathbf{L}_z,\mathbf{L}_x]$, and use them to show that $\mathbf{L}_z$ commutes with $\mathbf{L}^2=\mathbf{L}_x^2+\mathbf{L}_y^2+\mathbf{L}_z^2$.

3. If the potential of a particle is spherically symmetric and is given by $V(r)=-V_o$ for $r<R$ and $V(r)=0$ for $r>R$.. V_o is a positive value. Find the eigenenergies and eigenfunctions of the bound states. [You need spherical Bessel functions: $h_0(x)=e^{ix}/(ix)$; $h_1(x)=-e^{ix}(1+i/x)/x$; $j_0(x)=\sin x/x$; $j_1(x)=\sin x/x^2-\cos x/x$ and the recursion formulas $(2l+1)z_l(x)/x=z_{l-1}(x)+z_{l+1}(x)$ and $z_l'(x)=[l z_{l-1}(x)-(l+1)z_{l+1}(x)]/(2l+1)$]

4. If the motion of an electron in an atom can be described by an effective one-electron Schrödinger equation with a potential given by
$$V(r,\theta,\phi)=-Z_{eff}e^2/r+\alpha\sin\theta\cos\phi/r^2.$$
Obtain the eigenenergies and eigenfunctions of the electron in this atom. Treat the one-dimensional r integrals as parameters.

5. Based on the eigenenergies and eigenfunctions of a hydrogen-like atom to construct the first-order estimate of the eigenfunctions of the two electrons in the helium atom and estimate the corresponding eigenenergies. [Remember that the electron has a spin of $1/2\hbar$.]

國立中山大學八十九學年度碩博士班招生考試試題

科　目：物理所（電動力學）　　　　　　　　共 2 頁 第 1 頁

1. Two long, cylindrical conductors of radii a_1 and a_2 are parallel and separated by a distance d, which is large compared with either radius, Show that the capacitance per unit length is given approximately by

$$C \approx \pi\varepsilon_o \left(\ln\frac{d}{a}\right)^{-1}$$

where a is the geometrical mean of the two radii. (15%)

2. A point charge q inside a hollow, grounded, conducting sphere of inner radius a. The distance between the point charge and the center of the sphere is d ($d < a$), find the maximum and minimum value of the induced surface-charge density. (15%)

3. A total charge q uniformly distributed around a circular ring of radius a, located as shown in Fig. 1, with its axis the z axis and its center at z = b. Prove that

(a) the potential at point P is

$$\Phi(z=r) = \frac{q}{4\pi\varepsilon_o}\sum_{\ell=0}^{\infty}\frac{c^\ell}{r^{\ell+1}}P_\ell(\cos\alpha), \quad \text{for } r > c$$

$$\Phi(z=r) = \frac{q}{4\pi\varepsilon_o}\sum_{\ell=0}^{\infty}\frac{r^\ell}{c^{\ell+1}}P_\ell(\cos\alpha), \quad \text{for } r < c$$

(b) the potential at an arbitrary point P'is

$$\Phi(r,\theta) = \frac{q}{4\pi\varepsilon_o}\sum_{\ell=0}^{\infty}\frac{r_<^\ell}{r_>^{\ell+1}}P_\ell(\cos\alpha)P_\ell(\cos\theta)$$

where $r_<$ ($r_>$) is the smaller (larger) of r and c. (20%)

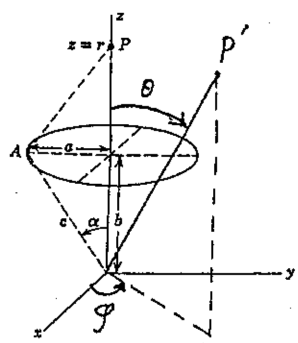

Fig. 1

4. Two concentric conducting spheres of inner and outer radii a and b, respectively, carry charges $\pm Q$. The empty space between the spheres is half-filled by a hemispherical shell of dielectric (of dielectric constant $\varepsilon/\varepsilon_o$), as shown in Fig. 2.
   (a) Find the electric field everywhere between the spheres.
   (b) Calculate the surface-charge distribution on the inner sphere.
   (c) Calculate the polarization-charge density induced on the surface of the dielectric at r = a. (15%)

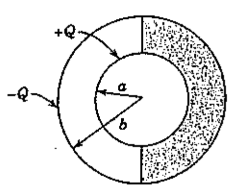

Fig. 2

5. A right-circular solenoid of finite length L and radius a has N turns per unit length and carries a current I. Show that the magnetic induction on the cylinder axis in the limit NL $\rightarrow \infty$ is

$$B_z = \frac{\mu_o NI}{2}(\cos\theta_1 + \cos\theta_2)$$

where the angles are defined in Fig. 3. (15%)

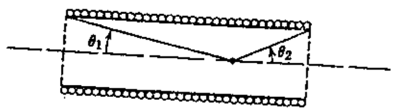

Fig. 3

6. Consider a plane electromagnetic wave

$$\mathbf{E} = E_o\hat{\mathbf{x}}e^{i(kz-\omega t)} + E_o'\hat{\mathbf{y}}e^{i(kz-\omega t+\phi)}$$

where k = ω/c and E_o, E_o' are real. (a) Determine the energy density u and the Poynting vector S. (b) Determine the polarization of this electric field, if $E_o' = 2E_o$, and $\phi = \pi/4$. (20%)