

國立中山大學八十八學年度碩博士班招生考試試題

科目：量子力學 (物理學系博士班)

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1. Describe three examples for physical phenomena that can not be explained satisfactorily in classical physics but can be explained by quantum theory. (12%)

2. Carefully define or explain:

- (a) Correspondence principle (3%)
- (b) Uncertainty principle (3%)
- (c) Probability current density (3%)
- (d) Pauli's exclusion principle (3%)
- (e) Fermi golden rule (3%)
- (f) Stark effect (3%)

3. (a) Determine the energy levels and the normalized wave functions of a particle in a "potential well". The potential energy  $V$  of the particle is

$$V = \infty, \quad x < 0 \quad \text{and} \quad x > a; \quad (10\%)$$

$$V = 0, \quad 0 < x < a.$$

(b) Calculate the expectation values of  $x$ ,  $p$ , and the uncertainty  $\Delta x$  and  $\Delta p$  for ground state. (16%)

(c) Show the condition, which let the above result agree with the corresponding classical result. (5%)

4. A system described by the Hamiltonian  $H = -\frac{\hbar^2}{2m}\nabla^2 + \frac{m}{2}(\omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2)$  is called an "anisotropic harmonic oscillator". Determine the possible energies of this system, and for the isotropic case ( $\omega_1 = \omega_2 = \omega_3 = \omega$ ) calculate the degeneracy of the level  $E_n$ . (12%)

5. Find the wave functions and energy levels of the stationary states of a plane rotator is equal to  $I = \mu a^2$ , where  $\mu$  is the reduced mass of the particles and  $a$  is their distance apart. (12%)

6. (a) Show that for a state  $\chi$  (spin 1/2 particle) with a well defined value of  $S_z$  (i.e.  $S_z \chi_{\pm} = \pm(1/2)\chi_{\pm}$ ), the average value of  $S_x$ ,  $S_y$  are equal to zero. (8%)

(b) The z-component of the electron spin is equal to 1/2. What is the probability that its component along a direction  $z'$  which makes an angle  $\theta$  with the z axis is equal to 1/2 or -1/2? (7%)

# 國立中山大學八十八學年度碩博士班招生考試試題

科目：電動力學 (物理學系博士班)

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1. Write down the Maxwell's equations. (8%)
2. A simple capacitor is a device formed by two insulated conductors adjacent to each other. If equal and opposite charges are placed on the conductors, there will be a certain difference of potential between them. The ratio of the magnitude of the charge on one conductor to the magnitude of the potential difference is called capacitance. Using Gauss's law, calculate the capacitance of (a) two large, flat, conducting sheets of area  $A$ , separated by a small distance  $d$ ; (b) two concentric conducting spheres with radii  $a, b$  ( $b > a$ ). (12%)

3. Using the method of images, discuss the problem of a point charge  $q$  inside a hollow, grounded, conducting sphere of inner radius  $a$ . Find (a) the potential inside the sphere; (b) the induced surface-charge density. (20%)

4. A spherical surface of radius  $R$  has charge uniformly distributed over its surface with a density  $Q/4\pi R^2$ , except for a spherical cap at the north pole, defined by the cone  $\theta = \alpha$ . (a) Show that the potential inside the spherical surface can be expressed as

$$\Phi = \frac{Q}{2} \sum_{\ell=0}^{\infty} \frac{1}{2\ell+1} [P_{\ell+1}(\cos\alpha) - P_{\ell-1}(\cos\alpha)] \frac{r^\ell}{R^{\ell+1}} P_\ell(\cos\theta)$$

where, for  $\ell = 0, P_{\ell-1}(\cos\alpha) = -1$ . What is the potential outside?

- (b) Find the magnitude and the direction of the electric field at the origin. (20%)

5. A cylindrical conductor of radius  $a$  has a hole of radius  $b$  bored parallel to, and centered a distance  $d$  from, the cylinder axis ( $d+b < a$ ). The current density is uniform throughout the remaining metal of the cylinder and is parallel to the axis. Use Ampere's law and principle of linear superposition to find the magnitude and the direction of the magnetic-flux density in the hole. (20%)

6. A transmission line consisting of two concentric circular cylinders of metal with conductivity  $\sigma$  and skin depth  $\delta$ , as shown below, is filled with a uniform lossless dielectric ( $\mu, \epsilon$ ). A TEM mode is propagated along this line. (a) Show that the time-averaged power flow along the line is

$$P = \left[ \frac{c}{4\pi} \right] \sqrt{\frac{\mu}{\epsilon}} \pi a^2 |H_0|^2 \ln\left(\frac{b}{a}\right)$$

where  $H_0$  is the peak value of the azimuthal magnetic field at the surface of the inner conductor. (20%)

