应数系

Ph.D. Entrance Examination
Numerical Analysis
May 2, 2014

Please write down all the detail of your computation and proof.
Twenty points for each problem.

1. (1) Write down the algorithm of Newton method to solve a root γ of the nonlinear equation $f(x) = 0$.

   (2) If γ is a simple root of $f(x)$ and the initial value is sufficiently close to γ, show that this iteration converges to γ quadratically by the fixed point theorem.

   (3) What happens if γ is a multiple root? Why?

2. How to find the Hermite polynomial that interpolates $n+1$ data (x_i, y_i, y_i') for $i = 0, 1, \cdots, n$ with all x_i distinct? State your method and show it works.

3. What is Gaussian quadrature for numerical integration? In what sense that it is best? Prove that it can achieve the best approximation.

4. Let T be an $n \times n$ matrix and $\mathbf{v}$ be an n dimensional column vector. Prove that the iterative method $\mathbf{x}^{(k+1)} = T\mathbf{x}^{(k)} + \mathbf{v}$ converges if, and only if, the spectral radius $\rho(T) < 1$. Please provide the detail for all theorems you use in the proof.

5. Let A be an $m \times n$ matrix with full rank and $m \geq n$. How to use (1) normal equation, (2) QR factorization, (3) singular value decomposition to obtain the least squares solution of $A\mathbf{x} = \mathbf{b}$? Explain why they work?