

## Part 1

動態體系裡的計量統計問題中，對於隨機干擾項(stochastic disturbance terms)  $\{\varepsilon_t\}$  的處理，通常都會從文獻上或教科書上看到作者作如此的假設：

(1) stationary stochastic process，或者，(2) serially uncorrelated，或者，(3) identically and independently distributed (iid) with some distributions (e.g. normal)，或者，(4) white noise 等，不一而足。

(一) 請問：以上(1)~(4)四種 alternative assumptions 彼此之間究竟有無差別？如有差別，其差異何在？如無差別，其理安在？

(二) 關於上述第(4)種假設，常見的中文翻譯都寫成「白色躁音」或「白音」，這不免產生了語意上的矛盾：既然是一種「躁音」，為何還會有「顏色」(白色)之別呢？(聲音應是無色才對！) 請就此點質疑，提出您個人的分析性看法。

(本大題共計 50 分)



## Part 2

1. The home ownership of a Kaohsiung city is distributed as follows, by annual income:

Annual Income	Home Owner		Total
	Yes	No	
\$ 0 – 200000	20	347	367
200001–1000000	60	124	184
1000001–3000000	272	50	322
Above 3000000	498	66	564
Total	850	587	1437

- a. Explain why home ownership and annual income are likely to show dependence in a chi-square test? (5 points)
- b. Test the hypothesis that home ownership and annual income are independent at a 5 percent significance level. [ $\chi^2(6, 0.05) = 12.59$ ,  $\chi^2(4, 0.05) = 9.488$ ,  $\chi^2(3, 0.05) = 7.815$ ] (5 points)
2. The monthly income,  $Y$ , as a function of age,  $X_1$  (expressed in year), and number of years on the job,  $X_2$ , is given by the following regression equation:

$$Y = 200 + 5X_1 + 10 X_2$$

- a. According to above regression equation, which of the following has a higher expected monthly income: a person who is 40 years of age and 10 years on the job or a person who is 35 years of age and 18 years on the job? (5 points)
- b. What must the deviation  $e_i$  from the regression plane be if the two people mentioned in part a actually earn \$ 450 per month? (5 points)
3. Let  $X_1, X_2, \dots, X_n$  denote a random sample from the distribution with p. d. f.:

$$f(x) = \theta^x (1-\theta)^{1-x}, x = 0, 1$$

$$= 0 \quad \text{elsewhere,}$$

Where  $0 \leq \theta \leq 1$ . Prove that  $\sum x_i/n$  is the maximum likelihood estimate of  $\theta$ . (10 points)

4. Let  $X_1, X_2, \dots, X_n$  denote a random sample from the distribution with p. d. f.:

$$f(x) = \theta^x (1-\theta)^{1-x}, x = 0, 1$$

$$= 0 \quad \text{elsewhere,}$$



Let  $Y_1 = X_1 + X_2 + \dots + X_n$ , prove that  $Y_1$  is a sufficient statistic for  $\theta$ ? (10 points)

5. The following Table provides data on long-term debt as a percentage of total invested capital in 2007 for 12 manufacturing companies. Assume that the 12 companies represent a sample taken from a very large population of companies.

Company Name	Long-term Debt(%)	Company Name	Long-term Debt(%)
A company	32.4	G company	37.9
B company	34.5	H company	29.3
C company	7.9	I company	33.2
D company	20.2	J company	59.5
E company	11.3	K company	33.5
F company	26.7	L company	35.9

- Solve the point estimate of the long-term debt as a percentage of the invested capital in the industry. (3 points)
- Solve the 95 percent confidence interval for your estimate, assuming that  $\sigma$  is known to be 10. (3 points)
- Solve the 95 percent confidence interval for your estimate, assuming that  $\sigma$  is not known. [ $t(11,0.05) = 2.201$ ,  $t(12,0.05) = 2.1788$ ] (4 points)



97 財管博士班經濟學

個體經濟部分

1. A student has an exam to take. From her knowledge of the material she would receive  $\bar{M}$  points but she can augment her points by spending time during the exam cheating from the student sitting next to her. If  $t$  denotes the time spent cheating, then the relationship between  $t$  and the points gained by cheating ( $M^*$ ) is given by  $M^*=At$  where  $A$  is a positive parameter. The student's probability of being detected cheating (call this probability  $\Pi$ ) increases with the amount of time spent cheating. The penalty for cheating is an exam score of zero.

Assume that the utility derived from her examination points does not depend on the state of the world in which it is experienced and assume that the student maximizes her expected utility where utility depends only on her score on the exam.

- (a) Derive the equilibrium condition for the optimal time spent cheating and give a verbal explanation of it.
- (b) Under what conditions will she not engage in cheating at all? (20 分)
2. Consider an economy where consumption takes place only in the future in one of two possible states which may arise: state 1 which occurs with probability  $\theta$  and state 2 which occurs with probability  $(1-\theta)$ . Each citizen owns a fruit-bearing tree which yields one unit of fruit if state 1 arises and 2 units in state 2. Half of these citizens have the utility function  $u=\ln(c)$  (where  $c$  is the consumption of fruit); the other half have the utility function  $u=\ln(1+c)$ . All citizens are expected utility maximizers.

Prior to learning which state will obtain, citizens may exchange state-contingent claims. Let  $b_1$  be the units of a "security" entitling the bearer to one unit of consumption if state 1 arises and 0 otherwise; define  $b_2$  similarly for state 2.

- (a) Find the competitive equilibrium price for each security and the equilibrium quantities purchased (or issued) by individuals of each type.
- (b) Suppose, in addition to these securities, "equities" (entitling the bearer to the entire output of one tree, whichever state arises) and "bonds" (entitling the bearer to one unit of output no matter which state arises) are also traded. Find the equilibrium price for both these securities.(15 分)
3. Ms. Mary Kelley has initial wealth  $W_0=\$1200$  and faces an uncertain future that she partitions into two states,  $s=1$  and  $s=2$ . She can invest in two securities,  $j$  and  $k$ , with initial prices of  $p_j=\$10$  and  $p_k=\$12$ , and the following payoff table:

Security	Payoff	
	$s = 1$	$s = 2$
$j$	\$10	\$12
$k$	20	8



- (a) If she buys only security  $j$ , how many shares can she buy? If she buys only security  $k$ , how many can she buy? What would her final wealth,  $W_s$ , be in both cases and each state?
- (b) Suppose Ms. Kelley can issue as well as buy securities; however, she must be able to meet all claims under the occurrence of either state. What is the maximum number of shares of security  $j$  she could sell to buy security  $k$ ? What is the maximum number of shares of security  $k$  she could sell to buy security  $j$ ? What would her final wealth be in both cases and in each state?
- (c) What are the prices of the pure securities implicit in the payoff table?
- (d) What is the initial price of a third security  $i$  for which  $Q_{i1}=\$5$  and  $Q_{i2}=\$12$ ?
- (e) Summarize the results of (a) through (d) on a graph with axes  $W_1$  and  $W_2$ .
- (f) Suppose Ms. Kelley has a utility function of the form  $U=W_1^6W_2^4$ . Find the optimal portfolio, assuming the issuance of securities is possible, if she restricts herself to a portfolio consisting only of  $j$  and  $k$ . How do you interpret your results? (15 分)

97 總體經濟部份：共五十分

1. 正斜率的總供給線可能由於(i)名目工資僵固(ii)實質工資僵固(iii)物價僵固(iv)實際物價與預期不同，請將四種情形的勞動市場各別以數學模型表示。  
(20 分)
2. (i) 古典學派認為貨幣政策對所得沒有影響，何故？(5 分)  
(ii) 古典學派可貸資金理論以債券供需決定利率，請寫出其數學模型及畫出圖形。(5 分)  
(iii) 依(ii)之架構，分析在古典學派假設之下央行公開市場操作是否會影響利率？是否會影響所得？(10 分)
3. Construct an IS-LM-BP model of a small open economy with flexible exchange rates and mobile capital flows. Suppose world interest rate decreases, explain the direction of and the reason for the change of the following variables in the small open economy:
  - (i) interest rates
  - (ii) exchange rates
  - (iii) income
  - (iv) trade balance
 (10 分)



**Ph.D. Entrance Exam 2008**

**--Financial Management**

1. (25%) Please compare the “Static Theory of Capital Structure” and the “Signaling Theory of Capital Structure”, in particular, their implication to firms’ capital structure decisions in practice.
2. (25%) Please describe the “Three Factor Model” developed by Fama and French, and further explain its application in asset pricing.
3. (25%) Please comment on the current financial issues concerning with “Structure Notes (連動債券)”, indicate the problems and propose your solutions.
4. (25%) Please give an example to compares similar and different attributes of the Net Present Value (NPV) method and Internal Rate of Returns(IRR) methods in capital budgeting decision.



本試題共計五大題，每大題20分，滿分100分。每題均應詳列推導過程

(一)

(a) Let  $D = \{(x, y) | x^2 + y^2 \leq 1\}$  and  $\phi(x, y) = \sqrt{x^2 + y^2}$  for  $(x, y) \in D$ .

(i) Find  $c$  such that  $c\phi$  is a probability density function on  $D$ .

(ii) Evaluate  $\int \int_D x^2 y^2 \phi(x, y) dx dy$ .

(b) Let  $a_1 = 2$  and  $a_{n+1} = 1/a_n + a_n/2$ ,  $n = 1, 2, \dots$ . Is  $\{a_n\}$  a convergent sequence? If yes, find its limit.

(二) Evaluate for  $|z| < 1$

$$\int_0^{2\pi} \frac{e^{it}}{z - e^{it}} dt.$$

(三) Let the mapping  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be defined as

$$T(x, y) = \left(\frac{3}{2}x + \frac{1}{2}y, \frac{1}{2}x + 2y\right).$$

Find  $e^T(x, y)$ , where

$$e^T(x, y) = \sum_{n=0}^{\infty} \frac{1}{n!} T^n(x, y),$$

where  $T^1(x, y) = T(x, y)$  and  $T^n(x, y) = T(T^{n-1}(x, y))$  for  $n \geq 2$ .

(四) Suppose that  $f$  is a real continuous function on a metric space  $S$ . Let  $P(f) = \{s \in S | f(s) > 0\}$ . Is  $P(f)$  a closed subset, open subset, or neither?

(五) Find the general solution of  $t^2 y''(t) - 2y(t) = e^t$ ,  $t > 0$ .