國立中山大學九十三學年度博士班招生考試試題

科目：統計學【財管系選考】

第一部份

1. 是非題 (請回答對錯並說明理由，每題 3 分)
   (1). 若(4,7)爲 μ 之 90%的信賴區間，則有 0.90 的機率 μ 落在(4,7)之中。
   (2). 假設隨機變數 X_1, X_2 爲常態分配，則 X_1+X_2 亦爲常態分配。
   (3). 若 X 與 Y 爲兩個隨機變數，則 X+Y 的變異數必大於 X-Y 的變異數。
   (4). 當我們進行假設檢定時，我們計算出的檢定統計值低於設定的臨界值時，則無法推翻虛無假設，此時我們可說是接受了虛無假設，因爲我們證明了虛無假設是真的。
2. 當二項式分配在何種大樣本與小 p 值情況下而產生稀有事件(rare event)時，可用何種離散機率分配去近似之(approximating)? 說明如何轉換並請自行假設參數值，舉出該機率分配之例子? (本題 8 分)(請寫出計算過程，不用計算出機率大小)
3. 假如你要進行一個假設檢定，步驟應該爲何，請簡要列出。(本題 5 分)
4. 一家電視製造公司生產的電視平均使用 5 年後才可能有嚴重的損害發生，製造商對嚴重的損害提供免費的保證，而這家公司要確定在保證期內不會有超過 30%的洗衣機有嚴重的損害，這家公司的保證期間應該訂多少年，假設損害的發生是呈卜瓦松分配。(請寫出計算過程，不用計算出保證期間) (本題 5 分)
5. 假設你想要估計高雄市家庭平均每月用多少錢投資共同基金，你希望有對結果有 90%的信心，希望估計值與實際投資的差不超過 10,000 元，平均每月投資共同基金的標準差爲 40,000 元，請問你採取的樣本規模該有多大？(請寫出計算過程與答案) (本題 5 分)
6. 假設你正進行一個比例的雙尾檢定，虛無假設是母體比例爲 0.4，對立假設是母體比例不爲 0.4。一樣本數爲 250 之隨機樣本產生一樣本比例數爲 0.44。令 α 爲 0.05，則 $\alpha/2$ 的 Z 查表值爲 1.96。從樣本資料計算而得之 Z 值爲 1.29，因此虛無假設不被拒絕。假若此時並不是已達成正確決定，而是犯了型二誤差，在此假設對立假設的母體比例數爲 0.36，此時犯下型二誤差的機率是多少？(請寫出計算過程與答案) (本題 5 分)
7. 寫請出貝氏定理(Bayes' Theorem)的公式，解釋其含意，並證明之。(本題 10 分)

A system will be called classical linear regression model if

$$\underset{(n\times 1)}{y} = \underset{(n\times k)}{\mathbf{X}}\ \underset{(k\times 1)}{\mathbf{B}} + \underset{(n\times 1)}{\varepsilon}$$

where $E(\varepsilon)=0$, $Var(\varepsilon)=\sigma^2\Omega$, and Ω is a known symmetric, positive definite matrix of the order of $(n\times n)$, σ^2 is an unknown scalar.

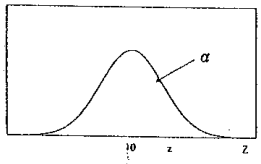
(1) To derive a best, linear, and unbiased estimator for **B**, and prove that it actually is a BLUE. (25 分)

(2) To derive an unbiased estimator for σ^2. (15 分)

(3) Let R^2 be the coefficient of determination for this model, prove that $R^2=\dfrac{(k-1)F}{(n-k)+(k-1)F}$, where F is a statistic. (10 分)

標準常態累加機率值表

$P(0 < Z < z) = \alpha$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

國立中山大學九十三學年度博士班招生考試試題

科目：經濟學【財管系選考】

93 財管所博士班

壹，總體經濟部份，共五十分

1. 一般而言，一國實質 GDP 持續成長，物價也持續上升。
   (a) 請以總供給-總需求模型說明爲何如此
   (b) 請列舉導致總供給線及總需求線右移的所有理論因素
   (c) 在真實世界，(b)中所列哪些因素最爲重要？爲何這些因素會持續增加？
   (15 分)

2. Suppose that the demand for real money balances depends on the interest rate, i, and on disposable income, Y-T; in other words, suppose that the correct way to write the LM equation is M/P=L(i, Y-T)
   (a) With this change to AS-AD model, can you tell whether a tax cut increases or decreases output? Assume a closed economy.
   (b) Redo part (a) assuming an open economy under the assumption that the exchange rate is floating, exchange rate expectations are static, and capital is perfectly mobile. (17 分)

3. Suppose the economy is described by linear IS and LM curves that are subject to disturbances: $y = \alpha - \beta i + \varepsilon_D$, $m - p = \gamma y - \delta i + \varepsilon_L$, where ε_D and ε_L are independent, mean-zero shocks with variances σ_D^2 and σ_L^2, and where $\beta > 0$, $\gamma > 0$, $\delta > 0$. Policy makers want to stabilize output, but they cannot observe y or the shocks, ε_D and ε_L. Assume p is fixed.
   (a) Suppose the policy maker fixes i at some level i^*. What is the variance of y?
   (b) Suppose the policy maker fixes m at some level m^*. What is the variance of y?
   (c) If there are only LM shocks (so $\sigma_D^2 = 0$), does money targeting or interest-rate targeting lead to lower variance of y?
   (d) If there are only IS shocks (so $\sigma_L^2 = 0$), does money targeting or interest-rate targeting lead to lower variance of y?
   (e) Explain your results in parts (c) and (d) **intuitively**. (18 分)

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科目：經濟學【財管系選考】

貳、個體經濟學

一、Consider an economy with N consumers and one product X. Consumers have unit demand for product X. Their valuations for product X are either V_H or V_L, with $V_H > V_L > 0$. The proportion of consumers who have valuation V_H is $\alpha \in (0,1)$. Suppose that X can be costless produced by a monopolist M. M sells the product to retailer R at wholesale price w, and then given w, R sells the product to consumers by optimally choosing retail price p.

(1) Suppose M and R have been vertically integrated. What are the optimal (w^*, p^*)? (10 分)

(2) Suppose M and R are separate entities and the game proceeds in the way described above. Determine the equilibrium (w^*, p^*) and compare the total channel profit here to that of part (1). (20 分)

二、假設你開了一家 VCD 出租店，最近有兩部片子即將發行：「魔戒 3」以及「托斯卡尼豔陽下」。經過調查，潛在消費者可以分為四群：A、B、C、D，每群消費都有 100 人，對於兩部電影的最高願付價格如下表所示：

消費群	魔戒 3	托斯卡尼豔陽下
A	30	90
B	40	60
C	60	40
D	90	30

(1)若兩部電影者邊際成本均為 0，你可以採取單一商品訂價(pure component)或是組合商品訂價法(pure bundle or mixed bundle)，請問如何訂價可以使你利潤達到最大？(5 分)

(2)承上題，若兩部電影者邊際成本均為 35 元，則最適訂價又為何？(10 分)

(3)請說明為何(1)與(2)的最適訂價方式有所不同。(5 分)

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科目：數 學 【財管系選考】

請按題號順序作答，並請列出推導過程。

1. Define a Cauchy sequence. Is a convergent sequence Cauchy sequence? If yes, prove it. It not, give a counterexample. 10%

2. Prove that $f(x)=x^3-3x+1$ has a zero in the interval $[0,1]$ 10%

3. Prove that $f(x)=\sqrt{x}$ is uniformly continuous in $[a,b]$. 10%

4. Approximate $\sqrt{9.2}$ and show the accuracy of your approximation. 10%

5. Prove that $\lim_{x\to 0}\frac{\cos x-1}{x}=0$ 10%

by i)L'Hopital Rule and by ii) Mean Value Theorem

6. Determine if each of the following is linearly independent. 10%

a. $\left\{\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},\begin{bmatrix}0 & 1\\ 0 & -1\end{bmatrix},\begin{bmatrix}0 & 0\\ 1 & -1\end{bmatrix}\right\}$ b. $\{1,\ e^x, e^{-x}\}$ in $C(-\infty,\infty)$.

7. Determine if each of the following is inner product for the indicated vector space. If not, list all properties violated. 10%

a. (p, q) = $a_0b_0+a_1b_1$, p(x) = a_0+a_1x, q(x) = b_0+b_1x in P_1.

b. (A, B) = det(A)det(B) where A, B are 2x2 matrices.

8. Find the rank of $\begin{bmatrix}1 & 0 & -1\\ 3 & 1 & 1\\ -1 & -1 & -3\end{bmatrix}$ 5%

9. Determine if the following set of vectors is orthogonal or orthonomal for the indicated vector space and inner product. 10%

$\{(\frac{1}{\sqrt{5}},\frac{2}{\sqrt{5}},0),(\frac{-2}{\sqrt{5}},\frac{1}{\sqrt{5}},0),(0,0,1))$ in $R^3;(u,v)=u_1v_1+u_2v_2+u_3v_3$

10. $T: R^3\to R^3$ is given by $T(x, y, z)=(3x+y-z,\ x+2y+z)$. Show that if $v_1=(3,-4,5), v_2=(1,4,7)$ are in the kernel of T. 5%

11. Find the eigenvalues and eigenvectors of

a. $A=\begin{bmatrix}1 & 1\\ 6 & 2\end{bmatrix}$ b. A^{-1} 10%

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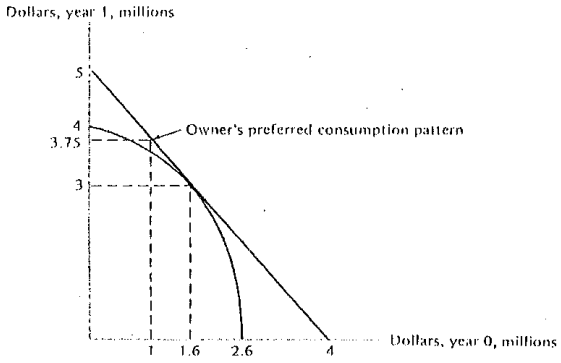
科目：財務管理【財管系選考】

Part I :Per question counts 10 points, total 50 points.

1. The following figure the sloping line represents the opportunities for investment in the capital market and the solid curved line represents the opportunities for investment in plant and machinery. The company's only asset at present is $2.6 million in cash.
   a. What is the interest rate?
   b. How much should the company invest in plant?
   c. How much will this investment be worth next year?
   d. What is the average rate of return on the investment in plant?
   e. What is the marginal rate of return?
   f. What is the present value of this investment?
   g. What is the net present value of this investment?
   h. What is the total present value of the company?
   i. How much will the individual consume today?
   j. How much will he or she consume tomorrow?



2. Consider projects A and B:

	CASH FLOWS,DOLLARS			
Project	C_0	C_1	C_2	IRR. Percent
A	-4,000	+2,410	+2,930	21
B	-2,000	+1,310	+1,720	22

a. The opportunity cost of capital is less than 10 percent. Use the IRR rule to determine which project or projects you should accept (i) if you can undertake both, and (ii) if you can undertake only one.

b. Suppose that project A has an NPV of $690 and project B has and NPV of $657. What is the NPV of the $2000 incremental investment in A?

3. Big Oil is wondering whether to drill for oil in Westchester County. The prospects are as follows:

Depth of Well, Feet	Total Cost, Millions of Dollars	Cumulative Probability of Finding oil	PV of Oil (if Found), Millions of Dollars
1000	2	.5	5
2000	2.5	.6	4.5
3000	3	.7	4

Draw a decision tree showing the successive drilling decisions to be made by Big Oil. How deep should it be prepared to drill?

4. Please briefly discuss the three most important ideas in Finance?
5. Please briefly discuss three unsolved problems in Finance?

Part II: 此部分共 4 題，共 50 分。

6. (15%) **Use the arbitrage-free argument and <u>the following notations</u> to prove MM Theorem of capital structure irrelevance**: There are 2 companies in the economy: U and L. Assume one-period economy, i.e., the economy will end in one year. U and L will generate the same future operating cash flow $\tilde{X}$ at year 1, respectively. Company U is unlevered (i.e., no debt) and the current value of equity of U is V_U. Company L is levered, in which the current value of equity of L is E_L and the current value of debt of L is D_L. Interest rate of debt of L is r_d. **Prove $V_U=D_L+E_L$.** Assume no tax, no information asymmetry, no transaction costs, and no market frictions.

7. (15%) **<u>In addition to stock abnormal return approaches</u>**, how to empirically examine whether or not a corporate event, for example, a merger or proxy fight, creates value ? You **<u>must</u>** use the following format to answer this question: **Step 1:..., Step 2:..., Step 3:..... etc.** 注意：你的答案必須是<u>股票異常報酬率以外</u>的方法。<u>答案不可超過 30 字，超過部分不計分</u>。

8. (10%，此爲單選題) Company ABC decides to borrow $100 millions to buy back stock. Assume no information asymmetry, no transaction costs, no tax and no market frictions. **<u>How many (有幾個)</u>** of the following of Company ABC **<u>will not change</u>** after ABC borrows money to buy back stock: stock price, stock's expected return, WACC, return on old debt, EPS at year 1 ? (A) 1 (B) 2 (C) 3 (D) 4 (E) 5.

9. (10%) Corporate governance is an increasingly important issue in Taiwan. Please briefly discuss 3 most effective corporate governance mechanisms in Taiwan. <u>答案不可超過 60 字，超過部分不計分</u>。