

# 國立中山大學九十一學年度博士班招生考試試題

科目：統計學【財管系】

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一、(本題 50 分)

在單一直線迴歸模型  $y = \underset{(n \times 1)}{X} \underset{(n \times k)}{B} + \underset{(k \times 1)}{\varepsilon}$  裡，假定模型中相關變數皆滿足 OLS 基本假設，則模型外顯參數 (explicit parameters)  $B$  的 OLS 估計式  $\hat{B} = (X'X)^{-1}X'y$  根據 Gauss-Markov 定理知，它是一個 BLUE；此外，模型內隱參數 (implicit parameter)  $\sigma^2$  的不偏估計式為  $\hat{\sigma}^2 = \frac{\hat{\varepsilon}'\hat{\varepsilon}}{n-k}$ ， $\hat{\varepsilon}$  是 OLS 迴歸式的殘差項，並且知： $\hat{\varepsilon} = My = M\varepsilon$ ，式中， $M$  稱為「最小平方基本自乘不變矩陣」(the fundamental idempotent matrix of least square)。請回答以下問題：

1. 證明 Gauss-Markov 定理。(10 分)
2.  $M$  具有哪些特性？並請逐一證明之。(40 分)

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2. Taico Corporation, a market leader in the production of specialized detergents, is considering the introduction of a new spot remover product. Its manufacturing facility can produce 50 thousand cases of the new product per year with a setup cost of \$100,000. The average variable cost per case of product is \$5. Orders above 50 thousand cases will be subcontracted overseas to a Chinese firm at a variable cost per case, of \$9. Whether the product is manufactured by Taico or by the Chinese vendor, there is a \$12 cost per case for marketing and overhead.

Taico's marketing manager is considering placing an order for an extra 50 thousand cases, 75 thousand cases, and 100 thousand cases. The actual level of sales will depend on the state of the economy. Sales estimates prepared by the marketing manager indicate that if the economy is strong, it will sell 100 thousand cases; if the economy is moderate, it will sell 70 thousand cases; and if the economy is weak, it will sell only 40 thousand cases. The marketing department forecasts a 50% chance of a strong market and a 15% chance of a weak market.

A case of the new product is priced to sell a \$40. Unsold units will be sold to an overseas distributor at a 50% discount. Before deciding upon a method and level of production, the production manager wishes to evaluate the possibility of hiring a local market research firm to conduct an economic survey. The manager has received a proposal from a local marketing survey firm. This firm has offered to conduct the survey for \$20,000. The historical performance of the survey firm in predicting economic activity is summarized as follows: Correctly predicted a strong economy 12 out of 16 times (2 times they predicted a moderate economy and 2 times a weak economy); correctly predicted a moderate economy 7 out of 10 times (2 times they predicted a strong economy and 1 time a weak economy); and correctly predicted a weak economy 9 out of 10 times (1 time they predicted a moderate economy).

The marketing manager wishes to know:

- (1) The optimal quantity of cases to produce. (10 points)
- (2) The number of cases to be produced by the Chinese vendor. (10 points)
- (3) The expected value of the optimal decision. (10 points)
- (4) The maximum the company should pay for the perfect information. (10 points)
- (5) What production policies should Taico employ based on the results of the market survey? (10 points)

It is assumed that there are no goodwill costs associated with failure to meet demand.

91 個體經濟部份(共五十分)

(一)(觀念題 15 分)

電影『美麗境界』(A Beautiful Mind) 中描述了賽局論大師納許(John Nash) 戲劇化的生平。在瘋狂與天才之間，納許留給世人許多有別於新古典經濟學(Neoclassical Economics)的思維，對於我們理解人類經濟活動有深遠的影響。

請簡要說明下列因納許而發展的觀念(請分別說明(1)數學公式(2)經濟意義(3)可能應用之情況)

- (i) Non-cooperative Nash Equilibrium(5 points)
- (ii) Cooperative Nash Solution(5 points)
- (iii) Bayesian Nash Equilibrium(5 points)

(二)(常識題 15 分)

為因應知識經濟的發展趨勢，行政院於民國 89 年 8 月 30 日第 2696 次院會，核定通過「知識經濟發展方案」，希望透過此一方案的實施，於十年內達到先進知識經濟國家水準。請簡要回答下列問題(每小題請限於 100 字之內)：

- (1) 我國推動「知識經濟發展方案」之經濟背景。(5 points)
- (2) 「知識經濟發展方案」之基本構想與內容。(5 points)
- (3) 「知識經濟發展方案」之具體措施(請以條例方式列出至少 5 項)。(5 points)

(三)(計算題 20 分)

某甲對風險的偏好可用一個預期效用(Bernoulli utility function)

$$u(x) = (x)^{0.5}$$

請回答下列問題

- (1) 假設某甲之期初財富  $w = 5$ ，請問 coefficient of Arrow-Pratt absolute 以及 relative risk aversion 各為何?(5 分)
- (2) 有一個具風險的遊戲可表達為(16,4; 0.5,0.5)(得到 16 或 4 之機率均為 0.5)，請計算某甲對此一遊戲之 certainty equivalent。(5 分)
- (3) 若風險遊戲的報酬變更爲(36,16; 0.5,0.5)，請問某甲之 certainty equivalent 會如何變動? 請分析此一變化之經濟意義。(10 分)

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## 總體經濟部份(共五十分)

1. Prove Marshall-Lerner Condition (12 分)
2. Critically evaluate the classical macroeconomic theory. (12 分)
3. Suppose that the demand for real money balances depends on the interest rate,  $i$ , and on disposable income  $Y-T$ ; in other words, suppose that the correct way to write the LM equation is  $M/P=L(i, Y-T)$ . With this change to the IS-LM-AS model, can one tell whether a tax cut increases or decreases output? Assume an open economy with floating exchange rates, static exchange rate expectations and perfectly mobile capital. (12 分)

4. Consider the following model:

$$y = \alpha(i - \dot{p}), \dot{m} - p = -\lambda i, \dot{p} = \theta y$$

Assume  $\alpha > 0$ ,  $\lambda > 0$ ,  $\theta > 0$ , and  $\alpha \theta < 1$ .  $m$ ,  $y$  and  $p$  are in logarithms. Assume that initially  $y = i = \dot{p} = \dot{m} = p = 0$ . Now suppose that at some time, time 0 for convenience, there is a permanent drop in  $m$  to some lower level  $m'$ . Assume that  $p$  can not jump at the time of the change.

- (i) Draw the path of  $y$  over time. Explain. (7 分)
- (ii) Suppose we measure the total amount of output volatility caused by the change in  $m$  as  $V = \int_{-\infty}^{\infty} y(t)^2 dt$ . How is  $V$  affected by an increase in the speed of price adjustment,  $\theta$ ? (7 分)

# 國立中山大學九十一學年度博士班招生考試試題

科目：財務管理【財管系】

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Please explain your answer clearly and concisely

1. Comparing hybrid debts with the traditional financing instruments: (10 points)
  - (1) What are the benefits to the issuing firms to raise capital with hybrid bonds which have imbedded options characteristics?
  - (2) If a securities firm chooses to issue stock index-linked subordinated bond to raise its capital, rather than issues a straight bond and shorts stock index futures at the same time, is there any difference between the two in term of the agency problems you know? Please explain.
  
2. The case of Enron exposed the corporate governance problems in the U.S. and led to downturn in U.S. stock market. The aftershocks remained to be further observed. Please briefly explain: (10 points)
  - (1) What are the major monitoring mechanisms for corporate governance?
  - (2) Briefly summarize the empirical evidence of each corporate governance mechanism.
  - (3) Traditionally, the value of the firm depends on its real assets in place, the separation of ownership and management leads to agency problem which is the core of corporate governance, i.e., to control the conflict between stockholders and management. Do you think the core of the traditional corporate governance mechanism still applies to the high-tech industry today? Why?
  
3. (1) 兩稅合一 has been implemented for several years, its effect is to reduce total taxes paid by shareholders. What do you think would be the change in taxation on the **capital structure** of the firm? Why? (15 points)
  - (2) What do you think would be the impact of the above change in taxation on the **dividend policy** of the firm? Please explain first your **hypotheses**, followed by empirical model(s) to test your hypotheses. Be sure to define the variables in your empirical models.
  
4. (15 points) The issuing firm in IPOs (initial public offerings) may choose auction or fixed price underwriting mechanism. In auction offering, investors bid for the shares, the highest offer gets allocated first. In fixed price offering, the offer price is fixed and the allocation is based on an open lottery mechanism if there is oversubscription. Generally, the IPO discount (the difference between offer price and initial market price) is smaller and the institutional investors participation is higher in auction offerings, compared with fixed price offerings in which most participants are individuals.  
Please analyze the factors that may influence the decision of the issuer whether to choose the auction or the fixed price method in IPOs:
  - (1) State first your hypotheses on the determinants of the issuing decision, and briefly explain your rationale,
  - (2) Construct empirical model(s) to test your hypotheses. Be sure to define the variables in the models.

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問答題(請詳加解釋並列出計算過程)

5. (15 points) 中油買了一家美國的油田探測公司, CPC Drilling Company 在阿拉斯加擁有一些荒地, 但不清楚是否有石油。若要鑽探測試油井(exploratory well), 成本為 2000 萬美元。有 80% 的機率鑽出的測試油井是乾的, 但不管測試油井是不是乾的, 只要在一年內裝上一億美元的生產設備(production capacity), 都能期獲得石油收益。設折現率為 10%。  
若測試油井鑽探成功, 亦即不是乾的, 年收入每年為 3000 萬美元直到永遠, 若不成功, 則為每年仍有收入 750 萬元直到永遠, CPC Drilling 是否應投資測試油井的鑽探?
- (a) 畫出 CPC drilling 投資測試油井兩期的決策樹, 並說明若在第一年已知測試油井不是乾的, 結果是否應投資一億美元的生產設備, 若測試油井為乾的, 結果是否仍應投資一億美元的生產設備?
- (b) CPC Drilling 今天是否應投資測試油井的鑽探?
6. (15 points) 永信製藥正開發一種最新的治療禿頭的藥。開發成本是 30 億元。開發此藥一年會帶來三種可能的成果:
- (1) 此藥很成功, 第一年到永久每年可預期賺取 6.5 億元到永久。  
(2) 此藥尚稱成功, 第一年起每年可多賺 5.5 億元直到永久。  
(3) 此藥完全失敗, 無任何收益。
- 在測試前三種可能出現的機率皆相同。假設無風險利率是 5%, 製藥市場收益之 Beta 是 0.6, 風險的市場價格是 8.4%。
- (a) 在三種情境下此一藥品的 NPV 是多少?  
(b) 在投資 30 億前可再做一測試, 測試可以看清此一新藥是會完全失敗(Case 3)或者是成功(Case 1 或 2)。但不能分清是 Case 1 或 Case 2, 因此測試後若傾向成功, 則很成功與尚成功之機率各 50%。你願意付多少錢做此測試?  
(c) 若測試結果能斷定此藥是非常成功(Case 1)或尚屬成功與完全失敗(Case 2 或 3)。但無法區分 Case 2 或 Case 3。因此 Case 2 與 Case 3 在測試後出現的機率各有 50%。你願意付多少錢?  
(d) 對永信而言, 最有價值的訊息是什麼(比較(c)與(b))  
(e) 若測試費時兩年, (b)的答案是否會改變?
7. (20 points) 甲、乙兩公司是同類型的公司, 做同一行業, 有同樣的現金流量(每年均賺 1500 萬元, 直到永遠), 同樣的風險, 同樣的資本結構, 100%權益融資。權益之必要報酬率同樣是 15%。
- (a) 甲、乙兩公司之總價值是多少?  
(b) 乙公司之財務長(台大 MBA 畢業)注意到他的公司可以發行 2000 萬元之債券, 年息 10%。他認為公司若發行債券把籌到的錢買回乙公司之股票護盤(以庫藏股方式或透過子公司買回), 則股價會上揚, 股東會有好處。你是否同意他的觀點, 權益總值在發行債券後是值多少?  
(c) 若發行債券後乙公司之權益總值是 7000 萬元, 債權之市值仍然是 2000 萬元。你是乙公司的財務顧問(中山財管畢業)該怎麼做?  
(d) 若甲公司發行 5000 萬市值之新股, 用獲得的現金買年息 10%之債券。甲公司在新投資後的資產報酬率是多少?  
(e) 由於現金增資使甲公司之 EPS 下降 11%, 你看甲公司之股價是否會因每股盈餘被稀釋而使股價下跌?

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一、Let  $S$  be a mutually exclusive and collectively exhaustive state space,  $F$  be a family set collection which is composed of all the subsets of  $S$ . Then a  $\sigma$ -field  $F$  on the space  $S$  is called a sample space and is denoted by  $(S, F)$ .

Answer the following questions:

- 1.(10%) What is  $\sigma$ -field? How to define it?
- 2.(15%) Could  $F$  be a  $\sigma$ -finite? Suppose you are asked to tossing two coins one time, listing your  $\sigma$ -finite  $F$  on your answer sheet.

二、A monotone sequence of sets  $E_n$  may be increasing or decreasing. Answer the following questions:

- 1.(10%) How do you to define the supreme of  $E_n$ , and, the inferior of  $E_n$ ?
- 2.(20%) Prove a fundamental probability limit theory:

$$\text{pro} \left( \lim_{n \rightarrow \infty} E_n \right) = \lim_{n \rightarrow \infty} \text{pro}(E_n)$$

三、Suppose a stochastic time series  $\{\varepsilon_t\}_{-\infty}^{\infty}$  is white noise. Answer the following questions:

- 1.(5%) Derive its covariance generating function.
- 2.(5%) Compute its spectrum.
- 3.(5%) Figure that a white noise has a flat spectrum, indicating that all frequencies between  $-\pi$  and  $+\pi$  are equally important in accounting for its variance.

四、Consider the conditional distribution:

$$f(y|x) = (\alpha + \beta x)^{-1} \exp\left(-\frac{y}{\alpha + \beta x}\right), \quad y \geq 0, \quad 0 \leq x \leq 1$$

- 1.(5%) Find the conditional mean  $E(y|x)$ .
- 2.(10%) Find  $E(y)$  and  $Cov(x, y)$ .
- 3.(15%) Find  $Var_x[E(y|x)]$ ,  $E_x[Var(y|x)]$  and  $Var(y)$ .

[Hints]: 1. This is an exponential distribution with  $\lambda = (\alpha + \beta x)^{-1}$  and the mean of an exponential distribution with parameter  $\lambda$  is  $\lambda^{-1}$ .

2. The marginal distribution of  $x$  is uniform distribution.