一、 (50%)

Analyzing Stock Market Trends and Forecasting Stock Prices

Many financial analysts believe that stocks in an industry group tend to show similar sensitivity to the movement of the general market over time. According to the capital asset pricing model (CAPM) , one parameter often used as a measure of sensitivity is called a beta factor (β). Beta factors are computed as the relationship of the change in a stock's price to the change in the general market (in either direction) over a time period. Another parameter used in measuring a stock's performance is called an alpha factor (α). The α factor expresses how much the stock price would have moved, on the average, every month over time, assuming that the general market remained unchanged during that period. An analysis of computed α and β factors could provide important insights into stock performance for both industry group and specific companies. The following table shows α and β factors for the insurance industry (16 companies) and steel industry (13 companies) for the period of performance from 1980 to 1985. Each of these companies is listed on the NYSE. Based on the data, answer these questions:

a. Is there a linear relationship between α and β for the insurance group? For the steel group? (12.5%)
b. Do insurance stocks show a similar sensitivity to changes in the general market? (12.5%)
c. How could this relationship be used in forecasting the future direction of an industry group? (12.5%)
d. Is there a linear relationship between the α factors for the two industry groups? (12.5%)

Table: The α and ß Factors for the Insurance and Steel Industries, 1980-85

Insurance Industry			Steel Industry		
NYSE Symbol	α	ß	NYSE Symbol	α	ß
ACIG	4.73	.33	AS	-.31	.99
AET	.01	.76	BS	-.34	1.22
AML	1.30	.57	BNY	.71	.66
BWD	-1.73	1.16	CRS	.56	1.12
CI	-.45	1.17	COS	.70	.99
CHUB	1.15	.44	IK	.56	.77
GEC	2.33	1.44	KSC	.71	.71
GRN	1.15	.90	LTV	.42	1.91
FBH	-.35	.52	NII	-.57	1.32
LNC	.33	.87	NX	-.24	1.51
MLC	1.56	.59	RS	-.20	.83
MMC	.24	.67	X	-.23	1.14
TMK	.57	.66	WHX	-.47	1.05
TIC	.45	.80			
FG	.45	.72			

二、For any random variables X and Y, let ρ_{XY} be the correlation coefficient of X and Y. Prove that

1) $-1 \le \rho_{XY} \le 1$ (10%)

2) $|\rho_{XY}| = 1$ if and only if there exist numbers $a \neq 0$ and b such that $\Pr(Y = aX + b) = 1$. If $\rho_{XY} = 1$ then $a > 0$, and if $\rho_{XY} = -1$ then $a < 0$. (15%)

三、Let the demand for money be governed by

$$m_t = p_t + ky + u_t$$

where $Eu_t = 0 = Eu_t m_t$. Here m_t is the log of the money supply, p_t the log of the price level, and y the constant level of the log of real income. Assume that Em_t^2 and Ep_t^2 exist.

Suppose that a researcher attempts to verify the absence of money illusion in this economy by estimating

$$m_t = \alpha p_t + \text{constant} + \text{residual}_t,$$

by least squares, and testing whether $\alpha = 1$. In arbitrarily large samples, will this procedure lead him to conclude the truth, namely that $\alpha = 1$? If this procedure is flawed provide a better one and defend it. (25%)

個體經濟部份（50 分）

1. 假設社會上有兩類人，兩種商品。A 類人的原賦向量是 $(\omega_1^A, 0)$，B 類人的原賦向量是 $(0, \omega_2^B)$。A 類人共有 N_A 人，其效用函數為

$$u = \alpha \ln \chi_1 + (1-\alpha)\ln \chi_2 \text{ ，}$$

B 類人共有 N_B 人，其效用函數為

$$u = \beta \ln \chi_1 + (1-\beta)\ln \chi_2$$

請解出競爭均衡之交換價格。（15 分）

2. 設稻米未來的單位價格可能是\$4 或\$8，機率各是 1/2。令稻米的生產成本為 $c(q)=1/2q^2$。若淨利為 π，設農夫的效用函數為 $u(\pi) = \ln(10+\pi)$。

1）請導出農夫的最適產量 q^*。（10 分）

2）在 q^* 之下，若有期貨市場的存在，請問廠商在期貨市場出售之最低願售價格為何？（10 分）

3. 某人的效用函數是 $u(\omega) = \ln \omega$，他要選擇一個最適賭注 χ，去

$$\max \ \rho u(\omega+\chi) + (1-\rho)u(\omega-\chi)$$

1） 求解最適 χ^*。（5 分）

2）χ^* 與其主觀猜測 ρ 之間有何關係？（10 分）

88 總體經濟部份(共五十分)

1. Assume a money market equilibrium M=P•L(Y,i) where L(Y,i) stand for real demand for money. Derive the rate of money growth as functions of growth rates of P,Y and i. (14 分)

2. Assume an intertemporal consumption function $U(C_1, C_2) = u(C_1) + u(C_2)/(1+\rho)$, where $u' > 0, u'' < 0$ and $\rho > 0$ is called the subjective rate of time preference. Denoting wealth by Ω, the budget constraint is $C_1 + C_2/(1+r) = \Omega$, r is the interest rate.

a. Derive the sign of $dC_1/d\Omega$, assuming $dr = 0$

b. Derive the sign of dC_1/dr, assuming $d\Omega = 0$ (18 分)

3. Assume that there are separate consumption and investment goods industries with respective prices P_c and P_i. Labor demands in these two sectors depend on wages relative to output prices: $d^c = d_0^c + d_w^c(W/P_c), d^i = d_0^i + d_w^i(W/P_i)$ and $d = d^c + d^i$. Assume that labor supply depends on real wages relative to the prices of consumption goods, so that $s = s_0 + s_w(W/P_c)$, and that labor market equilibrium prevails: $d = s$.

a. How will a change is relative prices, P_i/P_c, affect real wages and employment? Interpret your results.

b. Is there still a classical dichotomy between the demand and supply sides of the economy? (18 分)

一、(1) Which of the following statements are correct? Why?

(20%)

(*a*) The value of a share equals the discounted stream of future earnings per share.

(*b*) The value of a share equals the present value of earnings per share assuming the firm does not grow, plus the net present value of future growth opportunities.

(*c*) The value of a share equals the discounted stream of future dividends per share.

(2) Under what conditions does r, a stock's market capitalization rate, equal its earnings-price ratio EPS_1/P_0?

二、(10%)

Mr. Cyrus Clops, the president of Giant Enterprises, has to make a choice between two possible investments:

	CASH FLOWS, THOUSANDS OF DOLLARS			
Project	C_0	C_1	C_2	IRR, Percent
A	−400	+241	+293	21
B	−200	+131	+172	31

The opportunity cost of capital is 9 percent. Mr. Clops is tempted to take B, which has the higher IRR.

(*a*) Explain to Mr. Clops why this is not the correct procedure.

(*b*) Show him how to adapt the IRR rule to choose the best project.

(*c*) Show him that this project also has the higher NPV.

三、(15%)

Here are book and market value balance sheets of the United Frypan Company:

Book				Market			
Net working capital	$ 20	Debt	$ 40	Net working capital	$ 20	Debt	$ 40
Long-term assets	80	Equity	60	Long-term assets	140	Equity	120
	$100		$100		$160		$160

Assume that MM's theory holds with taxes. There is no growth and the $40 of debt is expected to be permanent. Assume a 40 percent corporate tax rate.

(*a*) How much of the firm's value is accounted for by the debt-generated tax shield?

(*b*) How much better off will UF's shareholders be if the firm borrows $20 more and uses it to repurchase stock?

(*c*) Now suppose that Congress passes a law which eliminates the deductibility of interest for tax purposes after a grace period of 5 years. What will be the new value of the firm, other things equal? (Assume an 8 percent borrowing rate.)

四、(20%) Define each of the following theories in a sentence or simple equation.
(*a*) Interest-rate parity theory
(*b*) Expectations theory of forward rates
(*c*) Law of one price
(*d*) International capital market equilibrium (relationship of real and nominal interest rates in different countries)

五、(15%) (*a*) What is the formula for the value of a 2-year, 5 percent bond in terms of spot rates?
(*b*) What is the formula for its value in terms of yield to maturity?
(*c*) If the 2-year spot rate is higher than the 1-year rate, is the yield to maturity greater or less than the 2-year spot rate?

六、(20%) What is an efficient market? Six lessons can be drawn from an efficient market, they are (1) markets have no memory; (2) trust market prices; (3) there are no financial illusions; (4) the do-it-yourself alternative; (5) seen one stock, seen them all; and (6) reading the entrails. Please explain the meanings of the six lessons respectively.

國立中山大學八十八學年度碩博士班招生考試試題

科　目：　數學　（財務管理學系博士班）　　共 / 頁 第 / 頁

1 (a) Let U be an uniformly distributed random variable on $[0, 1]$. Show that, for $\lambda > 0$,

$$\frac{-1}{\lambda}\log(U)$$

is an exponential random variable with parameter λ. (10%)

(b) Let X_0 and X_1 be independent exponentially distributed random variables with parameters λ_0 and λ_1 so that

$$P(X_i \le t) = 1 - e^{-\lambda_i t}, \quad \text{for } t \ge 0,\ i = 0, 1$$

Show that $\min(X_0, X_1)$ is exponentially distributed with parameter $\lambda_0 + \lambda_1$. (10%)

2 Let $\{B(t) : t \ge 0\}$ be a standard Brownian motion. We define

$$X = \int_0^t B(s)ds,$$
$$Y = \int_0^t e^{\alpha(t-s)}dB(s),$$
$$Z(t) = \exp(\theta B(t) - \theta^2 t/2),$$

where α, θ are real constants.

(a) Compute Var(X). (10%)

(b) Compute Var(Y). (10%)

(c) Show $\{Z(t) : t \ge 0\}$ is a martingale (A stochastic process $\{Z(t) : t \ge 0\}$ is said to be a martingale process if, for $s < t$, $E[Z(t)|Z(u), 0 \le u \le s] = Z(s)$.) (10%)

3. (30%) Define $\max[a, b] = \begin{cases} a & \text{if } a > b \\ b & \text{if } a \le b \end{cases}$ (a) Prove that max[x,0] is a convex function of x. The precise geometric characterization of a convex function is as follows. If we pick any pair of points M and N and join them by a straight line, the line MN must lie either above the curve, or along (coinciding with) the curve. Or, equivalently, $\alpha_1 f(x_1) + \alpha_2 f(x_2) \ge f(\alpha_1 x_1 + \alpha_2 x_2)$, where $\alpha_1 + \alpha_2 = 1$ and $0 \le \alpha_k \le 1$, k=1,2. (b) Prove that

$$\max\left[\sum_{k=1}^{N} \alpha_k x_k - c, 0\right] \le \sum_{k=1}^{N} \alpha_k \max[x_k - c, 0]$$ where c is a real constant.

$$\sum_{k=1}^{N} \alpha_k = 1$$

4. (20%) There are two possible states of the world at t=1. Probability of state 1= 0.4 Investments 1 and 2 have the following possible payoffs: (Payoffs can be thought of as the value of your "position", or investment.)

	State 1	State 2
Investment 1	1.5	0.5
Investment 2	0.5	1

(a) Are these two investments independent of each other ?

(b) You are at t=0 now and have \$100. Your utility (that is, satisfaction index) function is $E_{t=0}(u(\widetilde{x}))=E_{t=0}(\log(\widetilde{x}))$. What is your optimal allocation of your \$100 in Investments 1 and 2 ?