工程數學（單選題；每題 5 分，共 100 分）

1. Which one is the solution of $y' + y = x^2 - 2$

(A) $y = ce^{-x} + x^2 - 2x$ (B) $y = a\cos x + b\sin x$ (C) $y = e^x + ax^2 + bx + c$

(D) $y = e^{-x}(a\cos x + b\sin x)$ (E) None

2. Which one is the solution of $y''' = e^x$

(A) $y = ce^{-x} + x^2 - 2x$ (B) $y = a\cos x + b\sin x$ (C) $y = e^x + ax^2 + bx + c$

(D) $y = e^{-x}(a\cos x + b\sin x)$ (E) None

3. Which one in the following differential equations is the nonhomogeneous equation?

(A) $(1 - x^2)y'' - 2xy' + 6y = 0$ (B) $y'' - y = 0$ (C) $y'' + 4y = e^{-x}\sin x$

(D) $x(y''y + y'^2) + 2y'y = 0$ (E) None

4. Which one in the following differential equations is nonlinear?

(A) $(1 - x^2)y'' - 2xy' + 6y = 0$ (B) $y'' - y = 0$ (C) $y'' + 4y = e^{-x}\sin x$

(D) $x(y''y + y'^2) + 2y'y = 0$ (E) None

5. Which one is the solution of the initial value problem $y'' - y = 0$ with $y(0) = 4$ and $y'(0) = -2$?

(A) $y = 3x^2 - 2x + 4$ (B) $y = e^x + 3e^{-x}$ (C) $y = 2e^{2x} - \sin x + 2$

(D) $y = 4e^{2x} - 2\sin x + 2x^2$ (E) None

6. If matrices A and B are defined as $A = \begin{bmatrix} 9 & 3 \\ -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 \\ 2 & 5 \end{bmatrix}$, then the product $C = AB$ is

(A) $\begin{bmatrix} 15 & -21 \\ -2 & 8 \end{bmatrix}$ (B) $\begin{bmatrix} 15 & -2 \\ -21 & 8 \end{bmatrix}$ (C) $\begin{bmatrix} -15 & 2 \\ -21 & -8 \end{bmatrix}$ (D) $\begin{bmatrix} 15 & -21 \\ 21 & 18 \end{bmatrix}$ (E) None

7. 7. If matrices A and B are defined as $A = \begin{bmatrix} 9 & 3 \\ -2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -4 \\ 2 & 5 \end{bmatrix}$ and $C = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$, then the product $C = -A^T BC$ is (A) $\begin{Bmatrix} -21 \\ 8 \end{Bmatrix}$ (B) $\begin{Bmatrix} 21 \\ 8 \end{Bmatrix}$ (C) $\begin{Bmatrix} 21 \\ 6 \end{Bmatrix}$ (D) $\begin{Bmatrix} 36 \\ 6 \end{Bmatrix}$ (E) None

8. Which one is the eigen value solution pair of the matrix $A = \begin{bmatrix} -40 & 40 \\ -16 & 12 \end{bmatrix}$?

(A) $(\lambda_1 = 2; \lambda_2 = 4)$ (B) $(\lambda_1 = -2; \lambda_2 = -0.8)$ (C) $(\lambda_1 = -2; \lambda_2 = -4)$ (D) $(\lambda_1 = 2; \lambda_2 = 0.8)$ (E) None

9. Consider A, B and C are nxn matrices, which one in the following matrix operations is wrong?

(A) $(AC)^T = C^T A^T$ (B) $B(AB)^{-1} = A^{-1}$ (C) $(AC)^{-1} = A^{-1}C^{-1}$ (D) *in general*, $AB \neq BA$ (E) None

10. Let $v(x,y,x) = 3xz\vec{i} + 2xy\vec{j} - yz^2\vec{k}$ be a differentiable vector function, then the divergence of the vector $\nabla \cdot v$ is

(A) $3z\vec{i} + 2x\vec{j} - 2yz\vec{k}$ (B) $3z\vec{i} + 2x\vec{j} - yz^2\vec{k}$ (C) $3z + 2x - yz^2$ (D) $3z + 2x - 2yz$ (E) None

11. The Euler's formula is （A） $e^x = \sum_{n=0}^{\infty} \frac{1}{n!}x^n$ （B） $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$ （C） $e^{ix} = \cos(x) + i\sin(x)$

（D） $y'' + p(x)y' + q(x)y = 0$ （E） None

12. Determine the value of A so that the following equation is "exact"

$$Ay^2 + ye^{xy} + \left(4xy + xe^{xy} + 2y\right)y' = 0$$

（A）A=2 （B）A=5 （C）A=11 （D）A=18 （E） None

13. The Laplace Transform of the function $y(t) = 4t\sin 2t$ is （A） $Y(s) = \frac{16s}{(s^2+4)^2}$

（B） $Y(s) = \frac{2}{(s+2)^2}$ （C） $Y(s) = \frac{s^2-2}{s^2(s^2+2)-4s}$ （D） $Y(s) = \frac{s}{2s^2-4}$ （E） None

14. The inverse Laplace Transform of the function $Y(s) = \frac{5}{(s+7)^2}$ is （A） $y(t) = 5\cos(7t)$

（B） $y(t) = 5te^{-7t}$ （C） $y(t) = 5\sin(7t)$ （D） $y(t) = 5t\sin(7t)$ （E） None

15. The Lapace transform of the initial value problem, i.e. $y''+y=t;\ y(0)=1, y'(0)=0$, is

(A) $Y(s)=\frac{s}{s^2+1}$ (B) $Y(s)=\frac{1}{s^2(s^2+1)}$ (C) $Y(s)=\frac{1}{s^2(s^2+1)}+\frac{s}{s^2+1}$

(D) $Y(s)=\frac{1}{s^2(s^2+1)}-\frac{s}{(s^2-1)}$ (E) None

16. The cross product $\vec{F}\times\vec{G}$ of vectors $\vec{F}=\vec{i}+2\vec{j}-3\vec{k}$ and $\vec{G}=-2\vec{i}+\vec{j}+4\vec{k}$ is

(A) $2\vec{i}+11\vec{j}+5\vec{k}$ (B) $5\vec{i}+11\vec{j}+2\vec{k}$ (C) $2\vec{i}+5\vec{j}+11\vec{k}$ (D) $11\vec{i}+2\vec{j}+5\vec{k}$ (E) None

17. Three vectors are $\vec{F}=\vec{i}-\vec{j}-\vec{k}$, $\vec{G}=-3\vec{i}+4\vec{j}+6\vec{k}$ and $\vec{H}=-2\vec{i}-4\vec{j}+2\vec{k}$. The corresponding product of $\vec{H}\cdot(\vec{F}\times\vec{G})$ is (A) 2 (B) 5 (C) 11 (D) 18 (E) None

18. The first two Fourier coefficients of following equation are:

$$f(x)=\begin{cases}-4, & -\pi\le x\le 0\\ 4, & 0<x<\pi\end{cases}$$

(A) $\left(\frac{16}{\pi}\right)\left(\cos x+\frac{1}{3}\cos 3x+\cdots\right)$ (B) 4 (C) $\left(\frac{16}{\pi}\right)\left(\sin x+\frac{1}{3}\sin 3x+\cdots\right)$

(D) $\left(\frac{16}{\pi}\right)\left(\sin x+\frac{1}{2}\sin 2x+\cdots\right)$ (E) None

19. The curl $\nabla\times\vec{F}$ of the vector $\vec{F}=y\vec{i}+2xz\vec{j}+ze^x\vec{k}$ is

(A) e^x (B) 0 (C) $2x\vec{i}-ze^x\vec{j}+2\vec{k}$ (D) $-2x\vec{i}-ze^x\vec{j}+(2z-1)\vec{k}$ (E) None

20. Which of following differential equations is a linear equation

(A) $y''+4xy'=\cos y$ (B) $y'+x^2y\sin x=3x$ (C) $y''+\sin(y)y'+y^2=2$

(D) $yy''+xy'=3y$ (E) None