國立中山大學九十三學年度博士班招生考試試題

科目：工程數學【機電系】

[1] Interpret and solve the following initial-value problem. (20%)

$$0.2\frac{d^2x}{dt^2}+1.2\frac{dx}{dt}+2x=5\cos 4t$$

at t = 0, x = 0.5 & $\frac{dx}{dt}=0$

[2] Using the Laplace transform, solve $\frac{dy}{dt}-3y=e^{2t}$ subject to y(0) = 1. (20%)

[3] Find the eigenvalues and eigenvectors of (20%)

$$[A]=\begin{bmatrix}9 & 1 & 1\\ 1 & 9 & 1\\ 1 & 1 & 9\end{bmatrix}$$

[4] Find the directional derivative of $F(x, y, z) = xy^2 - 4x^2y + z^2$ at (1, -1, 2) in the direction of $6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. Also, find the maximum and minimum values of the directional derivative of F at (1, -1, 2). (20%)

[5] Evaluate

$$\oint_C \frac{z+1}{z^4+4z^3}dz$$

where C is the circle $|z| = 1$. (20%)