國立中山大學九十一學年度博士班招生考試試題

科目：**工程數學【機電系】**

Prob. #1（25%）

(1) Find the Laplace transform of $(t^3-3t+2)e^{-2t}$.

(2) Calculate the inverse Laplace transform of $\dfrac{1}{s(s-4)^2}$.

Prob. #2（25%）

(1) Let **A** be a constant vector, and let $\mathbf{R}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$. (a) Prove that

$\nabla(\mathbf{R}\cdot\mathbf{A})=\mathbf{A}$ (b) Prove that $\nabla\cdot(\mathbf{R}-\mathbf{A})=3$. (15%)

(2) Find a unit normal vector **n** of the cone of revolution $z^2=4(x^2+y^2)$ at the point P: (1, 0, 2). (10%)

Prob. #3（25%）

Solve $y''+y=xe^{2x}$.

Prob. #4（25%）

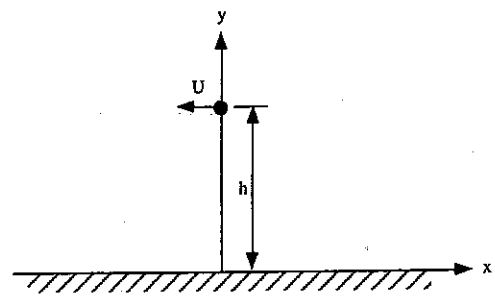
Integrate $\displaystyle\int_C \frac{\cos z}{z^3+z}dz$ over the given curve: $C:|z|=2$.

1 The pressure drop, Δp, for steady, incompressible turbulent flow with viscosity μ and density ρ in a horizontal round pipe of diameter D can be written in functional form as

$$\Delta P = F(V, D, l, \varepsilon, \mu, \rho)$$

where V is the average velocity, l is the pipe length, and ε is a measure of the roughness of the pipe wall, Please find the dimensional groups of this flow. 10%

2 An airplane is just leaving the ground with velocity U and a height h. In this case the airplane can be simulated as a moving circulation with velocity U and circulating strength G located at a distance h above a wall as shown in the figure below. Calculate the pressure acting on the ground and the velocity distribution. 20%



3 For a laminar flow past a flat surface, the velocity distribution in the x-direction u through the boundary layer can be approximated by a third order polynomial equation,

$$u/U = a_o + a_1(y/\delta) + a_2 (y/\delta)^2 + a_3 (y/\delta)^3$$

Please write down the boundary conditions of this case to determine the coefficiencies : a_0, a_1, a_2, and a_3. Then find the integral boundary layer solution for this flow and use the result to evaluate the local shear stress and friction drag coefficient. 20%

Note : The Integral equation for laminar flow can be written as

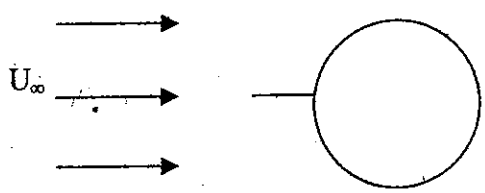
$$\frac{\partial}{\partial x}\int_0^\delta u^2 dy - U(x)\frac{\partial}{\partial x}\int_0^\delta \rho u dy = -\frac{\delta}{\rho}\frac{dP}{dx} - \nu\left(\frac{\partial u}{\partial y}\right)_o$$

where U(x) is freestream velocity, y is the coordinate in the y direction, ρ is the density of fluid, δ is the boundary layer thickness, ν is the kinematic viscosity of the fluid.

4. (20%) Consider the case of viscous fluid flow inside a duct with hydraulic diameter D_h. Answer the questions with short explanations.

   (a) What is the situation in which viscous dissipation becomes significant?
   (b) What is the situation in which streamwise diffusion can be negligible?
   (c) Whether air or water usually has higher viscous dissipation?
   (d) For identical velocity distribution, whether air or water has longer entrance length for temperature getting developed.
   (e) Why does liquid metal have high heat transfer coefficient?

5. (16%) As illustrated in the figure of cross-flow over a cylinder, with sufficient large Reynolds number (say Re_D>1000) using the given coordinates (x,y).



   (a) Write down the boundary layer equations assuming the flow is 2-D, incompressible constant properties and negligible viscous dissipation.

   (b) Assuming no injection or suction occurred on the cylinder surface, should one starts from the stagnation point (x=0) and travels along x-direction, where is the region in which those equations you have given in part (a) are valid? Where is not? Explain.

6. (14%) What role does the Knudsen number play in micro-channel flow? Suggest how to approach a problem with flow and heat transfer in a micro-channel.

國立中山大學九十一學年度博士班招生考試試題

科目：固體力學【機電系乙組】

1. Functional $I = \int_0^1 \left[\frac{1}{2}\left(\frac{d^2W}{dx^2}\right)^2 + 5W\right] dx - 2\frac{dW(1)}{dx}$ and

$$W(0) = \frac{dW(0)}{dx} = 0$$

Try to derive the corresponding governing equation and the natural boundary conditions. (20%)

2. In finite element method, how do you
(a) impose the specified essential boundary conditions
(b) calculate the nodal stresses for displacement-based formulations (15%)

3. Give detailed descriptions of the following terms
(a) natural frequency
(b) eigenvalue
(c) momentum
(d) principal stress (15%)

4. A matrix-valued quantity a_{ij} $(i,j=1,2,3)$ is given as follows:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 2 \\ 0 & 2 & 3 \end{pmatrix}$$

What are the values of (a) a_{ii}, (b) $a_{ij}a_{ij}$, (c) $a_{ij}a_{jk}$ when $i=1$, $k=1$ and $i=1$, $k=2$. (15%)

5. Express the following set of equations in a single equation using index notation.

$$e_{xx} = \frac{1}{E}[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})]$$

$$e_{yy} = \frac{1}{E}[\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx})]$$

$$e_{zz} = \frac{1}{E}[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})]$$

$$e_{xy} = \frac{1+\nu}{E}\sigma_{xy}, \quad e_{yz} = \frac{1+\nu}{E}\sigma_{yz}, \quad e_{zx} = \frac{1+\nu}{E}\sigma_{zx}$$ (15%)

6. In plane stress problems, the stress components $\sigma_{13} = \sigma_{23} = \sigma_{33} = 0$, and the remaining ones are functions of y_1 and y_2 only. If the body force components and body moments are equal to zero, find a relation between the nonzero stress components and a single function $\phi(y_1,y_2)$, so that all equilibrium equations are satisfied. (20%)

國立中山大學九十一學年度博士班招生考試試題

科目：自動控制【機電系丙組】

1. (50%) 一系統之open-loop transfer為$s^2/(s^2+4s+8)$，請利用Root-locus方法設計二種不同結構之控制器，讓其系統之安定時間小於0.2秒

2. (12%) Describe the physical meaning of phase margin. You should consider the interaction between time-delay and system stability.

3. (12%) Describe the relationship between the bandwidth and the speed of response of the system.

4. (13%) Is an ideal differentiator a low pass filter or a high pass filter? Based on your answer, determine whether an ideal differentiator is sensitive to noise or not.

5. (13%) Can you determine the frequency response for a nonlinear system? Why?

國立中山大學九十一學年度博士班招生考試試題

科目：機械設計與製造【機電系丁組】

1. 說明 (a) 車削 (Turning), (b) 銑削 (Milling), (c) 磨削 (Grinding), (d) 研光 (Lapping), (e) 拋光 (Polishing), (f) 機械化學拋光 (MCP), (g) 化學機械拋光 (CMP) 等加工原理及特徵 (含加工精度等級)。(35%)
2. 說明 (a) 共晶 (Eutectic), (b) 包晶 (Peritectic), (c) 偏晶 (Monotectic) 等反應過程。(15%)
3. 三角形 $A^1B^1C^1$, $A^1=(0, 0)$, $B^1=(1, 2)$, $C^1=(2, 0)$, 經一 2-D 轉換矩陣 $[T]_{3\times3}$ 轉換至 $A^2B^2C^2$ 的新位置：$A^2=(2, 5)$, $B^2=(4, 0)$, $C^2=(5, 2)$。求出這個矩陣 $[T]_{3\times3}$ (15%), 並說明這轉換是否為 Affine 轉換 (10%)。
4. 怎樣設計以平面四桿機構來導引一剛體在兩個指定位置間的運動？如增至三個位置又如何？又最多可以達到幾個位置？(25%)