（1）

Find a particular solution for the following equation.

$$y'' + 3y' + 2y = 10e^{3x} + 4x^2$$

(25%)

（2）（25%）

(a)

Find the poles of $f(z) = \dfrac{1}{(z^3 - 1)^2}$,

where z is a complex variable.

(b)

Integrate $f(z)$ in the counterclockwise sense around the circle $|z - 1| = 1$.

You may need the following theorem for the calculation.

Theorem: If C is a closed curve, and if $f(z)$ is analytic within and on C except at a finite number of singular points in the interior of C, then

$$\int_C f(z)\,dz = 2\pi i (r_1 + r_2 + \cdots + r_n)$$

where $r_1, r_2, \ldots, r_n$ are the residues of $f(z)$ at its singular points within C.

（3）

Please determine the eigenvalues and the corresponding eigenvectors (with norm of each eigenvector equals to one) for the following matrix.

$$\begin{bmatrix} 200 & 100 & 300 \\ 100 & 0 & 0 \\ 300 & 0 & 0 \end{bmatrix}$$ (25%)

（後面有題）

(4)

Please solve the following P.D.E. for F(x,y) :

$F_{xx} + F_{yy} = 0$ on region R
$F_x = y$ on x =a and x = -a
$F_y = -x$ on y =b and y = -b

(25%)
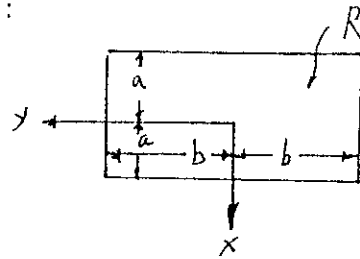
15%

1.(a) What is a fluid ?

(b) List the three basic dimensions used for a cold jet flow
(at room temperature) and the corresponding units in SI system.

(c) What is newtonian fluid? Give two examples

(d) Under what condition the pressure at a point is independent of direction.

(e) Determine the streamlines for the two-dimensional steady flow

$$\vec{V} = x\vec{i} - y\vec{j}$$

8%

2. The three components of velocity in a flow field are given by

$$u = x^2 + y^2 + z^2$$
$$v = xy + yz + z^2$$
$$w = -3xz - z^2/2 + 4$$

(a) Determine the volumetric dilatation rate and interpret the results.

(b) Determine an expression for the vorticity vector. Is this an irrotational flow field?

8%

3. An incompressible, viscous fluid is placed between horizontal, infinite, parallel places as is shown in Fig.3. The two plates move in opposite directions with constant velocities,U1 and U2 as shown. The pressure gradient in the x direction is zero and the only body force is due to the fluid weight. Use the Navier-Stokes equations to derive an expression for the velocity distribution between the plates. Assume laminar flow.

(a) Reduce the momentum equation $\rho \dfrac{d\vec{V}}{dt} = -\nabla p + \rho\vec{g} + \mu\nabla^2 V$ for this case, write down the differential equation for u(y)

(b) Write down the boundary condition

(c) Solve for the velocity

(d) Calculate the shear stress at each wall
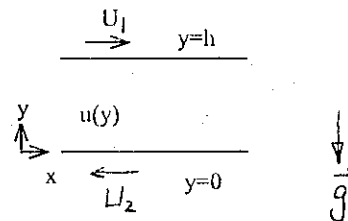
10%　4. When the wind blows over a chimney, vortices are shed into the wake. The frequency of vortex shedding f depends on the chimney diameter D, its length L, the wind velocity V, and the kinematic viscosity of air ν.

(a) Express the nondimensional shedding frequency in terms of its dependence on the other nondimensional groups.

(b) If a 1/10 scale model were to be tested in a wind tunnel and fully dynamic similarity was required:

    (i)　What air velocity would be necessary in the wind tunnel? If the wind velocity experienced by the full scale chimney is 1 m/s.

    (ii)　If the shedding frequency of the model test is measured to be 40Hz, what would be the shedding frequency generated by the full scale chimney?

9%　5. The results of a wind tunnel test to determine the drag on a body (see fig.) are summarized below. The upstream (section(1)) velocity is uniform at 30 m/s. the static pressures are given by $p_1 = p_2 = 1$ atm. The downstream velocity distribution, which is symmetrical about the centerline, is given by

$$u = 30 - 10(1 - |y|) \qquad |y| \le 1\text{m}$$
$$u = 30 \qquad\qquad\quad |y| > 1\text{m}$$

where u is the velocity in m/s and y is the distance on either side of the centerline in feet. Assume that the body shape does not change in the direction normal to the paper. Calculate the drag force (reaction force in x direction) exerted on the air by the body per unit length normal to the plane of the sketch.
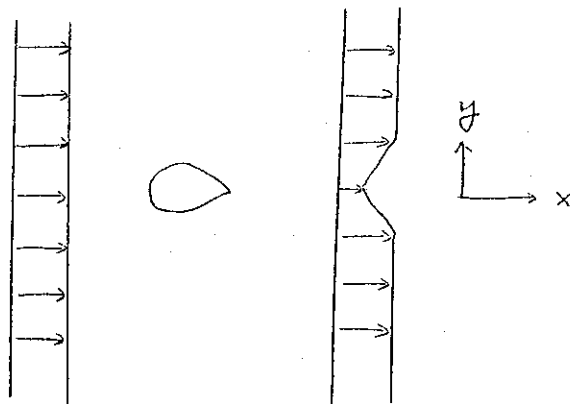
6. For a uniform flow over a flat plate, plot the local heat transfer coefficient along the plate (i.e. h-x diagram) and discuss the shape of the curve that you have drawn. (14%)

7. For a flow across a circular cylinder, plot the local Nusselt number, Nu_D, along the cylinder surface (i.e. Nu_D-θ diagram) for both Re=5×10^5 and 5×10^4. Discuss the shape of the curve that you have drawn. (15%)

8. Define a thermally fully developed laminar pipe flow. For thermally fully developed laminar pipe flow, prove that the Nusselt number (Nu) is constant, derive the relation between $\dfrac{\partial T}{\partial x}$ and $\dfrac{dT_m}{dx}$ for both constant wall heat flux and constant wall temperature cases. T_m is the bulk mean fluid temperature. What are the values of the Nusselt number , Nu_D , for both cases? (15%)

9. Define each of the following numbers, and state their physical interpretation.(6%)
   (1) Prandtl number, Pr
   (2) Grashof number, Gr
   (3) Peclet number, Pe

1. Derive the differential equations of equilibrium and describe the boundary conditions of the general three-dimensional elasticity problems.(15%)

2. In the case of two-dimensional problems, two types of stress distributions, namely, plane stress and plane strain are possible. Explain the plane stress and plain strain problems.(15%)

3. Show that

$$\phi = \frac{q}{8c^3}\left[x^2\left(y^3 - 3c^2y + 2c^3\right) - \frac{1}{5}y^3\left(y^2 - 2c^2\right)\right]$$

is a stress function, and find what problem it solves when applied to the region included in $y = \pm c$, $x = 0$, on the side x positive. (20%)

4. What is meant by the symbols a_i, a_j, a_{ij}, a_{ijk}, and $a_{ijk\ell}$? (10%)

5. What are the general stress-strain relationships (Hooke's Law) for homogeneous and isotropic materials due to tri-axial state of stress at elevated temperature? (10%)

6. Please list four failure criteria and make brief explanation for each criterion. (15%)

7. Metals are considered as ideal materials, i.e., homogeneous and isotropic, what are their independent material constants? How many independent material constants do nonhomogeneous and anistropic materials have? (15%)

(1). (25%)Consider a control system that has time delay at output. Please discuss the effect of this time delay on the control performance in details.

(2). (25%)Please derive and discuss the Nyquist stability criterion.

(3). (25%)A unity-feedback system has $KGH(s) = K(s^2+1)/[s(s + 2)]$

   (a). Concerning the root locus, when $K = \infty$, $s =$ __(i)__.

   (b). The breakaway point on real axis is $s =$ __(ii)__.
   (c). The angles of arrival to reaching the zeros are __(iii)__ (in degrees).

   (d). When damping ratio, $\zeta = 0.707$, is required, then the dominant poles are at $s =$ __(iv)__, when $K =$ __(v)__.

(4). (25%)A system is shown as follows with time constants T_1, $T_2 > 0$:

$$G(s)$$

R(s)   +   K $1/[s(T_1s + 1) (T_2s + 1)]$ C(s)

   −
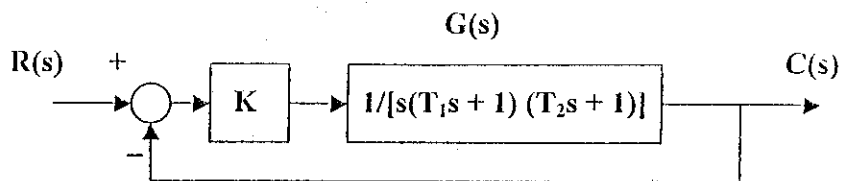
For a real small K, and a real large K, respectively, please discuss the open-loop and closed-loop system stability.

1. 試述高速切削加工之優點及其未來展望。　　　　　　(25%)

2. 試述機械與機電在設計及製造領域上之相關性。　　(25%)

3. 以相同的正多邊形所圍成的多面體稱為正多面體 (Regular polyhedron)，像是正立方體 (Cube) 即為其中之一種。

   A. 以三視圖表示出除正立方體以外之任一正多面體。如其邊長為1，則此多面體之表面積與體積各為何？　　(20%)

   B. 今需製作數件此多面體之模型以供教學使用。請提供初步之設計考量（含材料，尺寸，加工方式，夾具等）以供外包之工廠製作。　　　　　　　　　　　　(30%)