國立中山大學八十七學年度碩博士班招生考試試題

科 目：工程數學

1. (15%) Find the general solution of equation

$$x^2 y'' - 4xy' + 6y = 36\ln(x);\ \ y(1) = 2,\quad y'(1) = 3.$$

2. (20%) Solve

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},\ \ 0 < x < L,\ t > 0$$

Subject to

$$u(0,t) = 0,\quad u(L,t) = 0,\quad t > 0$$
$$u(x,0) = f(x),\quad 0 < x < L$$

3. (8%) (a) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 3 & 3 \\ 1 & 5 \end{bmatrix}$$

(12%) (b) Solve the system $X' = AX + G$, where A is given in (a), and

$$G = \begin{bmatrix} 8 \\ 4e^{3t} \end{bmatrix}.$$

4. (8%) (a) Find the Laplace transform of the function

$$f(t) = \begin{cases} 0 & if \quad 0 \le t < 3 \\ t & if \quad\quad t \ge 3 \end{cases}.$$

(12%) (b) Use (a) to solve the initial value problem

$$y'' + 4y = f(t);\qquad y(0) = y'(0) = 0.$$

5. (15%) Evaluate $\int_0^{2\pi} \frac{\cos(\theta)}{1 + 0.25\cos(\theta)} d\theta$.

6. Let **A** be a constant vector, and let $\mathbf{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

(5%) (a) Prove that $\nabla(\mathbf{R}\cdot\mathbf{A}) = \mathbf{A}$

(5%) (b) Prove that $\nabla\cdot(\mathbf{R} - \mathbf{A}) = 3$.

1.

20% (a) Explain Eulerian description and Lagrangian description

(b) What is Newtonian fluid? what is the unit of viscosity coefficient μ in the SI system?

(c) Determine the dimensions of the coefficients A and B which appear in the dimensionally homogeneous equation

$$\frac{d^2x}{dt^2} + A\frac{dx}{dt} + Bx = 0$$

where x is a length and t is time

(d) Give the restrictions on the Bernoulli equation

$$p + \frac{1}{2}\rho V^2 + \rho gz = \text{constant}$$ along a streamline

10% 2. A two-dimensional velocity field:

$$\vec{V} = x^2\vec{i} + (-2xy + 4x)\vec{j}\ \text{m/s} \qquad \text{(density= constant)}$$

Is this an irrotational flow? Does it satisfy continuity? If so, what is the stream function ψ ?

20% 3. A viscous fluid flows between two fixed parallel plates, as shown below. The velocity pattern is u=u(y) only, and the pressure drops linearly, p=-Cx+D. Assume the flow is laminar, two-dimensional, and incompressible.

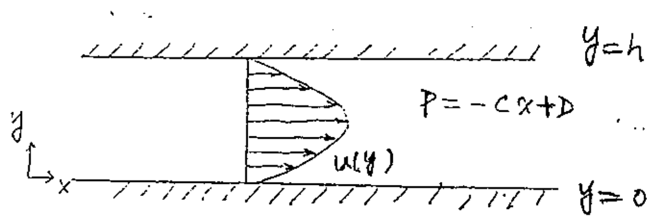
(a) Show that the velocity in the y-direction v=0

(b) Reduce the momentum equation $\rho\frac{d\vec{V}}{dt} = -\nabla p + \rho\vec{g} + \mu\nabla^2\vec{v}$ to fit this case, write down the differential equation for u(y)

(c) Write down the boundary conditions

(d) Solve for u=u(y)

(e) Calculate the shear stress at each wall



4.

15% (a) What is Fourier's law of thermal conduction?

(b) What is Stefan-Boltzmann Law?

(c) In deriving the heat conduction equation, what conservation you have to consider?

10% 5 Explain the following terms.

(a) diffuse surface

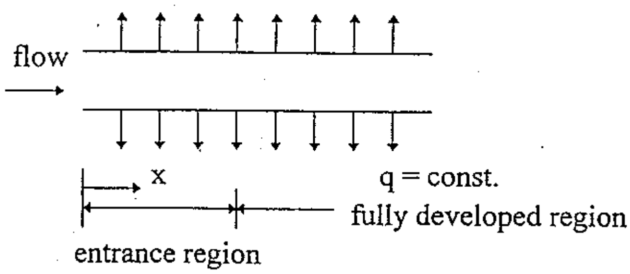
(b) black surface

(c) gray surface

(give the meaning of each symbol you use)

10% 6 Sketch the distribution of bulk temperature and surface temperature of a tube

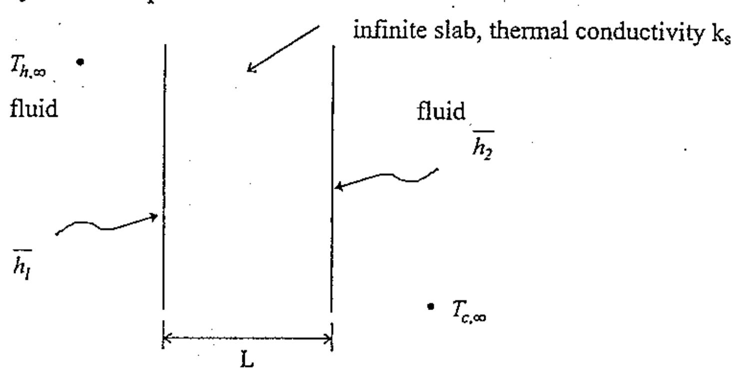
(a) in case the heat flux out of the tube is constant.



(b) in case the surface temperature is constant and the entrance temperature of the flow is higher than the surface temperature.

15% 7 Assume $\bar{h}_1 = \bar{h}_2 = \bar{h}$, no heat source, steady state. Sketch the temperature from $T_{h,\infty}$ to $T_{c,\infty}$, for the follow cases.

(1) $\frac{\bar{h}L}{k_s} << 1$, (2) $\frac{\bar{h}L}{k_s} >> 1$, k_s=constant, (3) $\frac{\bar{h}L}{k_s} >> 1$, k_s increases linearly with temperature.



國立中山大學八十七學年度碩博士班招生考試試題

科　目：固体力學

[1] A displacement field (u_1, u_2, u_3) is defined for all (x_1, x_2) and for $x_3 \geq 0$ by the relations

$$u_1 = -x_3 x_1,\quad u_2 = -x_3 x_2,\quad u_3 = \frac{1}{2}(x_1^2 + x_2^2) + \frac{1}{2} A x_3^2 \qquad (15\%)$$

where A is a constant. Determine the value of A which ensures that this field is a physically possible continuous field (an admissible field).

[2] Please determine whether the following stress field is admissible in an elastic body when the body forces are negligible:

$$\sigma_{xx} = -2x^2 - 3y^2 - 5z$$
$$\sigma_{yy} = -2y^2 + 7$$
$$\sigma_{zz} = 4x + y + 3z - 5 \qquad (15\%)$$
$$\sigma_{zx} = -3x + 2y + 1$$
$$\sigma_{yz} = 0$$
$$\sigma_{xy} = z + 4xy - 6$$

[3] Please explain what are "plane stress solution", "plane strain solution" and "generalized plane stress solution"? (20%)

[4] Please explain what are "Tresca yield criteria" and "von Mises yield criteria"? (15%)

[5] Given the following state of stress at a point

$\sigma_{xx} = 0$, $\sigma_{xy} = \sigma_{yx} = -800$ psi, $\sigma_{yy} = 300$ psi, $\sigma_{zz} = 0$, $\sigma_{xz} = \sigma_{zx} = 0$, $\sigma_{yz} = \sigma_{zy} = -500$ psi,

please determine the principal directions and the maximum shear stress.

(35%)

(橫書式) 國立中山大學八十七學年度碩博士班招生考試試題

科 目：自動控制 機械工程學系 共 1 頁 第 1 頁

1.(30%) Consider the following linear time-invariant system

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

with $\mathbf{x}(t_0) = \mathbf{x}_0$, where **A**, **B**, **C**, **D** are n x n, n x p, q x n, and q x p constant matrices. Find the time-domain solution for the state vector $\mathbf{x}(t)$, and the output vector $\mathbf{y}(t)$.

2.(30%) Describe the following terminologies:

(a). Lyapunov function

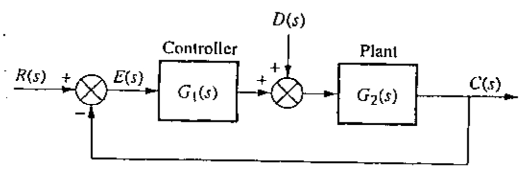
(b). pole placement

(c). asymptotical stability

(d). bounded-input bounded-output stability

(e). estimator (or observer)

3. (20%) Consider the following system, please apply the final value theorem to get the steady-state error due to system input R(s), and disturbance D(s), respectively.



4. (20%) Plot the s-plane pole locations for second-order control systems

$$G(s) = \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2)$$

which in overdamped , critically damped, underdamped , and undamped cases. And then, draw their step responses, respectively. (ζ is damping ratio, and ω_n is natural frequency)

國立中山大學八十七學年度碩博士班招生考試試題

科 目：機械設計與製造　　機械工程學系　共 / 頁 第 / 頁

I. 機械設計 (50%)

(1)孔(Hole)與軸(Shaft)的配合有哪幾種？ (4%)

(2)試說明機械元件受靜態負荷(Static loading)時，常以失效理論(Failure theory)為依據判定元件安全性的原因。 (6%)

(3)列出常用的機械元件失效理論。 (8%)

(4)試說明機械元件的強度(Strength)與應力(Stress)。 (6%)

(5)請說明疲勞(Fatigue)是高速機械中元件破壞主因的理由。(5%)

(6)實驗室所得到的元件疲勞強度(Se')並非機械元件的實際疲勞強度(Se)，請說明Se'與Se兩者間的關係。 (8%)

(7)齒輪常見的齒型有漸開線與擺線，試比較兩者的優缺點。 (8%)

(8)寫出五種常用的機械元件。 (5%)

II. 機械製造 (50%)

(1) Why single-crystal turbine blades are more reliable than conventional casting turbine blades? (10%)

(2) Give schematic illustrations to show the effect of roll radius on the type of residual stresses developed in flat rolling. (10%)

(3) Why knowledge of the temperature rise in cutting is important? (10%)

(4) Describe regenerative chatter (a type of self-vibration) in machining. (10%)

(5) List the broad categories of processing methods for materials. (10%)