The problem set is divided into two parts. The first part includes problems 1, 2, 3, and 4, and the second part includes problems 5, 6, 7, and 8.

1. (15%) 「Given a sequence $S = x_1, x_2, ..., x_n$ of elements, and an integer k with $1 \leq k \leq n$, find the kth-smallest element in S」. This problem is called **order statistics** or **selection**. If k is very close to 1 or very close to n, then we can find the kth-smallest by running the algorithm for finding the minimum (maximum) element k times. This approach requires approximately $O(kn)$ comparisons. Sorting would be better than this naive algorithm, unless k is $O(\log n)$ or n-$O(\log n)$. Please design a more efficient algorithm (e.g. $O(n)$ comparisons) than the above two methods to solve the selection problem.

2. (10%) Given two positive integers n and k, design an algorithm with $O(\log k)$ **multiplications** to compute n^k.

3. (15%) Let $G = (V, E)$ be an undirected graph where V is the set of vertices and E is the set of edges of G. A **valid coloring** of G is an assignment of colors to the vertices such that each vertex is assigned one color and no two adjacent vertices have the same color. Please write an algorithm to find a valid coloring of G with **three** colors.

4. (5%) What are **NP-Complete** problems?
   (5%) How do you prove a problem to be NP-Complete?

5. **(Logic Design)**

5.1. (5%) Draw the circuit diagrams (show how to construct the circuits from basic logic gates) of a D latch and a D flip-flop.

5.2. (5%) Draw the timing diagrams to explain in words the difference between a D latch and a D flip-flop.

5.3. (5%) Simplify the following Boolean function in sum-of-products form by means of a three-variable map. Note that in describing the boolean function, the variable x is the most significant bit, while z being the least significant bit.

$$F(x, y, z) = \Sigma (3, 5, 6, 7)$$

5.4. (5%) Implement the logic equation in Problem 5.3 with ONLY 2-input NAND gates. You need to minimize the number of gates.

5.5. (5%) Implement the logic equation in Problem 5.3 with ONLY 2-input NOR gates. You need to minimize the number of gates.

6. **(Data Representation)**

6.1. (5%) Which one of the following formats for 32-bit floating point numbers is better? And please explain why?

A format

1bit	8 bits	23 bits
Sign	Exponent	Significand

B format

1bit	23 bits	8 bits
Sign	Significand	Exponent

6.2. (5%) Why IEEE 754 floating point representation is biased (or biased exponent)? How much is the bias for the 8-bit exponent and 11-bit exponent respectively?

7. **(Performance)**

(5%) What's Amdahl's Law? Give an example by showing how Amdahl's Law can be applied to improve the performance of a computing system.

8. **(Pipelining)**

8.1. (5%) Explain the meaning of "data hazards" in a pipelined processor.

8.2. (5%) "Forwarding" and "Stalling" are techniques to resolve data hazards in a pipelined processor. Please define and compare these two techniques.

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